

UV properties of higher dimensional operators in Higgs Effective Field Theories from hidden symmetries

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We present a systematic approach to the classification of the UV properties of higher dimensional operators spanned by $\Phi^\dagger\Phi$ (where Φ is the Higgs doublet) and ordinary derivatives thereof (e.g. $\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$, $(\Phi^\dagger\Phi)^3$, $(\Phi^\dagger\Phi)^4$, ...) in Higgs Effective Field Theories. The procedure is purely algebraic and thus regularization-independent. It relies on a novel set of hidden symmetries that can be formulated in an extended field space where the singlet $\Phi^\dagger\Phi$ is treated as a dynamical variable. The resulting relations stemming from such symmetries are valid to all orders in the loop expansion for off-shell 1-PI Green's functions (and not only for S -matrix elements). One-loop applications are briefly discussed.

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1. Introduction

After the discovery at the LHC of a scalar particle whose properties are compatible with the Standard Model (SM) Higgs boson [1, 2] an important theoretical and experimental question is to constrain the potential triggering the electroweak spontaneous symmetry breaking (SSB). Effective Field Theories (EFTs) provide a useful tool to parameterize in a model-independent way the effects of Beyond-the-Standard-Model (BSM) physics induced by higher dimensional operators suppressed by inverse powers of a large energy scale Λ [3]. In the EFTs approach one enumerates all possible operators compatible with the symmetries of the theory by organizing them in layers of a given dimension. When dealing with physical quantities like S -matrix elements, some ambiguities are present, associated with the equivalence of operators differing by terms proportional to the equations of motions. These ambiguities can be fixed by choosing a particular operator basis [4]. EFTs are of course not power-counting renormalizable. Their UV properties can be understood within renormalizability in the modern sense *à la* Weinberg and Gomis [5], namely all possible UV divergences can be reabsorbed provided that all possible operators compatible with the symmetries of the theory are introduced, at the price of adding higher and higher dimensional operators as the loop number increases. It has been recently realized by explicit one-loop computations that in the SM extended with dimension-6 operators a surprising pattern of cancellations exists, yielding a much more constrained set of UV divergences than one would expect on the basis of symmetry arguments only [6, 7, 8]. These cancellations have been traced back to the remnants of supersymmetry in a (non-supersymmetric) EFT [9, 10]. A classification of one-loop non renormalization theorems for EFTs involving dimension six operators based on holomorphy arguments has been given in [11]. These results are valid at one-loop order for physical on-shell quantities since equations of motion are in general used in these computations. In [12] an alternative reformulation of the SSB sector of the SM has been proposed, based on the idea of using the singlet $\Phi^\dagger\Phi$ (after SSB) as the dynamical variable describing the physical scalar excitation. The advantage of this approach is the possibility to establish at one loop a diagrammatic separation between the SM and the BSM contributions to 1-PI Green's functions, which is in turn the consequence of some hidden functional symmetries.

The functional identities provide very strong constraints on the 1-PI Green's functions; these constraints are expressed in terms of 1-PI amplitudes involving insertions of a suitable set of external sources with a better UV behaviour than the quantized scalar fields. This in turn allows to use the techniques of Algebraic Renormalization [13, 14, 15, 16] in order to study the renormalization properties of the Higgs EFTs.

A unique deformation of the functional identities allows one to control the inclusion of the derivative operator $\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$ by modifying the 2-point function of the field describing the physical scalar excitation, while the interaction vertices are left unchanged.

2. Functional Identities

We denote by Φ the standard scalar field SU(2) doublet:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} i\phi_1 + \phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix},$$

where ϕ_a are the pseudo-Goldstone fields, ϕ_0 the would be Higgs field and v its expectation value. In order to use the combination $\Phi^\dagger\Phi$ (after SSB) as the dynamical variable associated with the singlet field X_2 , describing the physical scalar mode, we need to introduce a Lagrange multiplier X_1 enforcing on-shell the condition

$$X_2 = \frac{1}{v} \left(\Phi^\dagger\Phi - \frac{v^2}{2} \right) = \sigma + \frac{1}{2v} \sigma^2 + \frac{1}{2v} \phi_a^2. \quad (2.1)$$

The tree-level action of the model will be $S = S_{\text{SM}} + S_{\text{SSB}}$, where S_{SM} is the usual SM action involving the Yang-Mills, fermion, Yukawa, the gauge fixing and ghost terms and S_{SSB} replaces the SSB Higgs term and reads¹

$$S_{\text{SSB}} = \int d^4x \left[D_\mu \Phi^\dagger D^\mu \Phi - \frac{M^2}{2} X_2^2 - \bar{c} \square c + \frac{1}{v} (X_1 + X_2) \square \left(\Phi^\dagger\Phi - \frac{v^2}{2} - vX_2 \right) + \bar{c}^* \left(\Phi^\dagger\Phi - \frac{v^2}{2} - vX_2 \right) + V(X_2) \right], \quad (2.2)$$

with D the usual covariant derivative and $V(X_2)$ a BSM potential in the X_2 field. \bar{c}^* is an external source required to control the renormalization of the non-linear operator $\Phi^\dagger\Phi - \frac{v^2}{2} - vX_2$. In the present paper we consider the simplest case of a trilinear interaction $V(X_2) = g_6 X_2^3$. Then one needs to add to Eq.(2.2) a further external source R coupled to X_2^2 .

The dependence on X_1 and X_2 is fully controlled by the following functional identities (a subscript denotes differentiation w.r.t. the variable):

$$\Gamma_{X_1} = \frac{1}{v} \square \Gamma_{\bar{c}^*}, \quad \Gamma_{X_2} = \frac{1}{v} \square \Gamma_{\bar{c}^*} + 3g_6 \Gamma_R - \square X_1 - (\square + M^2) X_2 - v\bar{c}^* + 2RX_2. \quad (2.3)$$

By going on-shell with X_1 in Eq.(2.2) one recovers the classical SM action plus a dimension six operator $V(X_2) = g_6 X_2^3 \sim \frac{g_6}{v^3} (\Phi^\dagger\Phi - \frac{v^2}{2})^3$. One should notice that Eqs.(2.3) still holds without any further external source if one adds a kinetic term $\int \frac{z}{2} \partial_\mu X_2 \partial^\mu X_2$ for X_2 into Eq.(2.2), so that now the X_2 -equation reads

$$\Gamma_{X_2} = \frac{1}{v} \square \Gamma_{\bar{c}^*} + 3g_6 \Gamma_R - \square X_1 - ((1+z)\square + M^2) X_2 - v\bar{c}^* + 2RX_2. \quad (2.4)$$

By going on-shell with X_1 this is equivalent to the inclusion into the action Eq.(2.2) of the dimension six operator

$$\frac{z}{v^2} \partial^\mu (\Phi^\dagger\Phi) \partial_\mu (\Phi^\dagger\Phi).$$

Eqs.(2.3) and (2.4) are valid to all orders in the loop expansion and for any (symmetric) regularization scheme. It should be noticed that at $z \neq 0$ the identities connecting amplitudes involving X_2 -external legs with those without do not change with respect to the case $z = 0$ (the z -dependent term in the right hand side of Eq.(2.4) being confined at tree-level). However the UV divergences of X_2 -independent amplitudes change at $z \neq 0$. The reason is that the propagator Δ_{XX} of the combination $X = X_1 + X_2$ now falls off for large momenta as $1/p^2$ (instead as $1/p^4$, as it happens at

¹a further term $-\frac{m^2}{2v^2} (\Phi^\dagger\Phi - \frac{v^2}{2})^2$ is allowed in the above action [17] without altering the UV behaviour of the 1-PI amplitudes. Nevertheless the parameter m drops out of physical amplitudes [17] and thus for the sake of simplicity it is set here to zero.

$z = 0$):

$$\Delta_{XX} = \frac{i(-zp^2 + M^2)}{p^2[(1+z)p^2 - M^2]} \quad (2.5)$$

Then the derivative interaction vertices in the first line of (2.2) become dangerous and spoil the milder UV behaviour of the $z = 0$ theory. A complete classification of UV divergences at $z \neq 0$ is beyond the scope of the present work and is currently under investigation.

The action (2.2) is invariant under a U(1) BRST symmetry $s\bar{c} = S_{\bar{c}^*}$, $sX_1 = vx$, $sc = 0$ ensuring that the only physical scalar particle is described by the field X_2 , while the combination $\sigma' = \sigma - X_1 - X_2$ cancels out together with X_1 to all orders in the loop expansion [12, 17].

3. One-loop Effective Action

The advantage of the reformulation of the SSB action in Eq.(2.2) is that it allows a one-loop diagrammatic decomposition of the SM and BSM-contributions of the complete 1-PI amplitudes (not only of on-shell quantities) and a classification of UV divergences in terms of Green's functions involving the external sources R, \bar{c}^* .

Indeed the BSM coupling g_6 only appear in the trilinear interaction vertex for X_2 and thus it can never contribute at one loop to amplitudes without external X_2 -legs. On the other hand, amplitudes involving at least one external X_2 -leg are uniquely fixed by Eq.(2.4) in terms of amplitudes that do not involve X_2 and which are thus g_6 -independent.

In order to recover the 1-PI amplitudes in the standard formalism one needs to go on-shell with both X_1 and X_2 . This amount to replace X_2 by the solution (2.1) to the X_1 -equation of motion and by enforcing the tree-level X_2 -equation of motion (at zero external sources)

$$\square(X_1 + X_2) = -(z\square + M^2)X_2 + 3g_6X_2^2, \quad (3.1)$$

where X_2 is to be substituted by Eq.(2.1). Thus for example for any given amplitude involving n -insertions of the Higgs field σ in the target theory (the one in the standard formalism), one has the following contributions:

- the pure SM part, equal to the 1-PI Green's function $\Gamma_{\sigma(x_1)\dots\sigma(x_n)}^{(1)}$ in the $X_{1,2}$ -theory;
- the BSM contribution, obtained by summing up all amplitudes with a X_1 or X_2 -external leg that give rise upon the substitutions in Eqs.(2.1) and (3.1) to amplitudes with n -external σ -legs in the target theory.

The UV properties of the BSM contribution to these amplitudes are fixed by those of the kernels involving the external sources R, \bar{c}^* , which in turn can be easily classified. We refer the reader to [18] for a detailed application of this method and the complete classification of one-loop divergences of amplitudes involving the Higgs field in a SM Higgs EFT with an arbitrary derivative-independent potential $V(X_2)$.

4. Conclusions

At one-loop level Higgs EFTs exhibit interesting unexpected cancellations of UV divergences that cannot be ascribed to gauge-invariance. We have presented an approach that allows for a complete classification of one-loop divergences in the presence of a derivative-independent SSB potential depending only on $\Phi^\dagger\Phi$. The main technical tool is the reformulation of the SSB mechanism at work in the SM by using the invariant $\Phi^\dagger\Phi$ as the new dynamical variable (after SSB). The correspondence with the standard formalism is achieved by going on-shell with X_1 and X_2 . This in turn allows for a diagrammatic decomposition of the 1-PI amplitudes into a pure SM part and a BSM contribution, whose UV properties can be easily studied by classifying those of the kernels involving the external sources insertions. This provides a very effective way to compute the one-loop divergences of 1-PI amplitudes in the Higgs EFTs. We remark that the results hold for the full 1-PI amplitudes, without any restriction to on-shell physical quantities as done before in the literature.

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