Electroweak Physics at Future $e^+e^-$ Colliders

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With the discovery of the Higgs boson at the LHC in 2012, all fundamental parameters of the Standard Model (SM) have been experimentally measured. Then the global fits of the electroweak sector provide a powerful test of the internal consistency of the SM. Any observable sensitive to electroweak corrections can be unambiguously predicted and any deviation of the measurements with respect to the predictions would reveal the existence of new, weakly interacting particles. Thus precision electroweak measurements are a key tool for constraining theories describing physics beyond the SM. Although hadron colliders (Tevatron, LHC) have matched or occasionally exceeded the precision of LEP measurements, $e^+e^-$ colliders are unrivalled for electroweak precision measurements. The expected performance of the future $e^+e^-$ colliders is discussed in the perspective of indirect discovery of new physics. These machines are nevertheless well-suited for direct discoveries as well.

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1. Introduction

There are four projects of $e^+e^-$ colliders: 2 linear colliders (CLIC at CERN, ILC in Japan) and 2 circular colliders (FCC at CERN, CEPC in China). The circular colliders are expected to later evolve towards proton-proton colliders. Figure 1 shows the instantaneous luminosities as a function of the centre-of-mass energy in the most recent running scenarios [1] of the FCC, ILC and CLIC; the CEPC is not shown here as the original project has progressively turned into a version very similar to the FCC and the luminosity values have not been updated yet. The advantage of circular over linear colliders results from a much larger luminosity for centre-of-mass energies below 400 GeV, the possibility of several interaction points, a precise measurement of the beam energy through resonant transverse depolarization [2]. The centre-of-mass energy is expected to be known with a 100 keV accuracy at the FCC.

The ILC and CLIC physics programs cover two decades. Although none of these projects includes running at the Z pole or at the W-pair production threshold, the CLIC would be sensitive to anomalous gauge couplings, through WW Vector Boson scattering at 1.4 TeV and 3 TeV [3]. During a period of 14 years the FCC is expected to deliver a total of 150 ab$^{-1}$ at the Z pole over 4 years, 10 ab$^{-1}$ at the W-pair production threshold over 2 years, 5 ab$^{-1}$ at the Higgs production peak (in the covered energy range) over 3 years, 1.5 ab$^{-1}$ at the $t\bar{t}$ production threshold over 5 years. Although a GigaZ option exists for ILC [4], the FCC is the only project including the essential precision measurements at the Z pole and at the W-pair production threshold [5].

![Figure 1: Instantaneous luminosities as a function of the centre-of-mass energy in the most recent scenarios for the FCC, ILC and CLIC.](image)

The prospects for high precision electroweak measurements at the Z pole, at the W-pair production threshold and at the $t\bar{t}$ threshold are discussed in the next sections and conclusions on indirect searches for new physics are drawn. The prospects for the FCC as a Higgs factory are discussed elsewhere in these proceedings [6].
2. Physics at the Z pole

The precision measurements at the Z pole made by LEP will be revisited with a huge statistics (a few $10^{12} Z$s), leading to a 5 keV statistical uncertainty on the measurement of Z mass and width through a line-shape scan. The knowledge of the centre-of-mass energy is the dominant source of systematic experimental uncertainty. At LEP the depolarization resonance was very narrow [2], leading to a 100 keV intrinsic precision on each individual beam energy measurement [7], but the final systematic uncertainty was 1.5 MeV due to the transport of the calibration from dedicated polarization runs to the physics runs. At the FCC continuous calibration with dedicated bunches will be performed in order to suppress the transport uncertainty and obtain a beam energy uncertainty below 100 keV at the Z pole (and at the WW threshold), leading to 100 keV uncertainty on the Z mass and width.

The five orders of magnitude increase of statistics with respect to LEP will yield to considerably more precise measurements of other observables such as the Z partial widths and asymmetries, where systematic uncertainties largely cancel in the ratio of cross sections. For example the accurate measurement of $R_1$, the ratio between the hadronic and the leptonic width, leads to a significantly better precision in the determination of $\alpha_s(m_Z^2) (\approx \pm 0.0002)$. Improvements are also expected in the measurement of $R_2 (R_3)$, the ratio between the partial width into b-quarks (c-quarks) and the total hadronic width.

With a considerable increase of statistics with respect to LEP, the FCC might well give the final word to the long standing differences in asymmetries, whose measurements at LEP were statistically limited. A good example is the measurement of $\sin^2 \theta_W^{\text{eff}}$ from the forward-backward asymmetry for muon pairs, $A_{FB}(\mu\mu)$, with a factor 40 gain in precision with respect to LEP [8]. Detailed studies of lepton and quark asymmetries are also underway.

It was demonstrated [9] that $A_{FB}(\mu\mu)$ around the Z pole could also be used to directly measure the electromagnetic coupling constant, $\alpha_{\text{QED}}(m_Z^2)$, with an adequate accuracy to match the precision on the other input parameters of the Standard Model and exploiting the full potential of the FCC to constrain or fit the parameters of Beyond-the-Standard Model theories. The current estimation of $\alpha_{\text{QED}}(m_Z^2)$ is based on the precise measurement ($10^{-10}$) of $\alpha_{\text{QED}}(0)$ at zero momentum transfer extrapolated to the Z-mass scale with a $1.1 \times 10^{-4}$ uncertainty [10], dominated by the experimental determination of the hadronic vacuum polarization. The direct determination of $\alpha_{\text{QED}}(m_Z^2)$ at the Z-mass scale eliminates the need for the extrapolation from zero-momentum transfer and the related uncertainty. At the peak of the resonance, the $e^+e^- \rightarrow Z/\gamma^* \rightarrow l^+l^-$ process is dominated by the Z exchange and has no sensitivity to $\alpha_{\text{QED}}$, but away from the peak the photon exchange increases and provides sensitivity. In contrast to the cross section, the forward-backward asymmetry is a self normalizing quantity for which uncertainties on the integrated luminosity, detector acceptance and selection efficiency cancel. The relative uncertainty on $\alpha_{\text{QED}}$, proportional to the relative uncertainty on $A_{FB}(\mu\mu)$, is minimal at the two centre-of mass energies 87.9 and 94.3 GeV, as shown on Figure 2. As $A_{FB}(\mu\mu)$ changes sign on the opposite sides of the Z pole, most sources of uncertainties cancel by combining the measurements at these two centre-of-mass energies and a relative statistical accuracy of $3 \times 10^{-5}$ is achievable. The knowledge of the centre-of-mass energy is the dominant source of experimental systematic uncertainty. Missing electroweak orders are estimated to contribute to
the dominant theoretical systematic uncertainty at the level of a few $10^{-4}$, and a factor 10 improvement is required to match the experimental contribution to the total uncertainty on $\alpha_{\text{QED}}$.

Figure 2: Relative statistical uncertainty on $\alpha_{\text{QED}}$ as a function of the centre-of-mass energy for one year of running at any given centre-of-mass energy (left). Muon forward-backward asymmetry, $A_{\text{FB}}(\mu\mu)$, in $e^+e^+ \rightarrow \mu^+\mu^-$ as a function of the centre-of-mass energy (right).

An accurate measurement of the number of quark-lepton generations at the FCC would be a powerful test of the unitarity of the PMNS matrix or of the existence of right-handed neutrinos [11]. At LEP the number of neutrino families, inferred from the total hadronic cross section at the $Z$ peak, was measured 1.4-2 standard deviation below the standard-model value of 3. This measurement was limited by the accuracy of the theoretical estimation of the small-angle Bhabha cross section used for the luminosity normalisation. A possible improvement might come from the precise measurement of the luminosity from the $e^+e^+ \rightarrow \gamma\gamma$ process at the FCC. This uncertainty is expected to be 0.0003 (0.0046 at LEP) still dominant over the statistical precision of 0.00008. Another method, using the radiative return process at centre-of-mass energies above the $Z$ peak is discussed in the next section.

3. Physics at the WW production threshold

The large number of W pairs collected at ($\approx 4\times10^7$ around 161 GeV) and above ($\approx 8\times10^7$ above 240 GeV) the WW production threshold will provide precise measurements of W-boson properties.

At LEP2, the W mass was measured at the production threshold with a statistically limited accuracy of 210 MeV [12]. A better precision of 34 MeV was obtained through direct reconstruction of the W decay products [12]. This uncertainty was reduced to 15 MeV by the combination with Tevatron results [13]. Recently the ATLAS collaboration published a measurement with a 19 MeV uncertainty [14]. These uncertainties have to be compared to the 8 MeV accuracy of the prediction of the electroweak fit [15]. Figure 3 shows the W-pair cross section as a function of the centre-of-mass energy [16] with W mass and width set at the average measured values [17], with 1 GeV variation bands of the mass and width central values. Whilst a variation of the mass induces a shift of the cross section lineshape along the energy axis, a
variation of the width changes the slope of the lineshape and the W width dependence shows a crossing point where the cross section is insensitive to the W width.

Figure 3: W-pair production cross section as a function of the centre-of-mass energy. The central (red) curve corresponds to the predictions obtained for $m_W = 80.385$ GeV and $\Gamma_W = 2.085$ GeV. Purple (green) bands show the cross section curves obtained by varying the mass (width) by $\pm$ 1 GeV.

With taking data at a single energy point, the minimum statistical uncertainty on the W mass is achieved 600 MeV above the threshold. With assuming LEP selection quality [19], the expected statistical precision is 400 KeV for 10 ab$^{-1}$ accumulated in one year at the FCC. To match such precision, the relative uncertainties must be less than 2x10$^{-3}$ for the background, 10$^{-4}$ for the acceptance and the luminosity, 5x10$^{-6}$ for the beam energy, 10$^{-4}$ for the theoretical cross section. If two cross section measurements are done at two centre-of-mass energies, both the W mass and width can be extracted from a two-parameter fit to the cross section lineshape. The optimal centre-of-mass energies $(E_1, E_2)$ and fraction $f(E_1)$ of integrated luminosity at the lowest energy have been determined from a three-dimensional scan of these parameters. The configuration which minimizes the sum of the statistical uncertainties on the mass and width would be $E_1 = 157.1$ GeV, $E_2 = 162.3$ GeV, $f(E_1)=0.4$. The statistical uncertainties would then be 0.62 MeV for the mass and 1.5 MeV for the width; in the same configuration, measuring only the mass would yield a slightly better uncertainty of 0.56 MeV, to be compared to the 0.4 MeV expected from a single measurement at the optimal centre-of-mass energy (161.4 GeV).

W mass and width can also be measured at and above the WW production threshold from direct reconstruction of the final state with a comparable statistical uncertainty. Better detectors, better understanding of jet energy scale and angular resolution, improved Monte-Carlo simulations might lead to a 1 MeV systematic uncertainty.

The lepton universality can be tested through the measurement of the leptonic branching fractions with a relative statistical accuracy of $\approx 4 \times 10^{-4}$, a factor 50 smaller than the LEP measurement ($\approx 2\%$). The hadronic branching fraction of the W boson is expected to be measured with a $10^{-4}$ relative uncertainty ($4 \times 10^{-5}$ at LEP), resulting into an absolute uncertainty on $\alpha_s(m_W^2) \approx \pm 0.0001$, a factor 40 improvement with respect to LEP [12] and even 10 with respect to the current measurement [13].

At and above WW production threshold, the radiative return to the $Z$, $e^+ e^- \rightarrow Z \gamma$, leading to a very clean sample of photon-tagged on-shell Z bosons, can be used to determine the number of
neutrino families. This method was statistically limited at LEP, but after 5 years of running at the FCC the expected accuracy is 0.001 number of families, 10 times better than LEP [7]. This process could also be used to search for sterile neutrinos [11].

4. Physics at the t\bar{t} production threshold

The top mass is currently determined at hadron colliders from the invariant mass of the decay products with an experimental precision of 0.5% [18], dominated by the theoretical interpretation of the experimental results with an inherent uncertainty of \( \approx 500 \text{ MeV} \) [19]. With \( 10^6 \) top-quark pairs produced at and above the production threshold in a clean experimental environment and at a precisely defined centre-of-mass energy, unaffected by bremsstrahlung effects expected at linear colliders, the top mass can be extracted from the measurement of the \( \text{e}^+\text{e}^- \to t\bar{t} \) inclusive cross section. From the precise measurement of this cross section in a scan of the threshold, the top-quark mass can be determined with a statistical accuracy of 10 MeV with 200 fb\(^{-1}\) [20]. The experimental uncertainty from the knowledge of the energy and of the beam energy spread is expected to be lower than 5 MeV. The threshold behaviour of the cross section significantly depends on the strong coupling constant and its precise measurement at the Z pole and at the WW production threshold would reduce the impact of this parametric uncertainty to about 20 MeV. By using the N\(^4\)LO formula, which is based on the four-loop relation between the M\(^\overline{\text{S}}\) and on-shell quark mass, the theoretical systematic uncertainty resulting from the translation of mass scheme is about 10 MeV [21]. Other scale uncertainties are under study. The expected total uncertainty expected at the ILC/CLIC is about 50 MeV [22]. These expectations have to be compared to the present precision of 600 MeV at hadron colliders [13].

The ILC & CLIC can also measure the top-quark mass even well above the threshold using different methods such as:

- direct reconstruction: with 100 fb\(^{-1}\) at 500 GeV, a statistical uncertainty of 80 MeV is expected [23]. This measurement can suffer from significant theory uncertainty when converting to a particular mass scheme.
- radiative events: even at energies well above threshold, there is still sensitivity to the t\bar{t} threshold in radiative events by measuring the rate of energetic ISR photons and FSR gluons. With 500 fb\(^{-1}\) at 380 GeV a statistical accuracy of 100 MeV is expected. With 3.5 ab\(^{-1}\) at 1 TeV the uncertainty increases to \( \approx 400 \text{ MeV} \) [24].
- other methods are also considered such as b-jet energy distribution [25] or event-shape analysis [26].

The electroweak couplings of the top-quark to the Z boson and to the photon may be very sensitive to effects from massive unknown particles. Their precise measurement would open the way to new physics discoveries well beyond the energy scale accessible to direct discoveries or would set constraints to standard model extensions. An example is shown on Figure 4 for composite Higgs models, where large deviations appear in the left-handed and right-handed couplings of the top-quark to the Z boson.
Electroweak Physics at future $e^+e^-$ Colliders

Elizabeth Locci

Figure 4: Deviations from the standard model predictions of the left-handed and right-handed couplings of the top quark to the Z boson in several Beyond-Standard-Model scenarios [27,28].

It has been shown [29] that, with the sole use of the lepton angular and energy distributions in semi-leptonic $tar{t}$ events ($e^+e^- \rightarrow l
\nu qar{q}bar{b}$) and modest detector performance, the lack of incoming beam polarization is compensated by the polarization of the final state top quarks and a significantly large integrated luminosity (Figure 5). The best accuracy on the 6 CP-conserving form factors is obtained for a centre of mass energy of 365-370 GeV. A relative statistical uncertainty of $10^{-2}$ to $10^{-3}$ is expected without initial state polarization.

Figure 5: (Modified from ref [30]). Statistical uncertainties on CP-conserving top-quark form factors expected at ILC (blue) and LHC(red). The figure was modified to include CLIC and FCC projections. The results for LHC assume an integrated luminosity of 300 fb$^{-1}$ at a centre-of-mass energy of 14 TeV. The results of ILC assume an integrated luminosity of 500 fb$^{-1}$ at a centre-of-mass energy of 500 GeV, with beam polarizations of $P = \pm 0.8$ and $-P' = \pm 0.3$. The ILC projections are obtained from the measurements of the total top-quark pair production cross section and forward-backward asymmetry. The FCC projections are obtained at a centre-of-mass energy of 365 GeV, with unpolarized beams and with an integrated luminosity of 2.4 ab$^{-1}$, from the sole lepton energy and angular distributions.

5. Conclusion

The high luminosity $e^+e^-$ circular collider FCC would perform measurements of electroweak observables from around the Z pole to the $t\bar{t}$ threshold and beyond with unrivalled precision. If the projected accuracies are achieved, the contour line in the ($m_t,m_W$) plane could evolve as...
shown on Figure 6, where both the results of the direct mass measurements and the indirect constraints from the precision measurements at the Z pole are included.

**Figure 6**: The 68% c.l. contour line in the \((m_t, m_W)\) plane as expected for the FCC and other colliders. The blue line indicates the expected contour from direct W and top quark mass measurements, while the red line gives the expected precision from a fit to the Z-pole observables.

Such a dramatic increase of precision could provide discriminating power for new physics. For example, in the effective lagrangian approach, contributions to new physics come from higher dimensional operators \(O_i\). In the chosen basis 10 operators contribute to electroweak precision observables. The result of the fit to electroweak precision data, assuming only one operator at a time is generated by new physics, is shown here for present data as well as projections for future colliders. Future colliders could probe scales up to 40 to 100 TeV [31].

**Figure 7**: Projected sensitivities to dimension 6 interactions at future colliders (1 operator at a time). Different shades of the same colour denote results including or neglecting future theory uncertainties.
Electroweak Physics at future $e^+e^-$ Colliders

Elizabeth Locci

Acknowledgements

I wish to thank all the contributors to the preparation of the talk and particularly Patrick Janot who carefully read the manuscript.

References


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