PROCEEDINGS OF SCIENCE

Shannon entropy and hadronic decays

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How much information is added to the Review of Particle Physics when a new decay branching ratio of a hadron is measured and reported? This is quantifiable by Shannon's information entropy. It may be used at two levels, against the distribution of decay-channel probabilities, or against the distribution of individual quantum-state probabilities (integrating the phase space of those states provides the former). We illustrate the concept with some examples.

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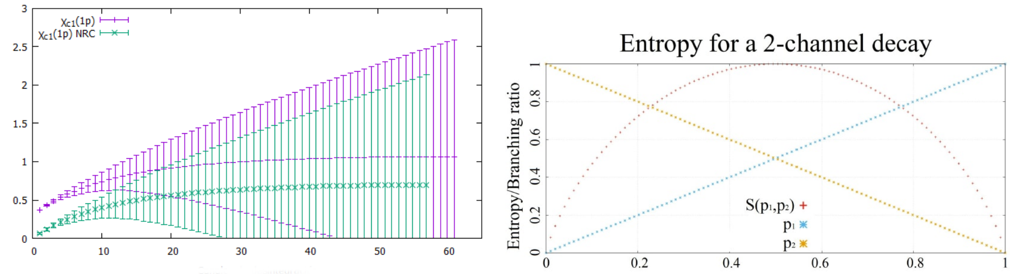

Figure 1: *Left:* Shannon entropy against the number of channels for the decay distribution of $\chi_{c1}(1p)$. For comparison, the entropy has also been calculated without radiative channels, denoted as NRC (No Radiative Channels). *Right:* entropy in bits (with the log in base 2) of a two-channel decay $-p_1 \log_2 p_1 - p_2 \log_2 p_2$ (curve, crosses) reaching a maximum when both channels are equally likely.

A measure of the surprise when, in the transmission of a message, a new letter is received, is provided by Shannon's entropy [1] of the message's alphabet distribution. If each letter has probability p_i, then that entropy is $S = \sum -p_i \ln p_i$. The same concept can be applied to the distribution of decays of an unstable particle (or nucleus or any other compound). The decay distribution as described by branching ratios BR_i satisfying $\sum \mathrm{BR}_i = 1 = \sum \Gamma_i/\Gamma$ (as defined by the total Γ and partial Γ_i widths) provides a natural definition of classical entropy

$$S(\text{decay}) = -\sum_i \mathrm{BR}_i \ln \mathrm{BR}_i \tag{1}$$

that is represented in figure 1 for the $\chi_{c1}(1p)$ meson decay distribution, with data taken from [2] and the channels ordered with decreasing width towards the righ (also plotted is S for a toy distribution of two channels).

We learn that entropy is maximum (in fact, equal to $\ln N$, with N the number of channels) when all decays are equally likely. For the typical decay shown, the entropy saturates to a value smaller than that maximum once most of the biggest decay channels have been accounted for. As the number of discovered channels increases towards the right of the plot, we see that the added information entropy is decreasingly smaller. It may be that the discovery of a new channel is important because it questions a symmetry or opens a new alley of investigation, but entropy is a fair predictor of how relevant it is for the knowledge of the decaying particle at a coarse level.

A technical aspect is that when part of the total width is unaccounted for, one does not know how many channels yet to be discovered share into that width. For the time being, we have opted for bunching them in only one "unknown" channel such that, by definition, $\mathrm{BR}_{\text{unknown}} = 1 - \sum_i \mathrm{BR}_i$ carries all missing probability. When a new channel is actually discovered, its probability is discounted from this unknown one. The procedure is illustrated in figure 2.

A further interesting property of Shannon's entropy is its additivity. If a part of the decay "alphabet" of N channels is divided into subsets N_1, N_2 (for example: a former decay to two charged hadrons A^+A is found to divide into $\pi^+\pi^-$ and K^+K^-), then $S(N) = S(N_1) + S(N_2)$ where $S(N_i)$ is the remaining entropy once it is known that the decay falls into the i group. Decay channels are not infinitely indivisible as the smallest unit is precisely one quantum state of specific particles,

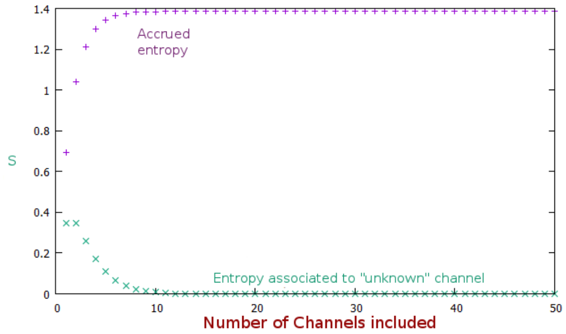

Figure 2: Top function (straight crosses): entropy of a decay distribution where all the unknown branching fraction is grouped into one "unknown" channel. Bottom function (diagonal crosses): the entropy associated to that unknown channel. The fake distribution is in the geometric progression $\Gamma_i/\Gamma \propto 2^{-i}$.

spin, momenta, etc. We can count the number of such quantum states with phase space, as briefly recounted in [3]. Figure 3 exemplifies the variation of entropy with the two-body phase space of the decay distribution for the a_2 meson.

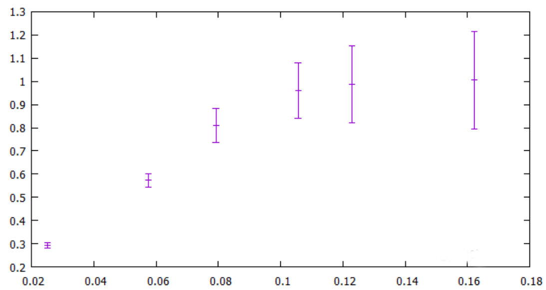

Figure 3: Entropy against the phase space $\sum_i \rho_i$ accrued upon including further two–body decay channels of the $a_2(1320)$ meson, taken from larger to smaller branching fraction.

We wish to propose simple criteria to quantify what information the discovery a new branching fraction provides. A simple one is to represent the actual importance of a new channel by the separation of the entropy from its maximum value $\ln N$ by the normalized entropy increment, defined as

$$\frac{\Delta S(N)}{\Delta log(N)} = \frac{S(N+1)-S(N)}{log(N+1)-log(N)} \tag{2}$$

that is plotted in figure 4.

On reasoning that the maximum information extractable from a random process is the difference between the maximum entropy and the measured value, $I_{\max} = S_{\max} - S = \ln N - S$ one can also, normalizing by $\ln N$ and subtracting before and after discovering a new decay channel, define a “degree of likeness” $\Theta = \frac{S(N+1)}{log(N+1)} - \frac{S(N)}{log(N)}$ that can play a similar role. We defer to our upcoming longer work for further discussion [5].

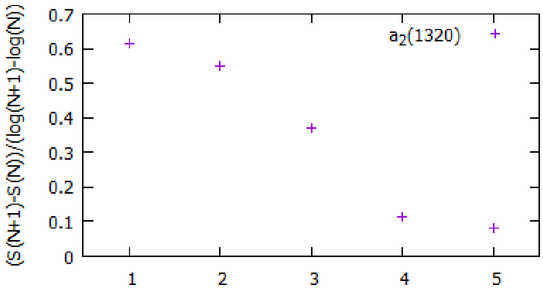

Figure 4: Increase of the normalized entropy for the decay distribution of the $a_2(1320)$ meson as function of the number of channels. Additional ones can be seen to contribute decreasingly less, so this quantity could be taken as a measure of the amount of information contained in each newly reported channel.

In summary, we have extended the known uses of Shannon's entropy as applied to axion physics [6], to the postdiction of the Higgs mass [7, 8] from a maximum entropy principle, or to quantifying the information in parton-splittings in jets [9] in particle physics. Our contribution has centered in deploying the entropy, and some derived functions, as measuring the amount of information gained when a new particle decay is discovered. This is motivated by the already vast information contained in the available PDG tables.

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