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The turnaround radius as a probe of dark energy and modified gravity

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The turnaround radius in cosmology is investigated as a possible probe of dark energy and modified gravity scenarios.

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1. Introduction

We propose a novel approach to the problem of the turnaround radius of a large structure in an accelerating universe. This new approach is based on the Hawking quasilocal energy of General Relativity (GR) and is part of a broader program aiming at applying this formal concept to cosmology. Thus far, the quasilocal energy has been applied to Newtonian N-body simulations of large scale structure formation [1], and to the decade-old problem of lensing by the cosmological constant [2].

2. Turnaround radius with Hawking mass in GR

Consider the present accelerated era of the universe and the largest bound objects in the sky. It was suggested recently that the turnaround radius is a possible probe of dark energy or modified gravity scenarios [3, 4, 5, 6] but the concept of turnaround radius is older [7].

In an accelerated FLRW universe with one spherical inhomogeneity, massive test particles with zero radial initial velocity cannot collapse if they lie outside a critical radius R_c (*turnaround radius*),[1] but they only expand. Dust shells with $R < R_c$ collapse but, if you are in geodesic motion outside R_c, you will never fall back.

Over the years, the turnaround radius has been studied in exact solutions of the Einstein equations, including the Schwarzschild-de Sitter, Lemaître-Tolman-Bondi (LTB), and McVittie spacetimes. In the heuristic Schwarzschild-de Sitter space with line element

$$ds^2 = -\left(1 - \frac{2M}{R} - H^2R^2\right)dt^2 + \frac{dR^2}{1 - \frac{2M}{R} - H^2R^2} + R^2 d\Omega^2_{(2)} \tag{2.1}$$

where $H = \sqrt{\Lambda/3}$, the turnaround radius

$$R_c = \left(\frac{3GM}{\Lambda}\right)^{1/3} \tag{2.2}$$

is obtained by setting to zero the acceleration along radial timelike geodesics. In LTB models with dust and a cosmological constant Λ

$$ds^2 = -dt^2 + \frac{R'(t,r)}{1+f(r)}dr^2 + R^2(t,r)d\Omega^2_{(2)} \tag{2.3}$$

(where $' \equiv d/dr$ and $f(r)$ is related to the initial density profile), radial timelike geodesics obey

$$\ddot{R} = -\frac{G\mathscr{M}(r)}{R^2} + \frac{\Lambda R}{3} \tag{2.4}$$

and the authors of [5] obtain the turnaround radius

$$R_c = \left(\frac{3G\mathscr{M}(r_c)}{\Lambda}\right)^{1/3}, \tag{2.5}$$

[1] R denotes the areal radius, a geometric quantity which is covariant and gauge-invariant.

where

$$\mathscr{M}(r) \equiv \int_0^R dR\, R^2 \rho \tag{2.6}$$

is the Lemaître mass.

While exact solutions of the Einstein equations are toy models, a realistic description of the universe uses the perturbed Friedmann-Lemaître-Robertson-Walker (FLRW) line element

$$ds^2 = a^2(\eta)\left[-(1+2\phi)\,d\eta^2 + (1-2\phi)\left(dr^2 + r^2 d\Omega_{(2)}^2\right)\right]. \tag{2.7}$$

The timelike radial geodesics obey [5]

$$\ddot{R} = \frac{\ddot{a}}{a} - \frac{G\mathscr{M}(r)}{R^2}, \tag{2.8}$$

and it is suggested implicitly in [5] that $\mathscr{M}(r)$ is the Lemaître mass (2.6), from which one obtains the turnaround radius [5]

$$R_c = \left(\frac{3\mathscr{M}}{4(3w+1)\pi\rho_{DE}}\right)^{1/3}. \tag{2.9}$$

Two questions arise: the issue of the gauge-invariance of this result and, more important, it is not clear what the "mass contained in a sphere of radius R" is. Should it include the dark energy density ρ_{DE}? If not, why? Should it include only $\rho_{perturbation}$? If so, why? The Hawking-Hayward quasilocal energy M_{HH} of GR, which includes all energy forms, is suitable for answering both questions. In spherical symmetry it reduces to the Misner-Sharp-Hernandez mass [8]. We assume that GR is valid, we restrict to first order in the metric perturbation which (as customary in the literature on the turnaround radius) is assumed to be spherically symmetric ($\phi = \phi(r)$), and we assume that the background universe is a spatially flat FLRW one accelerated by dark energy with density ρ_{DE} and pressure $P_{DE} = w\rho_{DE}$.

The physical Hawking-Hayward quasilocal energy [9], based on the idea that the total mass in a region bounded by a 2-surface S is measured by its effect on null geodesics at S, is defined as follows [9]. Let S be a closed spacelike orientable 2-surface, $\mathscr{R}$ the induced Ricci scalar on S, $\theta_{(\pm)}$ the expansions of outgoing/ingoing null geodesic congruences from S, $\sigma_{ab}^{(\pm)}$ their shear tensors, ω^a the anholonomicity (*i.e.*, the projection on S of the commutator of the null normal vectors to S), μ the volume 2-form on S, and A the area of S. Then,

$$M_{HH} \equiv \frac{1}{8\pi}\sqrt{\frac{A}{16\pi}}\int_S \mu\left(\mathscr{R} + \theta_{(+)}\theta_{(-)} - \frac{1}{2}\sigma_{ab}^{(+)}\sigma_{(-)}^{ab} - 2\omega_a\omega^a\right). \tag{2.10}$$

By computing this quantity for the line element (2.7) we find that, to first order, it decomposes as [10]

$$\tilde{M}_{HH} = \underbrace{\Omega M_{HH} - \frac{R\Omega_{,\eta}}{4\pi}\frac{\Omega_{,\eta}}{\Omega}\int_S \mu\,\phi_N +}_{\text{local}} \underbrace{\frac{R^3}{2}\frac{\Omega_{,\eta}^2}{\Omega}}_{\text{cosmological}}. \tag{2.11}$$

In spherical symmetry this decomposition reduces to [1, 11, 2]

$$M_{HH} = ma + \frac{H^2R^3}{2}(1-\phi) \simeq ma + \frac{H^2R^3}{2}, \tag{2.12}$$

where

$$m \equiv \frac{1}{4\pi} \int d^3\vec{x} \nabla^2 \phi \tag{2.13}$$

is the Newtonian mass of the perturbation. Our new criterion identifies the turnaround radius by equating the two contributions to the quasilocal mass, which yields

$$R_c(t) = \left(\frac{2ma}{H^2} \right)^{1/3} . \tag{2.14}$$

The Friedmann equation $H^2 = 8\pi G \rho_{DE}/3$ then gives $R_c(t) = \left(\frac{3ma}{4\pi\rho_{DE}} \right)^{1/3}$. If, in addition, the equation of state parameter w of the dark energy is constant, one obtains $R_c = \left(\frac{3ma}{4\pi\rho_0} \right)^{1/3} a^{(3w+4)/3}$. For comparison, the ratio of the turnaround radius (2.14) to that of [5] is

$$\frac{R_c}{R_c^{(PTT)}} = \left(\frac{|3w+1|}{2} \right)^{1/3} \approx 1 \qquad \text{if } w \approx -1 \, . \tag{2.15}$$

Although the difference is numerically small, in our approach there is no ambiguity in the concept of "mass inside a sphere of radius R_c" and the description is gauge-invariant. We have, therefore, a rigorous derivation of turnaround radius.

The turnaround radius may provide a way to constrain the equation of state parameter w of dark energy. One can express it as a function of redshift $R_c(z)$ and obtain [5]

$$\int dz \frac{w(z)+1}{z+1} = \ln \left[\left(\frac{3ma}{4\pi\rho} \right)^{1/3} \frac{1}{R(z)} \right] . \tag{2.16}$$

If $w =$ const., this expression reduces to

$$w = -1 + \frac{\ln \left[\left(\frac{3ma}{4\pi\rho_0} \right)^{1/3} \frac{1}{R_c(z)} \right]}{\ln(z+1)} \tag{2.17}$$

which, in principle, allows one to constrain w if ma and R_c are known.

3. Turnaround radius in scalar-tensor gravity

Consider now scalar-tensor gravity. Because there is no universally accepted notion of quasilocal energy in this class of theories (see, however, the proposals [12]), we proceed with the usual (gauge-dependent) method of setting to zero the acceleration along radial timelike geodesics. The perturbed FLRW spacetime is now described by the line element

$$ds^2 = a^2(\eta) \left[-(1+2\psi)\, d\eta^2 + (1-2\phi) \left(dr^2 + r^2 d\Omega^2_{(2)} \right) \right] \tag{3.1}$$

with two perturbation potentials $\phi = \phi(r), \psi = \psi(r)$. The radial timelike geodesics are described by

$$\frac{du^0}{d\tau} + \frac{a_\eta}{a} (u^0)^2 + 2\psi' u^0 u^1 + \frac{a_\eta}{a} (1-2\phi-2\psi) (u^1)^2 = 0 \tag{3.2}$$

(3.3)

$$\frac{du^1}{d\tau} + \psi' (u^0)^2 + \frac{2a_\eta}{a} u^0 u^1 - \phi' (u^1)^2 = 0 \, , \tag{3.4}$$

where $u_c u^c = -1$. The areal radius is $R(t,r) = ar\sqrt{1-2\phi} \simeq ar(1-\phi)$. Manipulations yield [13]

$$\frac{d^2R}{dt^2} = \left[\ddot{a}r + \frac{\dot{a}u^1}{au^0} + \frac{1}{au^0}\frac{d}{d\tau}\left(\frac{u^1}{u^0}\right)\right](1-\phi)\,. \tag{3.5}$$

The criterion $d^2R/dt^2 = 0$ locating the (unique) turnaround radius yields

$$\ddot{a}r - \frac{\psi'}{a} = 0 \tag{3.6}$$

or, in terms of the *areal* turnaround radius, $R_c = a(t)r_c[1-\phi(r_c)]$ or again, using the gravitational slip $\xi \equiv (\phi - \psi)/\phi$,

$$\ddot{a}R_c(1+\phi_c) - \phi_c'(1-\xi_c) + \phi_c\xi_c' = 0\,. \tag{3.7}$$

4. Conclusions

The turnaround radius constitutes a potential probe of gravity and/or of the ΛCDM model. For spherical perturbations of a FLRW universe in GR, the (covariant) Hawking-Hayward/Misner-Sharp-Hernandez mass M_{HH} splits, to first order, into a local part and a cosmological part. This covariant splitting allows a rigorous derivation of the turnaround radius R_c and provides a small numerical correction to the value of R_c in the literature, but a much needed clarification of "mass contained in a sphere of radius R".

In modified (scalar-tensor or $f(R)$) gravity, there is no universally accepted notion of quasilocal mass M_{HH}. In this case, the usual criterion $\ddot{R} = 0$ along radial timelike geodesics yields an expression of R_c in this class of theories.

Is the turnaround radius important? A recent claim by astronomers [6] that the upper bound set by GR on R_c is exceeded by far in the galaxy group NGC 5353/4 has been taken back due to the previously underestimated error introduced by the non-sphericity of the system [14]. However, the verdict on the usefulness of the turnaround radius as a cosmological probe is still out and requires further and careful study.

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