**SO(N)** models and Higgs extensions

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We discuss previous studies on **SO(N)** linear sigma models (LσM) and some limits of phenomenological interest. These models suffer a spontaneous symmetry breaking (SSB) down to \(SO(N-1)\), with the appearance of an associated vacuum expectation value (vev) \(f\), a heavy scalar degree of freedom (dof) with mass \(M\) and \(N-1\) massless Nambu-Goldstone bosons (NGB). These models are of a high interest for beyond Standard Model extensions where the Higgs boson is identified with a pseudo Nambu-Goldstone boson (pNGB) that appears in the \(SO(N)/SO(N-1)\) SSB. It gains a non-zero mass \(m\) due to soft explicit \(SO(N)\) symmetry breaking (ExSB) terms in the Lagrangian. In particular, we will focus on the soft breaking pattern \(SO(N)\) \(\xrightarrow{\text{ExSB}} SO(4) \times SO(P) \xrightarrow{\text{SSB}} SO(3) \times SO(P-1)\), with \(4 + P = N\), e.g., via new beyond Standard Model (BSM) gauge boson loops. The \(SO(4)/SO(3)\) are the electroweak (EW) chiral/custodial groups and the associated SSB is exactly the Standard Model (SM) one, giving mass to the \(W^\pm\) and \(Z\) gauge bosons while avoiding large corrections to the oblique \(T\) parameter. The comparison of this type of models with the current phenomenological situation, close to the SM \((m = 0.125\ \text{TeV}, \text{EW vev} v = 0.246\ \text{TeV}, M \gtrsim 0.1\ \text{TeV}, g_{\text{HWW}} \approx g_{\text{SMHWW}}^\text{SM})\) sets important constraints on the LσM parameters: there is a very small mixing between the heavy and light LσM massive scalars and the pNGB \(h\) is essentially SM-like, the low-energy effective field theory (EFT) couplings are very close to the SM ones, and a large hierarchy \(\xi = \frac{v^2}{f^2} \ll 1\) is needed in these LσM near the \(SO(N)\) limit (and \(\xi\) much smaller than a certain ratio \(\frac{\lambda_2}{\lambda_1}\) of quartic LσM couplings in the general case). Likewise, we note the existence of strongly coupled scenarios with a hierarchy \(m^2 \sim v^2 \ll f^2 \ll M^2\).

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1. BSM extensions of the SM scalar sector through an $SO(N)\,\text{L}\sigma\text{M}$

In this proceedings we review and discuss some results on theories with the SSB \cite{1, 2} \footnote{In the case $P = 2$ one has a discrete parity $Z_2$ in the place of $SO(P - 1)$. It is worthy to note that $SO(6)/SO(5) \sim SU(4)/Sp(4)$ provides the minimal coset of this type with an ultraviolet (UV) completion of fermions in a complex representation of the gauge group, and represents the minimal $SO(N)$ realization of an UV-complete pNGB composite Higgs model \cite{3}. The often denoted as minimal coset $SO(5)/SO(4)$ lacks a four dimensional UV completion.}

\begin{equation}
\text{w/o ExSB: } \quad SO(N) \xrightarrow{\text{SSB, vev } f} SO(N - 1),
\end{equation}

\begin{equation}
\text{w/ ExSB: } \quad SO(N) \xrightarrow{\text{ExSB}} SO(4) \times SO(P) \xrightarrow{\text{SSB, vev } (v, \nu)} SO(3) \times SO(P - 1),
\end{equation}

The $SO(N)$ ExSB turns one of $N \! - \! 1$ NGB from the symmetric case into a pNGB with mass proportional to the explicit breaking Lagrangian parameters; the other $N \! - \! 2$ remain as NGB, being the three of them associated to the $SO(4)/SO(3)$ SSB the standard EW Goldstones. \footnote{The experimental absence of massless scalars implies that all the remaining $P \! - \! 1 = N \! - \! 5$ NGB gain mass through some mechanism not discussed in these proceedings, such as Higgsing or some BSM non-perturbative dynamics.}

In order to study the implementation of this symmetry pattern in generic BSM scenarios, it is interesting to discuss its realization through an $SO(N)$ toy-model renormalizable L$\sigma$M with a real scalar multiplet $\Sigma^T = (\phi_1, \phi_2, \phi_3, \phi_4, S_1, S_2, ... S_P)$ in the fundamental representation \cite{2}:

\begin{equation}
\mathcal{L} = \frac{1}{2} (D_\mu \Sigma)^2 - \left[ V_0(\Sigma) + V_1(\Sigma, \bar{\xi}) \right],
\end{equation}

\begin{equation}
V_0(\Sigma) = -\frac{\mu_1^2}{4} \Sigma^2 + \frac{\lambda_1}{16} \Sigma^4, \quad V_1(\Sigma, \bar{\xi}) = \Sigma^4 \left( \frac{\mu_1^2 - \mu_2^2}{4 \Sigma^2} \bar{\xi} + \frac{\lambda_1 + \lambda_2 - 2\lambda_3}{16} \bar{\xi}^2 + \frac{\lambda_3 - \lambda_1}{8} \bar{\xi} \right),
\end{equation}

with $\Sigma \equiv |\bar{\Sigma}|$, $\phi \equiv |\phi|$, $S \equiv |\bar{S}|$ and the field $\bar{\xi} = 1 - \xi = S^2/\Sigma^2 \in [0, 1]$. $V_0 + V_1$ develops the vev $\langle \phi^2 \rangle = v^2 = \frac{2(\lambda_1 \mu_1^2 - \lambda_2 \mu_2^2)}{\lambda_2 \lambda_3}$ and $\langle S^2 \rangle = v_0^2 = \frac{2(\lambda_1 \mu_2^2 - \lambda_2 \mu_1^2)}{\lambda_2 \lambda_3}$, with $\lambda_1 \equiv \lambda(1 + \delta_j)$, $\mu_1^2 \equiv \mu^2(1 + \Delta_1)$; an arbitrarily small deviation from $SO(N)$ may lead to scenarios with either $v^2 \ll v_0^2$, $v^2 \sim v_0^2$ or $v^2 \gg v_0^2$.

One may induce an ExSB in an $SO(N)$ invariant L$\sigma$M through the gauging of just a few components of $\Sigma$, e.g., the gauging of the $S$ components under a BSM group: $SO(P)$ gauge bosons $A_{\mu}^s$ with coupling $e_s$ explicitly break the $SO(N)$ symmetry and induce a one loop contribution to $V_1 \text{ à la Coleman-Weinberg (CW) proportional to powers of} e_s^2$ while gaining a mass $M_{A^s} = e_s v_1$ \footnote{These results corresponds to the Landau gauge and $R$ is the renormalization scale in an appropriate scheme. There are further corrections $\sim \frac{e_s^2}{(4\pi)^2} S^4$ and $\sim \frac{e_s^4}{(4\pi)^2} S^6$ if one considers a different gauge. SM loops introduce further $SO(N)$ ExSB terms, as the SM only couples to the $\phi$ components.}. For instance, the $A_{\mu}^s$ loops induce for $P = 2$ an effective potential of the form $V_1^{A^s-\text{loop}} = \frac{3}{64\pi^2} S^4 \ln \frac{S^2}{R^2}$ \footnote{The experimental absence of massless scalars implies that all the remaining $P \! - \! 1 = N \! - \! 5$ NGB gain mass through some mechanism not discussed in these proceedings, such as Higgsing or some BSM non-perturbative dynamics.}: the $V_1$ potential is no longer flat for the field $\xi$ and one Goldstone $h$ becomes a pNGB with mass proportional to powers of the ExSB parameter $e_s$.}

In the case $P = 2$ one has a discrete parity $Z_2$ in the place of $SO(P - 1)$. It is worthy to note that $SO(6)/SO(5) \sim SU(4)/Sp(4)$ provides the minimal coset of this type with an ultraviolet (UV) completion of fermions in a complex representation of the gauge group, and represents the minimal $SO(N)$ realization of an UV-complete pNGB composite Higgs model \cite{3}. The often denoted as minimal coset $SO(5)/SO(4)$ lacks a four dimensional UV completion.
In the general broken case, the $\phi$–$S$ mixing parameter $\omega \in [0, 1]$ \cite{2} ($h \approx \phi$ for $\omega \approx 0$ and $h \approx S$ for $\omega \approx 1$) and the masses are related to the model parameters in the form
\[ M^2, m^2 = \frac{\bar{M}^2}{2} \left( 1 \pm \sqrt{1 - \frac{4\bar{m}^2}{\bar{M}^2}} \right), \quad \lambda_2 v^2, \lambda_1 v^2 = \frac{\bar{M}^2}{2} \left( 1 \pm |1 - 2\omega| \sqrt{1 - \frac{4\bar{m}^2}{\bar{M}^2}} \right), \quad (1.4) \]
\[ |1 - 2\omega| = \left( 1 - \frac{4\bar{m}^2}{\bar{M}^2} \right)^{1/2} \left( 1 - \frac{4\lambda_1 \bar{m}^2}{\lambda_1^{\text{eff}} \bar{M}^2} \right)^{1/4}, \quad \bar{M}^2 = \frac{1}{2} (\lambda_1 v^2 + \lambda_2 v^2), \quad \bar{m}^2 = \frac{\lambda_1^{\text{eff}} \lambda_2 v^2 v_s^2}{4M^2}. \]

2. Low energy limit and Effective Field Theory

In the limit of a large mass gap $m^2 \ll M^2$ —which we will assume from now on—, one has $m^2 \approx \bar{m}^2, M^2 \approx \bar{M}^2$ and $\lambda_1 v^2 + \lambda_2 v^2 \approx 2M^2$, up to corrections $\mathcal{O}(m^2/M^2)$. This alone does not imply a hierarchy between $v^2$ and $v_s^2$. However, in general, the $hW \bar{W}$ coupling is related to the mixing in the exact form $\omega = 1 - \left( \frac{\bar{m}_{\mu \mu}}{\bar{m}_{\mu \mu}} \right)^2$ \cite{2}, leading to the relations
\[ \text{SM} \cong \text{EFT} \quad \gamma \ll 1 \iff \text{Mixing} \quad \omega \ll 1 \iff \frac{4\lambda_1 m^2}{\lambda_1^{\text{eff}} M^2} \ll 1 \iff \lambda_1 v^2 \approx \frac{2\lambda_1 m^2}{\lambda_1^{\text{eff}}} \ll \lambda_2 v_s^2 \approx 2M^2, \quad (2.1) \]
with the positive parameter $\gamma = \frac{\lambda_1 v^2}{2\lambda_1 \mu_1^2}$ \cite{2}, and up to $\mathcal{O}(m^2/M^2)$ and $\mathcal{O}(\omega)$ corrections. Thus, there is a large $v_s^2 \ll v_s^2 \approx f^2$ hierarchy when $\lambda_1 \approx \lambda_2$. In the limit (2.1), the low-energy EFT is organized in powers of $\gamma \approx \frac{(\lambda_1 - \lambda_1^{\text{eff}}) m^2}{\lambda_1^{\text{eff}} M^2} \ll 1$, such that, up to $\mathcal{O}(\gamma)$, one finds \cite{2}, e.g.,
\[ V(h)_{\text{EFT}} = \frac{m^2 h^2}{2} + \left( 1 - \frac{3\gamma}{2} \right) \frac{m^2 h^3}{2v} + \left( 1 - \frac{25\gamma}{3} \right) \frac{m^2 h^4}{8v^2} - \frac{\gamma}{3} \frac{m^2 h^5}{2v^3} - \frac{\gamma}{12} \frac{m^2 h^6}{v^4}, \quad (2.2) \]
\[ \mathcal{F}_C(h)_{\text{EFT}} = 1 + \left( 1 - \frac{\gamma}{2} \right) \frac{2h}{v} + \left( 1 - 2\gamma \right) \frac{2h}{v} + \left( 1 - 2\gamma \right) \frac{2h}{v} + \left( 1 - 2\gamma \right) \frac{2h}{v} + \left( 1 - 2\gamma \right) \frac{2h}{v}, \quad (2.3) \]
with the low-energy potential $V(h)_{\text{EFT}}$ and $\Delta \mathcal{L} = \left( \frac{g^2}{4} W^\mu W_\mu + \frac{(g^2 + g_s^2) v_s^2}{4} Z^\mu Z_\mu \right) \times \mathcal{F}_C(h)_{\text{EFT}}$ the Lagrangian providing the interaction vertices $W^+W^-, ZZ \to h, hh \ldots$ (the SM corresponds to the value $\gamma = 0$). Experimentally $\lambda_1^{\text{eff}} \approx \frac{2m^2}{v_s^2} \approx 0.5$ and $0 \leq \gamma \approx \omega = 1 - \left( \frac{\bar{m}_{\mu \mu}}{\bar{m}_{\mu \mu}} \right)^2 \leq 0.2$ for an $hW \bar{W}$ coupling in the range $0.9 \leq \frac{\bar{m}_{\mu \mu}}{\bar{m}_{\mu \mu}} \leq 1$.

In terms of $v_s^2, v_s^2 \neq 0$ and the $\lambda_{1,2,3}$, one approaches the $SO(N)$ invariant limit when $|\delta_j| \ll 1$. Thus, $\lambda \sim \lambda_1^{\text{eff}} / \delta_j$ can become non-perturbative near the $SO(N)$ symmetric limit, for small enough $\delta_j$: e.g., for $|\delta_j| \ll \frac{1}{10 M}$ one has $\lambda \gtrsim 8\pi^2$. We have performed a numerical analysis for the benchmark points (BP) of the form $\delta_2 = \lambda, \delta_1 = \delta_3 = -\delta$ with $0 \leq \delta \leq 1/2$ and such that $\lambda^{\text{eff}} \approx 0.5$. In order to have a solution for $\delta$ one needs $\lambda \geq 4\lambda_1^{\text{eff}} = 2$. In Fig. 1, we have plotted $\lambda \lambda_s^2/24\pi^2$ vs. $M$ and $\lambda \lambda_s^2/24\pi^2$ vs. $\xi$ for arbitrary values of $v, v_s$. We fix $\delta = 0.64 \times 10^{-2}$ (soft ExSB), 0.15 (moderate ExSB), 0.04 (large ExSB) for the benchmark points $A, B$ and $C$, respectively, which correspond to $\lambda = 8\pi^2$, 4, 2. It is illustrative to note that, in the strongly coupled case $\lambda = 8\pi^2$, one has $M \approx 3.6$ TeV ($M \approx 6.5$ TeV) for $\xi = 1/4$ ($\xi = 1/16$). The results are exact and no expansion is performed here.
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Figure 1: BP A (thick black), B (light green) and C (red), from bottom to top. For the plot for \( \xi \), the lines for the BP A and B— and all the BP in between— are essentially superimposed and very approximately coincide with the straight line \( \xi = \frac{\lambda_1 v^2}{2 M^2} \). This linear relation is approximately fulfilled for any \( \delta \) in this type of BP. We note that \( m_2^2 \rightarrow 0 \) for either \( \frac{\lambda_1 v^2}{2 M^2} \rightarrow 0 \) or 1, so the Higgs mass is linked to the EW SSB.

In conclusion, the symmetry pattern \( SO(N) \xrightarrow{\text{ExSB}} SO(4) \times SO(P) \xrightarrow{\text{SSB}} SO(3) \times SO(P - 1) \) naturally recovers the SM at low energies provided the ExSB potential \( V_1 \) generates a vev \( \langle \zeta \rangle = \xi \ll 1 \) (obviously, far from trivial). We would like to point out in these proceedings the existence of strongly interacting scenarios with a large coupling \( \lambda \) and a scale hierarchy of the type \( m_2^2 \sim v^2 \ll f^2 \ll M^2 \approx \frac{\lambda_2 f^2}{\lambda_1} \) near the \( SO(N) \) limit, and \( \xi \ll \frac{\lambda_2}{\lambda_1} \) in general. Other works consider variants of this symmetry pattern with \( N = 6 \): \( SO(6) \xrightarrow{\text{SSB}} SO(4) \times SO(2) \), which gives places to 8 NGB [5]; a non-linear realization of \( SO(6) \xrightarrow{\text{SSB}} SO(5) \) where one of the 5 NGB is proposed as as a dark matter candidate [1]; lattice simulations of the \( SU(4)/Sp(4) (\sim SO(6)/SO(5)) \) spectrum properties [3]; a non-linear realization of the latter [6], where a large deviation from the SM is found for \( g_{hhb} \); variations of the ExSB \( V_1 \) based on fermion-loop estimates of the CW potential [7]. All of them point out \( SO(N) \) models as appropriate BSM extensions which naturally generate a light pNGB \( h \) and reproduce the SM phenomenology and its \( SO(4)/SO(3) \) chiral/custodial EW structure at low energies, deserving further studies in the future.

References