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Formulating electroweak pion decays in functional methods

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In dense environments, like neutron star mergers, dynamical backcoupling between the strong and weak interactions becomes important. We therefore use non-perturbative functional methods to investigate the β-decay as the dominant dynamical process. Especially, we use both Dyson-Schwinger/Bethe-Salpeter equations and Functional-Renormalization-Group equations to control systematic errors. We present the necessary setup and outline relevant challenges.

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In the past few years binary neutron star mergers became a focus of investigation, as the expected gravitational waves should provide insight into the structure of neutron stars. That this is possible has been finally confirmed by the recent successful detection of this signal [1]. Various analysis show that in binary neutron star mergers the dynamical backcoupling of the electroweak interaction to the neutron matter is relevant (see e.g. [2] and references therein). To describe this, a coupled non-perturbative treatment of both sectors is necessary.

We employ for this purpose a combination of functional methods: Dyson-Schwinger/Bethe-Salpeter-equations (DSEs/BSEs) [3] and the Functional-Renormalization-Group (FRG) [4]. Since both types of approaches have different systematics, this allows to control systematic error sources.

A full treatment of the electroweak interactions is rather complex. Furthermore, the relevant energy scales in neutron stars are low compared to the Fermi-scale. We therefore approximate them by Fermi-theory, introducing an effective low-energy 4-Fermi-interaction between the quarks and leptons of the first generation, which violates C and P like the electroweak interactions. Together with QCD, the theory reads

$$\begin{aligned}\mathscr{L} =& \mathscr{L}_{\text{QCD}} + \mathscr{L}_{\text{4-Fermi}} + \mathscr{L}_{e,\text{free}} + \mathscr{L}_{\nu,\text{free}} \\ \mathscr{L}_{\text{4-Fermi}} =& g_w \left\{ \left[\overline{\psi}^L_\nu \gamma^\mu \psi^L_e\right] \left[\overline{\psi}^L_u \gamma^\mu \psi^L_d\right] + \left[\overline{\psi}^L_e \gamma^\mu \psi^L_\nu\right] \left[\overline{\psi}^L_d \gamma^\mu \psi^L_u\right] \right\}\end{aligned} \tag{1}$$

where $\psi^L = \frac{1}{2}(1-\gamma^5)\psi$ are the left-handed fermion fields and g_w the effective electroweak coupling. Because of the conservation of baryon and lepton number this does not change the propagation of individual quarks or leptons, and in particular does not lead to flavor oscillations. However, bound states, like hadrons, can now decay weakly. The simplest such hadron is the pion, which therefore will be our test subject of choice.

The pole position of the pion is described by its homogeneous BSE, i. e. the corresponding four-point function evaluated at the pole. This takes the form of an eigenvalue equation for the Bethe-Salpeter-Amplitude Ψ. The relevant BSEs in our approximation, essentially a rainbow-ladder truncation in pure QCD, are shown in figure 1 in diagrammatic form. The first equation is

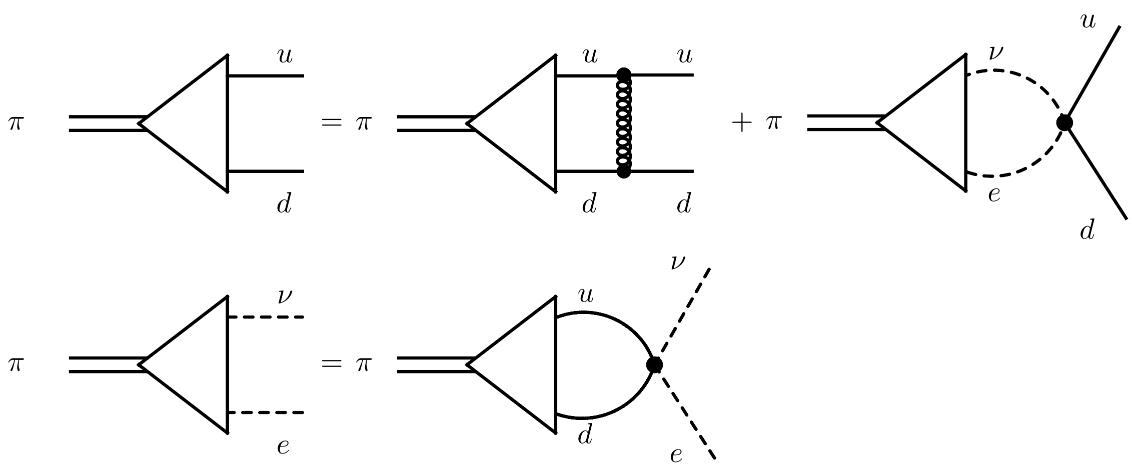

Figure 1: Diagrammatic representation of the BSEs. The solid and dashed lines represent the quarks and the leptons respectively. The double solid line represents the pion and the triangle stands for the Bethe-Salpeter-Amplitude. The wiggly line represents a gluon. The propagator equations are not shown.

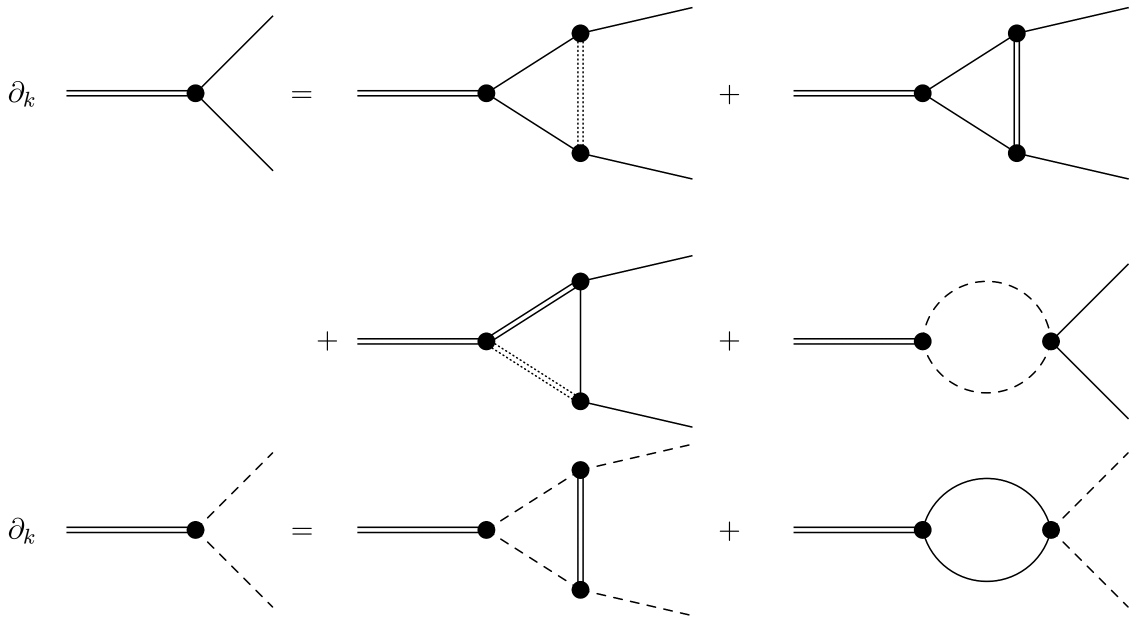

Figure 2: Diagrammatic representation of the flows. The solid and dashed lines represent the quarks and the leptons respectively. The double solid and dashed lines represent the pion and the sigma respectively. In each internal loop one of the propagator has a regulator insertion, which is suppressed for brevity. The derivative is with respect to the regulator scale k. The propagator equations and the equation for the σ are not shown.

the usual description in pure QCD, but has now an additional term describing that the pion has a leptonic intermediate state. The second equation provides the actual decay of the pion into leptons. Note that there is no non-trivial equation for a pole in the four-lepton channel in our approximation.

This system has to be solved self-consistently. In pure QCD the pion is stable, and the pole is on the real axis. The additional decay amplitude pushes this pole into the 2^{nd} Riemann sheet. This complexifies the BSEs, leading to search for poles in a plane in contrast to an axes. However, aside from the technical complexity, this is not qualitatively different from the usual BSEs, and under investigation.

In the FRG framework the bound states are most straightforwardly described using dynamical hadronization [5]. Thereby, we avoid a proliferation of interaction operators, as interactions are canceled at every step of the renormalization flow. However, in contrast to the BSE approach, this requires to also include the σ-meson in addition to the pions to realize all relevant symmetries appropriately. At the same time, we can avoid the complexity of the gauge interaction by integrating the gluons out, to arrive at an effective quark-meson model with dynamically generated bound states [6], supplemented by the same Fermi term as in (1).

The resulting FRG equations are shown in figure 2. Again, we have a QCD-like equation with an additional intermediate state and an additional equation for the decay itself. These equations are now for the full three-point functions $\Gamma^{(3)}$. These 3-point-functions agree with the Bethe-Salpeter-

Amplitudes at the poles

$$\Psi = \Gamma^{(3)}\Big|_{\text{Pole}}, \tag{2}$$

and therefore connect the mesons with their decay products, encoding even more information than in the BSE case. This now also includes the decay of the σ into two pions [6]. However, while the pole position in the BSEs arises directly from the self-consistency condition, in the FRG it is necessary to extract the pole position from the resulting pion propagator [7]. Particular care has to be taken in this process to avoid spurious divergences due to the regulator, but this is possible.

This outlines the necessary formalism to determine the weak decay of the pion self-consistently in Fermi theory. This is a major step towards a dynamically fully-backcoupled description of neutron star matter at the level of all invoked elementary degrees of freedom. The use of both DSEs/BSEs and FRG ensures best systematic control. At the moment, these equations are solved, and the results will be reported elsewhere.

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