Update on the hadronic vacuum polarisation contributions to muon $g - 2$ and $\alpha(m_Z^2)$

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An update of the hadronic vacuum polarisation contributions to the muon magnetic anomaly and to the running of the electromagnetic couplings constant at the Z-boson mass is presented. Newest $e^+e^- \to$ hadrons cross-section data mainly from the BABAR and VEPP-2000 experiments have been included. For the muon $(g-2)/2$, the lowest-order hadronic contribution is evaluated to be $(693.1 \pm 3.4) \cdot 10^{-10}$, improving the precision of our previous evaluation by 21%. The full Standard Model prediction differs by 3.5 σ from the experimental value. The five-quark hadronic contribution to $\alpha(m_Z^2)$ is evaluated to be $(276.0 \pm 0.9) \cdot 10^{-4}$.

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The Standard Model (SM) predictions of the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)/2$, with g_μ the muon gyromagnetic factor, and of the running electromagnetic coupling constant, $\alpha(s)$, a crucial ingredient of electroweak theory, are limited in precision by hadronic vacuum polarisation (HVP) contributions. The dominant hadronic terms can be calculated with a combination of experimental cross-section data, involving e^+e^- annihilation to hadrons, and perturbative QCD. They are used to evaluate energy-squared dispersion integrals ranging from the $\pi^0\gamma$ threshold to infinity. The kernels occurring in these integrals emphasise low photon virtuality, owing to the $1/s$ descent of the cross-section, and, in case of a_μ, to an additional $1/s$ suppression. In the latter case, about 74% of the lowest order hadronic contribution and 59% of the total uncertainty-squared are given by the $\pi^+\pi^-(\gamma)$ final state, while this channel amounts to only 12% of the hadronic contribution to $\alpha(s)$ at $s = m_Z^2$.

In this proceedings in common with that of PhiPsi 2017, we report an update, detailed in Ref. [1], of the lowest-order hadronic contribution, $a_\mu^{\text{had,LO}}$, to the muon magnetic anomaly, and the hadronic contribution, $\Delta\alpha_{\text{had}}(m_Z^2)$, to $\alpha(s)$ at the Z-boson mass using newest $e^+e^- \to$ hadrons cross-section data.

Exclusive bare hadronic cross-section measurements are integrated up to 1.8 GeV over the relevant dispersion kernels using the software package HVPTools [2]. In this update, 39 channels are included, as compared to only 22 in our previous work from 2011 [3]. Channels having important improvements (changes) are shown in Table 1.

For the dominant $\pi^+\pi^-$ channel, the computation of the dispersion integral over the full $\pi^+\pi^-$ spectrum requires to extend the available data to the region between threshold and 0.3 GeV, for which a fit as described in Ref. [2] is used. A tension between the BABAR and KLOE measurements is observed at the above the $\rho(770)$ peak region [1]. The local uncertainty rescaling applied increased the combined $a_\mu^{\text{had,LO}}$ uncertainty by 15% in the channel. In spite of this problem, the precision of the $\pi^+\pi^-$ channel of the new reevaluation is improved by 9%, which includes recent measurements from KLOE-2012 [4] and BESIII-2015 [5].

Channel	$a_\mu^{\text{had,LO}}$ [10^{-10}] 2017	$a_\mu^{\text{had,LO}}$ [10^{-10}] 2011	$\delta a_\mu^{\text{had,LO}}$ reduction
$\pi^+\pi^-$	$507.14 \pm 1.13 \pm 2.20 \pm 0.75$	$507.80 \pm 1.22 \pm 2.50 \pm 0.56$	-9%
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$13.35 \pm 0.10 \pm 0.43 \pm 0.29$	-42%
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$18.01 \pm 0.14 \pm 1.17 \pm 0.40$	-56%
K^+K^-	$22.81 \pm 0.24 \pm 0.28 \pm 0.17$	$21.63 \pm 0.27 \pm 0.58 \pm 0.36$	-46%
$K_S^0 K_L^0$	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$12.96 \pm 0.18 \pm 0.25 \pm 0.24$	-38%
$K\bar{K}\pi$	$2.45 \pm 0.06 \pm 0.12 \pm 0.07$	$(2.39 \pm 0.07 \pm 0.12 \pm 0.08)$	-6%
$K\bar{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$(1.35 \pm 0.09 \pm 0.38 \pm 0.03)$	-86%
Missing (%)	0.09 ± 0.02	0.69 ± 0.07	
R_{QCD} [1.8–3.7 GeV]$_{uds}$	$33.45 \pm 0.28 \pm 0.59_{\text{dual}}$	33.45 ± 0.28	
Sum	$693.1 \pm 1.2 \pm 2.6 \pm 1.7 \pm 0.1_\Psi \pm 0.7_{\text{QCD}}$	$692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2 \pm 0.3$	-21%

Table 1: Comparison of the new evaluation with our previous one and their relative uncertainty reduction. Where three (or more) uncertainties are given, the first is statistical, the second channel-specific systematic, and the third common systematic, which is correlated with at least one another channel. For the contribution computed from QCD, only the total uncertainties are given, which include effects from the α_s uncertainty, the truncation of the perturbative series at four loops, the FOPT vs. CIPT ambiguity, and quark mass uncertainties. The additional uncertainty dubbed "dual" estimates possible quark-hadron duality violating effects in the QCD estimate between 1.8 and 2.0 GeV.

The $2\pi^+2\pi^-$ channel includes an updated measurement from BABAR-2012 [6] using its full available data sample and a new measurement from CMD3-2017 [7]. The total uncertainty of 0.31 (in units of 10^{-10} unless otherwise specified) on the corresponding combined HVP contribution is reduced by -42%. The precision of the $\pi^+\pi^-2\pi^0$ channel is improved by 56% with the final BABAR-2017 data [8] that have its systematic uncertainty reduced to 3.1% below 2.7 GeV from about 10% in its preliminary version.

Including new measurements from BABAR-2013 [9] and SND-2016 [10] reduces the uncertainty of the K^+K^- channel by -46%. However both data, in agreement with each other, show a discrepancy with the former SND data that exceeds the quoted systematic uncertainty. The $K^0_S K^0_L$ channel has received two new data sets from BABAR-2014 [11] and CMD3-2016 [12]. The precision of this channel is improved by 38%.

In previous hadronic polarisation analyses the available exclusive $e^+e^- \to K\bar{K}$+pions data were incomplete. Missing channels were constrained based on assumptions about the process dynamics and isospin symmetry [3] leading to considerable uncertainty. This procedure became unnecessary since the BABAR experiment produced cross-section results for three channels contributing to the final state $K\bar{K}\pi$ and six channels contributing to $K\bar{K}\pi\pi$. This together with many processes with smaller cross sections measured from VEP-2000 and BABAR explains the improvements for the $K\bar{K}\pi$, $K\bar{K}\pi\pi$ and the missing channel in Table 1.

The continuum contribution in the non-resonant energy ranges above 1.8 GeV is calculated using four-loop perturbative QCD including $\mathcal{O}(\alpha_s^2)$ quark mass corrections. In the energy range of $1.8 - 3.7$ GeV, an uncertainty of 0.59 is added. This corresponds to the full difference between the perturbative calculation and the sum of exclusive measurements in the narrow energy interval $1.8 - 2.0$ GeV. It accounts for possible low-mass quark-hadron duality violation affecting the perturbative QCD approximation.

The sum of all lowest-order hadronic contributions, $a_\mu^{\text{had,LO}} = 691 \pm 3.4$, is improved by 21% in precision and is still dominated by experimental systematic uncertainties. Including the contributions from higher order hadronic loops, -9.87 ± 0.09 (NLO) and 1.24 ± 0.01 (NNLO) [13], hadronic light-by-light scattering, 10.5 ± 2.6 [14], as well as QED, $11\,658\,471.895 \pm 0.008$ [15], and electroweak effects, 15.36 ± 0.10 [16], the complete SM prediction amounts to, $a_\mu^{\text{SM}} = 11\,659\,182.3 \pm 3.4 \pm 2.6 \pm 0.2\,(4.3_{\text{tot}})$, where the uncertainties account for lowest and higher order hadronic, and other contributions, respectively. The result deviates from the experimental value, $a_\mu^{\text{exp}} = 11\,659\,209.1 \pm 5.4 \pm 3.3$ [17], by $26.8 \pm 7.6\,(3.5\sigma)$. A compilation of recent SM predictions for a_μ compared with the experimental result is given in Fig. 1.

Similar reevaluation is also performed for $\Delta\alpha_{\text{had}}(m_Z^2)$ with the result of $(275.3 \pm 0.9) \cdot 10^{-4}$ [1]. The uncertainty is dominated by data systematic effects $(0.7 \cdot 10^{-4})$ and the uncertainty in the QCD prediction $(0.6 \cdot 10^{-4})$.

To conclude, using newest available $e^+e^- \to$ hadrons cross-section data, the uncertainty of 0.5% on $a_\mu^{\text{had,LO}}$ is now reduced to about half the current uncertainty of the a_μ measurement, and has improved by more than a factor of two during the last thirteen years. The discrepancy between measurement and complete Standard Model prediction remains at a non-conclusive $3.5\,\sigma$ level. The forthcoming experiments at Fermilab [20] and J-PARC [21], aiming at up to four times better ultimate precision, have the potential to clarify the situation.
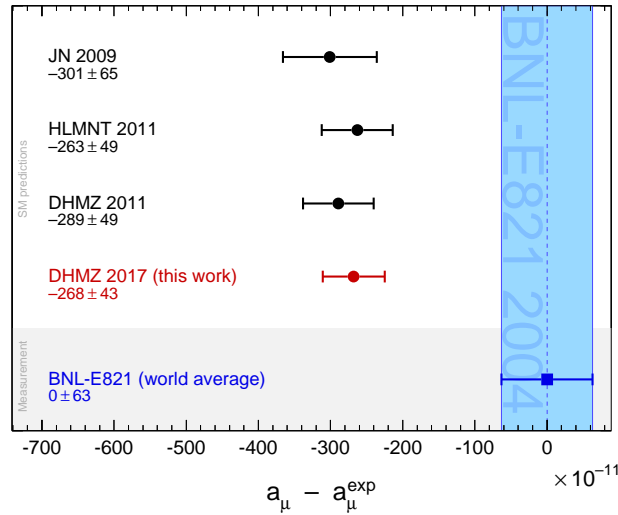
Figure 1: Compilation of recent results for a_μ^{SM} (in units of 10^{-11}), subtracted by the central value of the experimental average. The shaded vertical band indicates the experimental uncertainty. The representative SM predictions are taken from JN 2009 [18], HLMNT 2011 [19], DHMZ 2011 [3], and DHMZ 2017 [1].

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