Constraints from the $1/N_c$ Expansion on Properties of Exotic Tetraquark Mesons

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Scrubtining the scattering of ordinary mesons in the limiting case of the number of colour degrees of freedom $N_c$ of quantum chromodynamics approaching infinity, we formulate Feynman-diagram selection criteria and from these deduce rigorous self-consistency conditions for the manifestation of a tetraquark, a two-quark–two-antiquark bound state, as a pole in the corresponding amplitudes. Our constraints bear rather far-reaching consequences: In particular, all flavour-exotic tetraquarks, composed of four (anti)quarks of disparate flavour, must come in, at least, two variants differing in (and thus readily identifiable by) the large-$N_c$ behaviour of their couplings to two ordinary mesons. Quite generally, irrespective of their flavour composition, all tetraquarks prove to be narrow. Their decay rates behave, for large $N_c$, like $1/N_c^2$ and thus decrease faster than those of ordinary mesons.

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1. Criteria for the Potential Existence of Polyquark Hadron States in Large-$N_c$ QCD

Large-$N_c$ QCD [1] is an element of a class of quantum field theories obtained as generalizations of quantum chromodynamics by allowing $N_c$, the number of the colour degrees of freedom of QCD, to differ from $N_c = 3$. It is defined by considering the limit of $N_c$ increasing beyond bounds, $N_c \to \infty$, with correlative decrease of the strong coupling $\alpha_s \equiv g^2_s/4\pi$, $\alpha_s \propto 1/N_c$, and all quarks transforming according to the $N_c$-dimensional fundamental representation of its underlying gauge group $SU(N_c)$.

Polyquarks are exotic hadrons that cannot be interpreted as just quark–antiquark or three-quark bound states; among these, tetraquarks are considered to be essentially composed of two quarks and two antiquarks. Within the framework of large-$N_c$ QCD, only recently, after going through a kind of chequered history, the theoretical opinion about existence and observability of tetraquarks started to consolidate: At the leading order of their $1/N_c$ expansion, all QCD Green functions are saturated by free mesons as intermediate states [2]. Tetraquarks may be formed only at $N_c$-subleading orders [3]; unlike ordinary mesons, tetraquarks were expected not to survive the limit $N_c \to \infty$ as stable mesons. Mistakenly, the latter has been regarded as an argument for the tetraquarks’ non-existence in nature.

From the point of view of experimental evidence, however, the relevant feature of tetraquarks is the large-$N_c$ behaviour of their decay rates [4], which must not grow with $N_c$: observable tetraquarks should be narrow states. Tetraquarks with all quark flavours different exhibit decay rates decreasing like $1/N_c^2$ [5]; so they are even narrower than ordinary mesons, whose decay rates are of order $1/N_c$.

In that situation, we embarked on a systematic analysis [6,7] of the necessary conditions for the appearance of tetraquarks, $T$, with masses $m_T$ assumed to remain finite for $N_c \to \infty$, in form of poles corresponding to bound states of four (anti)quarks, with masses $m_1, m_2, m_3$, and $m_4$, in the $s$ channel of scattering processes of two ordinary mesons into two ordinary mesons. To this end, neglecting all reference to spin or parity irrelevant in the present context, we study four-point correlation functions of operators $j_{ij} \equiv \bar{q}_i q_j$, bilinear in the quarks and interpolating the scattered mesons, $M_{ij}$, consisting of antiquark $\bar{q}_i$ and quark $q_j$ (of flavour $i, j = 1, 2, 3, 4$), that is, having nonvanishing matrix elements between vacuum and meson state, $\langle 0|j_{ij}|M_{ij}\rangle \equiv f_{M_{ij}} \neq 0$, which rise for large $N_c$ like $f_{M_{ij}} \propto \sqrt{N_c}$ [2].

With respect to the contributions to the expansion of the corresponding scattering amplitudes in powers of $1/N_c$ and $\alpha_s$, our foremost task is to provide the characterization of the “tetraquark-phile” Feynman diagrams (with subscripts $T$ indicating their correlator contributions) that might develop a tetraquark pole at $s = m_T^2$ by two basic criteria, expressed in terms of meson momenta $p_1$ and $p_2$ [6]:

1. The Feynman diagram should depend in a nonpolynomial way on the variable $s \equiv (p_1 + p_2)^2$.

2. The Feynman diagram should admit adequate intermediate four-quark states with branch cuts starting at branch points $s = (m_1 + m_2 + m_3 + m_4)^2$. The potential presence or absence of such an intermediate-state threshold may be clearly decided by means of the Landau equations [8].

2. Rigorous Self-Consistency Conditions from Large-$N_c$ Ordinary-Meson Scattering

In order to carve out characteristics of tetraquarks for large $N_c$, we identify our tetraquark-phile Feynman diagrams and derive the tetraquark–two-ordinary-meson amplitudes $A(T \leftrightarrow M M)$ that fix the tetraquarks’ decay widths $\Gamma(T)$. Since we take into account all conceivable scattering processes, we expect to encounter “flavour-preserving” ones, where the flavour composition of initial and final ordinary mesons is identical, and “flavour-rearranging” ones, proceeding by a reshuffling of quarks.
2.1 Flavour-exotic tetraquarks: bound states of quarks involving four different flavour types

For genuinely flavour-exotic tetraquarks, the rightmost plots in Fig. 1 reveal that the $\mathcal{N}_c$-leading terms in tetraquark-phile flavour-preserving and flavour-reshuffling four-point correlation functions exhibit a different large-$\mathcal{N}_c$ behaviour. In this case, the resulting constraints cannot be fulfilled under the premise of the existence of a single tetraquark; rather, they require pole contributions of, at least, two tetraquark states, called $T_A$ and $T_B$, differing in their large-$\mathcal{N}_c$ couplings to two ordinary mesons. Expressed in terms of tetraquark–two-meson transition amplitudes $A$, generic decay constants $f_M$ of ordinary mesons, and propagator poles at $p^2 = m_{T_A,B}^2$, the pole contributions then read, symbolically,

\[
\langle j_1^i j_3^j j_2 j_4 \rangle_T = f_M^4 \left( \frac{|A(M_{12} M_{34} \leftrightarrow T_A)|^2}{p^2 - m_{T_A}^2} + \frac{|A(M_{12} M_{34} \leftrightarrow T_B)|^2}{p^2 - m_{T_B}^2} \right) + \cdots = O(\mathcal{N}_c^0),
\]

\[
\langle j_1^i j_3^j j_2 j_4 \rangle_T = f_M^4 \left( \frac{|A(M_{14} M_{32} \leftrightarrow T_A)|^2}{p^2 - m_{T_A}^2} + \frac{|A(M_{14} M_{32} \leftrightarrow T_B)|^2}{p^2 - m_{T_B}^2} \right) + \cdots = O(\mathcal{N}_c^0),
\]

\[
\langle j_1^i j_3^j j_2 j_4 \rangle_T = f_M^4 \left( \frac{A(M_{12} M_{34} \leftrightarrow T_A) A(T_A \leftrightarrow M_{14} M_{32})}{p^2 - m_{T_A}^2} \right. + \left. \frac{A(M_{12} M_{34} \leftrightarrow T_B) A(T_B \leftrightarrow M_{14} M_{32})}{p^2 - m_{T_B}^2} \right) + \cdots = O(\mathcal{N}_c^{-1}).
\]

The amplitudes of leading order in $\mathcal{N}_c$ then govern the total decay rates of $T_A$ and $T_B$; these, due to an inherent flavour symmetry of the above, prove to show a parametrically identical dependence on $\mathcal{N}_c$:

\[
A(T_A \leftrightarrow M_{12} M_{34}) = O(\mathcal{N}_c^{-1}), \quad A(T_A \leftrightarrow M_{14} M_{32}) = O(\mathcal{N}_c^{-2}) \quad \Longrightarrow \quad \Gamma(T_A) = O(\mathcal{N}_c^{-2}),
\]

\[
A(T_B \leftrightarrow M_{12} M_{34}) = O(\mathcal{N}_c^{-2}), \quad A(T_B \leftrightarrow M_{14} M_{32}) = O(\mathcal{N}_c^{-1}) \quad \Longrightarrow \quad \Gamma(T_B) = O(\mathcal{N}_c^{-2}).
\]

Of course, as bound states composed of the same set of constituents, $T_A$ and $T_B$ will undergo mixing.

![Diagram](image_url)

**Figure 1:** Flavour-preserving, $\langle j_1^i j_3^j j_2 j_4 \rangle$ (a), or -rearranging, $\langle j_1^i j_3^j j_2 j_4 \rangle$ (b), four-point correlators.
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Wolfgang Lucha

Figure 2: Flavour-preserving, \(\langle j_{12} I_2^i j_{23} I_3^i \rangle\) (a), or -rearranging, \(\langle j_{13} I_3^i j_{22} I_2^i \rangle\) (b), four-point correlators; the four purple crosses at the quark propagators indicate the constituents of the intermediate-state tetraquarks.

2.2 Flavour-cryptoexotic tetraquarks: four-quark states exhibiting merely two open flavours

For a quark–antiquark pair with same flavour different from the other two, the relations (Fig. 2)

\[
\langle j_{12} I_2^i j_{23} I_3^i \rangle_T = O(N_c^0), \quad \langle j_{13} I_3^i j_{22} I_2^i \rangle_T = O(N_c^0), \quad \langle j_{13} I_3^i j_{22} I_2^i \rangle_T = O(N_c^0)
\]

are satisfied by a single tetraquark with two-meson couplings implying again a narrow decay width:

\[
A(T \leftrightarrow M_{12} M_{23}) = O(N_c^{-1}), \quad A(T \leftrightarrow M_{13} M_{22}) = O(N_c^{-1}) \quad \implies \quad \Gamma(T) = O(N_c^{-2}).
\]

Due to the same total flavour, such tetraquark may mix with an ordinary meson \(M_{13}\). The pole terms

\[
\langle j_{12} I_2^i j_{23} I_3^i \rangle_T = f_M^2 \left( \frac{A(M_{12} M_{23} \rightarrow T)}{p^2 - m_T^2} g_{TM_{13}} \frac{A(M_{13} \rightarrow M_{12} M_{23})}{p^2 - m_{M_{13}}^2} \right) + \cdots = O(N_c^0)
\]

then restrict the large-\(N_c\) behaviour of their associated mixing strength \(g_{TM_{13}}\) to \(g_{TM_{13}} \leq O(1/\sqrt{N_c})\).

2.3 Resulting narrowness and number of tetraquarks: “always two there are, . . . no less” [9]

In summary, at large \(N_c\), we expect (1) any flavour-exotic tetraquarks to be formed pairwise and (2) all (compact) tetraquarks to be narrow, with decay widths decreasing not slower than \(1/N_c^2\) [6,7].

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References