



# Soft gluon resummation for the associated production of a top quark pair with a W boson at the LHC

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We present our results on soft gluon resummation in the invariant mass threshold applied to the associated production of a top quark pair with a W boson at the LHC in the Mellin space formalism.

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# 1. Introduction

The measurements of associated production of a vector boson with a top-antitop quark pair provide an important test for the Standard Model at the LHC [1, 2]. In particular these are the key processes to measure the top quark couplings to W/Z bosons. Furthermore they are relevant in searches for new physics due to both being directly sensitive to it and providing an important background. They also play an important role as a background for the associated Higgs boson production process  $pp \rightarrow t\bar{t}H$ . It is therefore necessary to know the theoretical predictions for  $pp \rightarrow$  $t\bar{t}W/Z$  with high accuracy. Fixed order cross sections up to next-to-leading order in  $\alpha_S$  are already known for some time [3, 4]. They were recalculated and matched to parton showers in [5, 6, 7, 8, 9]. Furthermore QCD-EW NLO corrections have been obtained [10]. While NNLO calculations for this particular type of 2 to 3 processes are currently out of reach, a class of corrections beyond NLO from the emission of soft and/or collinear gluons can be taken into account with the help of resummation methods. This was done in the framework of the soft-collinear effective theory (SCET) for  $pp \rightarrow t\bar{t}W$  [11] and with SCET-formulas expressed in Mellin-space for  $pp \rightarrow t\bar{t}W/Z$ [12, 13].

In the following we present results for threshold-resummed cross sections in the invariant mass kinematics, obtained using the Mellin-space approach at NLL accuracy. The calculations are then improved beyond NLL by including non-logarithmic hard contributions of the order  $\mathcal{O}(\alpha_S)$ .

## 2. Resummation at invariant mass threshold

Here we treat the soft gluon corrections in the invariant mass kinematics, i.e we consider the limit  $\hat{\rho} = \frac{Q^2}{\hat{s}} \rightarrow 1$  with  $Q^2 = (p_t + p_{\bar{t}} + p_{W/Z})^2$ . The logarithms resummed in the invariant mass threshold limit have the form

$$\alpha_{S}^{m} \left( \frac{\log^{n} \left( 1 - \hat{\rho} \right)}{1 - \hat{\rho}} \right)_{+} \qquad m \le 2n - 1 \tag{2.1}$$

with the plus distribution  $\int_0^1 dx (f(x))_+ = \int_0^1 dx (f(x) - f(x_0))$ . The Mellin moments of the differential cross section  $\frac{d\sigma_{ij \to v \bar{t} W/Z}}{dQ^2}$  are taken with respect to the variable  $\rho = \frac{Q^2}{S}$ . At the partonic level this leads to

$$\frac{\mathrm{d}\hat{\sigma}_{ij\to t\bar{t}W/Z}}{\mathrm{d}Q^2}(N,Q^2,m_t,m_{W/Z},\mu_R^2,\mu_F^2) = \int_0^1 \mathrm{d}\hat{\rho}\hat{\rho}^{N-1}\frac{\mathrm{d}\hat{\sigma}_{ij\to t\bar{t}W/Z}}{\mathrm{d}Q^2}(\hat{\rho},Q^2,m_t,m_{W/Z},\mu_R^2,\mu_F^2) \quad (2.2)$$

for the Mellin moments for the process  $ij \rightarrow t\bar{t}W/Z$  with i,j denoting two massless colored partons. In Mellin space the threshold limit  $\hat{\rho} \rightarrow 1$  corresponds to the limit  $N \rightarrow \infty$ . Since the process involves more than 3 colored partons, the resummed cross section involves color matrices. In Mellin space the resummed partonic cross section has the form [14, 15]

$$\frac{\mathrm{d}\hat{\sigma}_{ij\to t\bar{t}W/Z}}{\mathrm{d}Q^2} = \mathrm{Tr}[\mathbf{H}_{ij\to t\bar{t}W/Z}\mathbf{S}_{ij\to t\bar{t}W/Z}]\Delta_i\Delta_j,\tag{2.3}$$

where  $\mathbf{H}_{ij \to t\bar{t}W/Z}$  and  $\mathbf{S}_{ij \to t\bar{t}W/Z}$  are color matrices and the trace is taken in color space. We describe the evolution of color in the s-channel color basis, for which the basis vectors are

$$c_1 = \delta_{a_i, a_j} \delta_{a_k, a_l} \quad c_8 = T^c_{a_i, a_j} T^c_{a_k, a_l} \tag{2.4}$$

for the  $q\bar{q}$  initial state and

$$c_{1} = \delta_{a_{i},a_{j}} \delta_{a_{k},a_{l}} \quad c_{8S} = d^{c,a_{i},a_{j}} T^{c}_{a_{k},a_{l}} \quad c_{8A} = f^{c,a_{i},a_{j}} T^{c}_{a_{k},a_{l}}$$
(2.5)

for the gg initial state. This choice of color basis leads to a diagonal soft anomalous dimension matrix in the absolute threshold limit  $\frac{(2m_t+m_{W/Z})^2}{\hat{s}} \rightarrow 1$ , which is a special case of the invariant mass threshold limit.  $\mathbf{H}_{ij \rightarrow t\bar{t}W/Z}$  describes the hard scattering contributions projected on the color basis, while  $\mathbf{S}_{ij \rightarrow t\bar{t}W/Z}$  represents the soft wide angle emission. The (soft-)collinear logarithmic contributions form the initial state partons are taken into account by the functions  $\Delta_i$  and  $\Delta_j$ .

At NLL accuracy the evolution of the soft matrix  $S_{ij \to t\bar{t}W/Z}$  is given by the one-loop anomalous dimension matrix, see e.g. [18]. Since the soft anomalous dimension matrix is not diagonal in the invariant mass threshold, we use the method proposed in [15] to diagonalize the soft anomalous dimension matrix in a basis *R*. Then  $S_{ij \to t, \bar{t}W/Z}$  is given by [15]:

$$\mathbf{S}_{ij \to t\bar{t}W/Z,R} = \mathbf{S}_{ij \to t\bar{t}W/Z,R}^{(0)} \exp\left[\int_{\mu}^{Q/N} \frac{\mathrm{d}q}{q} (\lambda_{R,I}^* + \lambda_{R,J})\right]$$
(2.6)

where  $\lambda_i, \lambda_j, ...$  are the eigenvalues of the soft anomalous dimension matrix and  $\mathbf{S}_{ij \to t\bar{t}W/Z,R}^{(0)}$  is

$$\left(\mathbf{S}_{ij \to t\bar{t}W/Z}^{(0)}\right)_{IJ} = \operatorname{Tr}\left[c_{I}^{\dagger}c_{J}\right]$$
(2.7)

transformed into the R basis.  $\mathbf{H}_{ij \to t\bar{t}W/Z}$  can be calculated perturbatively

$$\mathbf{H}_{ij \to t\bar{t}W/Z} = \mathbf{H}_{i,j \to t,\bar{t},W/Z}^{(0)} + \frac{\alpha_S}{\pi} \mathbf{H}_{ij \to t\bar{t}W/Z}^{(1)} + \dots$$
(2.8)

For NLL accuracy it is sufficient to include  $\mathbf{H}_{i,j\to t,\bar{t},W/Z}^{(0)}$ , which is given by the leading order cross section. To improve the predictions beyond NLL we can also include  $\mathbf{H}_{ij\to t\bar{t}W/Z}^{(1)}$ . This factor collects non-logarithmic contributions of  $\mathcal{O}(\alpha_S)$  in the large *N* limit [19, 20]. In particular it includes the virtual loop corrections, which are numerically extracted from the PowHel implementation [8, 9]. The initial state jet functions  $\Delta_i$  and  $\Delta_j$  have been known for a long time [16, 17] and depend only on the emitting parton.

## 3. Numerical results

The numerical results were obtained using  $m_t = 173 \text{ GeV}$ ,  $m_W = 80.385 \text{ GeV}$  and MMHT14 PDF sets [21] and for the center of mass energy  $\sqrt{S} = 13 \text{ TeV}$ . The one-loop hard contributions to  $\mathbf{H}_{ij\to t\bar{t}W/Z}^{(1)}$  and the NLO cross sections were calculated with the PowHel implementation [8, 9]. We use  $\mu = \frac{M}{2} = m_t + \frac{m_W}{2}$  and  $\mu = Q$  for the scales  $\mu = \mu_R = \mu_F$ . Total cross section results were obtained by integrating the resummed differential cross section  $\frac{d\tilde{\sigma}}{dQ^2}$ . These resummed results are then matched to fixed order NLO predictions [22].

In table 1 we show the total cross section for  $t\bar{t}W^{+/-}$  production at the two different central scale choices and their scale uncertainty calculated with the seven point method. Figures 1, 2, 3 and 4 show the scale dependence of the  $t\bar{t}W^{+/-}$  production cross section by varying simultaneously the renormalization and factorization scale  $\mu = \mu_R = \mu_F$ . In all figures the NLO results are compared

| process         | $\mu_0$       | NLO                            | NLO + NLL                      | NLO+NLL w $\mathbf{H}^{(1)}$   |
|-----------------|---------------|--------------------------------|--------------------------------|--------------------------------|
| $t\bar{t}W^+$   | Q             | $329.9^{+12.5\%}_{-11.1\%}$ fb | $332.1^{+12.5\%}_{-11.2\%}$ fb | $341.1^{+10.7\%}_{-8.6\%}$ fb  |
| $t\bar{t}W^+$   | $\frac{M}{2}$ | $422.1^{+12.8\%}_{-11.5\%}$ fb | $423.5^{+13.2\%}_{-11.5\%}$ fb | $418.4^{+12.8\%}_{-10.0\%}$ fb |
| $t\bar{t}W^{-}$ | $\bar{Q}$     | $168.5^{+12.7\%}_{-11.2\%}$ fb | $170.0^{+12.1\%}_{-10.7\%}$ fb | $175.3^{+9.9\%}_{-8.4\%}$ fb   |
| $t\bar{t}W^{-}$ | $\frac{M}{2}$ | $215.6^{+13.4\%}_{-11.8\%}$ fb | $216.4^{+13.8\%}_{-11.6\%}$ fb | $214.4^{+13.4\%}_{-10.1\%}$ fb |

**Table 1:** Total  $t\bar{t}W^{+/-}$  cross sections for the two different central scale choices and their scale uncertainty, which was calculated with the seven point method

with the resummed NLO + NLL and NLO + NLL with  $\mathbf{H}_{ij \to t\bar{t}W}^{(1)}$  results for two different central scale choices  $\mu = Q$  and  $\mu = \frac{M}{2}$ . The resummed NLL matched to NLO with  $\mathbf{H}_{ij \to t\bar{t}W}^{(1)}$  cross section is less sensitive to scale variation as compared to the NLO result. At large scales the inclusion of  $\mathbf{H}_{ij \to t\bar{t}W/Z}^{(1)}$  has a bigger impact on the cross section than the logarithmic contributions. At central scale the cross section is increased by 3.4%  $(t\bar{t}W^+)$  and 4%  $(t\bar{t}W^-)$  for  $\mu_0 = Q$  and decreased by 0.9%  $(t\bar{t}W^+)$  and 0.6%  $(t\bar{t}W^-)$  for  $\mu_0 = m_t + \frac{m_W}{2}$ . The resummation reduces the scale uncertainty and brings the predictions for the two different scale choices closer together.



**Figure 1:** Scale dependence of the total  $pp \to t\bar{t}W^+$  cross section at NLO, NLL matched to NLO and NLL matched to NLO improved with  $\mathbf{H}_{ij\to t\bar{t}W/Z}^{(1)}$  for the central scale  $\mu_0 = Q$ .

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**Figure 2:** Scale dependence of the total  $pp \rightarrow t\bar{t}W^+$  cross section at NLO, NLL matched to NLO and NLL matched to NLO improved with  $\mathbf{H}_{ij \rightarrow t\bar{t}W/Z}^{(1)}$  for the central scale  $\mu_0 = \frac{M}{2}$ .



**Figure 3:** Scale dependence of the total  $pp \to t\bar{t}W^-$  cross section at NLO, NLL matched to NLO and NLL matched to NLO improved with  $\mathbf{H}_{ij\to t\bar{t}W/Z}^{(1)}$  for the central scale  $\mu_0 = Q$ .

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**Figure 4:** Scale dependence of the total  $pp \rightarrow t\bar{t}W^-$  cross section at NLO, NLL matched to NLO and NLL matched to NLO improved with  $\mathbf{H}_{ij\rightarrow t\bar{t}W/Z}^{(1)}$  for the central scale  $\mu_0 = \frac{M}{2}$ .

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