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Generalized Gounaris-Sakurai formula and $\rho^0(770)$, $\rho^0(1450)$ and $\rho^0(1700)$ masses and widths

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It is demonstrated that Gounaris-Sakurai model of the pion electromagnetic form factor is based on the P-wave iso-vector $\pi\pi$ scattering phase-shift given by a generalized effective-range formula of the Chew-Mandelstam type, valid exclusively only at the elastic region up to $1\,\mathrm{GeV}^2$. Therefore the Gounaris-Sakurai model is justified to be used in a determination of the $\rho(770)$ meson parameters from existing data, however, in no case in a determination of the inelastic $\rho(1450)$ and $\rho(1700)$ resonance parameters.
We propose the pion electromagnetic form factor model found on the analyticity in the complex energy plane in which all three resonances $\rho(770)$, $\rho(1450)$, $\rho(1700)$ are defined on equal level as poles on unphysical sheets of the corresponding Riemann surface. The $\rho(770)$ meson parameters obtained in a such way coincide with the parameters obtained in the framework of the GKPY Roy-like equations analysis.

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1. Methodology and results

The cleanest determination of the ρ-meson family parameters comes from $e^+e^- \to \pi^+\pi^-$ process [1], whereby

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3t}\beta_\pi^3 \left| F_\pi^{\mathrm{EM},I=1}(t) + R e^{i\Phi} \frac{m_\omega^2}{m_\omega^2 - t - i m_\omega \Gamma_\omega} \right|^2, \tag{1.1}$$

$$\Phi = \mathrm{atan}\frac{m_\rho \Gamma_\rho}{m_\rho^2 - m_\omega^2}, \quad R = 6\frac{\Gamma^{1/2}_{\omega\to 2\pi}\Gamma^{1/2}_{\omega\to e^+e^-}}{\alpha m_\omega \beta_\pi^{3/4}}.$$

A number of theoretical properties of $F_\pi^{\mathrm{EM},I=1}(t)$ simplifies investigations of existing data (the most precise up to now are in [2, 3]) to be commonly presented in the form of $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$.

First we pay the attention to parameters [1] $m_\rho = 775.26 \pm 0.25\,\mathrm{MeV}$; $\Gamma_\rho = 149.1 \pm 0.8\,\mathrm{MeV}$ resonance $\rho(770)$, which for the most part have been specified analysing the pion EM FF by means of the extensively quoted Gonaris-Sakurai model [4] to be found by assuming that for a wide elastic energy region ($t < 1\,\mathrm{GeV}^2$) the P-wave iso-vector $\pi\pi$ scattering phase shift $\delta_1^1(s)$ satisfies a generalized effective-range formula of the Chew-Mandelstam type [5]

$$q^3/\sqrt{t}\cot\delta_1^1(t) = a + bt + h(t), \qquad h(t) = \frac{2}{\pi}\frac{q^3}{\sqrt{t}}\ln\left(\frac{\sqrt{t}+2q}{2}\right), \tag{1.2}$$

where $q = \sqrt{\frac{t-4}{4}}$, $m_\pi = 1$.

On the other hand, defining $\rho(770)$ resonance as a pole on unphysical sheets of the Reimann surface and exploiting all known theoretical properties of the pion EM FF, the Unitary & Analytic (U&A) model has been constructed [6] on the four sheeted Riemann surface, which analysing existing data on $e^+e^- \to \pi^+\pi^-$ process produces the $\rho(770)$-meson parameters $m_\rho = 763.85 \pm 0.20\,\mathrm{MeV}$; $\Gamma_\rho = 144.18 \pm 0.10\,\mathrm{MeV}$ to be lower in comparison with the parameters obtained by the Gounaris-Sakurai expression.

Which of these sets of $\rho(770)$-meson parameters are correct?

For a solution of this puzzle the fully solvable mathematic problem, elaborated in [7, 8], is applied.

The latter method provides the $\rho(770)$-meson parameters from up to now the most precise P-wave iso-vector $\pi\pi$ scattering phase shift data (see Fig. 1) with theoretical errors, which is generated from all existing data on $\delta_1^1(t)$ by the Garcia-Martin-Kamiński-Peláez-Yndurain Roy-like equations with imposed crossing symmetry condition.

The analyticity of $F_\pi(t)$ in t-plane with $F_\pi(t)\big|_{|t|\to\infty} \sim t^{-1}$ (through Cauchy formula) allows to derive dispersion relation with one subtraction at $t = 0$, which in combination with the elastic $F_\pi(t)$ unitarity condition leads to phase representation with one subtraction

$$F_\pi(t) = P_n(t)\exp\left[\frac{t}{4}\int_4^\infty \frac{\delta_1^1(t)}{t'(t'-t)}\mathrm{d}t'\right]. \tag{1.3}$$

As the $t = 4m_\pi^2$ is a square-root type, by means of $q = \sqrt{\frac{t-4}{4}}$, two-sheeted Riemann surface of $F_\pi(t)$ is mapped into one q-plane in elastic region of which $F_\pi(q)$ has only "poles" and "zeros".

PoS(EPS-HEP2017)730

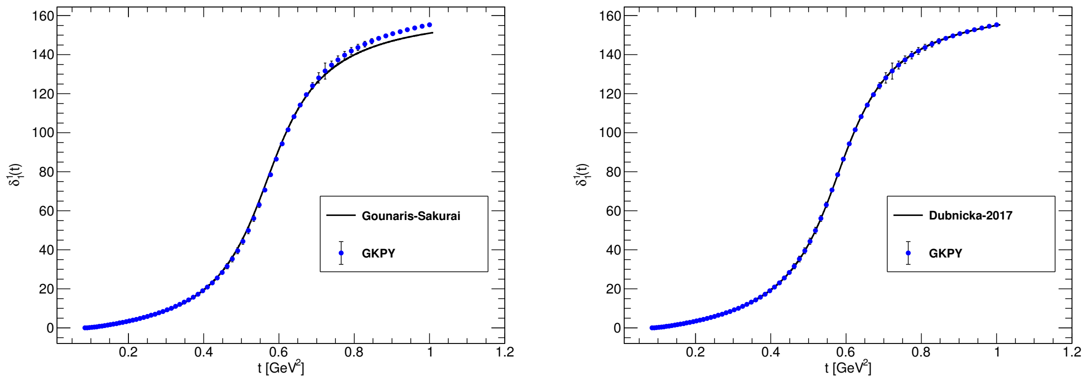

Figure 1: GKPY phase shift $\delta_1^1(s)$ data and the fits of Gounaris-Sakurai model (left) and our 5–parametric model of $\delta_1^1(s)$ (right).

Such considerations lead to the model independent $\delta_1^1(t)$ phase representation

$$\delta_1^1(q) = \arctan \frac{A_3 q^3 + A_5 q^5 + \dots}{1 + A_2 q^2 + A_4 q^4 + \dots} \tag{1.4}$$

$A_1 \equiv 0$ in order to secure the threshold behaviour of $\delta_1^1(q)$. An optimal description (the full line in Fig. 1) is achieved with nonzero coefficients A_2, A_3, A_4, A_5 with $\chi^2/\mathrm{ndf} = 0.0244$.

Substituting an equivalent form to (1.4)

$$\delta_1^1(q) = \frac{1}{2\mathrm{i}} \ln \frac{(1 + A_2 q^2 + A_4 q^4) + \mathrm{i}(A_3 q^3 + A_5 q^5)}{(1 + A_2 q^2 + A_4 q^4) - \mathrm{i}(A_3 q^3 + A_5 q^5)} \tag{1.5}$$

into (1.3), one finds an explicit algebraic form of $F_\pi(q)$ and from it $m_\rho = 763.56 \pm 0.51\,\mathrm{MeV}$ and $\Gamma_\rho = 143.09 \pm 0.82\,\mathrm{MeV}$.

These values, obtained from $\delta_1^1(q)$ in Fig. 1 by a completely model independent way, unambiguously confirm correctness of the parameters obtained by U&A model of $F_\pi(t)$.

An excess of the parameters from Gounaris-Sakurai formula can be naturally explained by the fact, that (1.2), which is a background of the Gounaris-Sakurai formula, is able to describe the data on $\delta_1^1(t)$ optimally only with $\chi^2/\mathrm{ndf} = 2.4499$.

If Gounaris-Sakurai model of $F_\pi^{\mathrm{EM},I=1}(t)$ is unable to provide correct values of the elastic $\rho(770)$-meson parameters, hardly one can expect good results by means of its generalization

$$F_\pi(t) = \frac{1}{1+\beta+\gamma}\left[\mathrm{BW}^{\mathrm{GS}}_{\rho(770)}(t).\left(1 + \delta \frac{t}{m_\omega^2}\mathrm{BW}_\omega(t)\right) + \beta \mathrm{BW}^{\mathrm{GS}}_{\rho(1450)}(t) + \gamma \mathrm{BW}^{\mathrm{GS}}_{\rho(1700)}(t)\right], \tag{1.6}$$

for excited states $\rho(1450)$ and $\rho(1700)$ far away in inelastic region.

The results for all three ρ'-meson resonances by (1.6) an the U&A $F_\pi^{\mathrm{EM},I=1}(t)$ model are presented in Table 1.

	m_{ρ^0} **[MeV]**	$m_{\rho^{0\prime}}$ **[MeV]**	$m_{\rho^{0\prime\prime}}$ **[MeV]**
G.-S.	774.81 ± 0.11	1497.60 ± 1.74	1848.40 ± 4.86
U&A	763.85 ± 0.04	1326.61 ± 3.46	1770.41 ± 5.56
PDG	775.26 ± 0.25	1465.00 ± 25.00	1720.00 ± 20.00
	Γ_{ρ^0} **[MeV]**	$\Gamma_{\rho^{0\prime}}$ **[MeV]**	$\Gamma_{\rho^{0\prime\prime}}$ **[MeV]**
G.-S.	149.21 ± 0.21	441.87 ± 8.48	322.12 ± 16.03
U&A	144.18 ± 0.01	325.89 ± 12.30	263.51 ± 11.26
PDG	149.10 ± 0.80	400.00 ± 60.00	250.00 ± 100.00

Table 1: Numerical values of masses and widths for ρ-meson resonances for Gounaris-Sakurai, Unitary & Analytic model and PDG average values.

Conclusions

Based on the recent P-wave iso-vector $\pi\pi$ scattering phase-shift data, we showed the limitation of Gounaris-Sakurai model to describe them, in comparison with our model independent way.

In the case of e^+e^- data Gounaris-Sakurai model is valid only in the region below 1 GeV^2 and it cannot be used for higher ρ-meson resonances $\rho(1450)$, $\rho(1700)$ as the received parameters from Gounaris-Sakurai model are higher than in U&A model of π electromagnetic form factor.

We can conclude that our approach independently interprets the data for the phase shift δ_1^1 and cross-section (alias π form factor) data with the similar central value of m_ρ and Γ_ρ, and Gounaris-Sakurai model is applicable only on $\rho(770)$ resonance.

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