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Polarization effects in the reactions $p+{}^3He \to \pi^+ + {}^4He$, $\pi^+ + {}^4He \to p+{}^3He$ and quantum character of spin correlations in the final $(p, {}^3He)$ system

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The general consequences of T invariance for the direct and inverse binary reactions $a+b \to c+d$, $c+d \to a+b$ with spin-1/2 particles a, b and unpolarized particles c, d are considered. Using the formalism of helicity amplitudes, the polarization effects are studied in the reaction $p+{}^3He \to \pi^+ + {}^4He$ and in the inverse process $\pi^+ + {}^4He \to p+{}^3He$. It is shown that in the reaction $\pi^+ + {}^4He \to p+{}^3He$ the spins of the final proton and ${}^3He$ nucleus are strongly correlated. A structural expression through helicity amplitudes, corresponding to arbitrary emission angles, is obtained for the correlation tensor. It is established that in the reaction $\pi^+ + {}^4He \to p+{}^3He$ one of the "classical" incoherence inequalities of the Bell type for diagonal components of the correlation tensor is necessarily violated.

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1. Consequences of T invariance for binary reactions

Let us consider the reaction $a+b \to c+d$, where a and b are the spin-1/2 particles and the particles c, d have arbitrary spins. The structure of the effective cross-section $\sigma_{a+b\to c+d}$ in the c.m. frame of the particles a and b, summed over spin projections of the final particles c and d, is as follows [1]:

$$\sigma_{a+b\to c+d}(\mathbf{k}_a, \mathbf{P}^{(a)}, \mathbf{P}^{(b)}; \mathbf{k}_c) = \sigma_0(E,\theta)\, L(\mathbf{k}_a, \mathbf{P}^{(a)}, \mathbf{P}^{(b)}; \mathbf{k}_c), \tag{1.1}$$

where $\sigma_0(E,\theta)$ is the respective cross-section for unpolarized particles a, b and L is the linear function of the polarization vectors $\mathbf{P}^{(a)}$ and $\mathbf{P}^{(b)}$:

$$L(\mathbf{k}_a, \mathbf{P}^{(a)}, \mathbf{P}^{(b)}; \mathbf{k}_c) = 1 + A(E,\theta)(\mathbf{P}^{(a)}\mathbf{n}) + B(E,\theta)(\mathbf{P}^{(b)}\mathbf{n}) + C(E,\theta)(\mathbf{P}^{(a)}\mathbf{P}^{(b)}) +$$
$$+ D(E,\theta)(\mathbf{P}^{(a)}\mathbf{l})(\mathbf{P}^{(b)}\mathbf{l}) + F(E,\theta)(\mathbf{P}^{(a)}\mathbf{m})(\mathbf{P}^{(b)}\mathbf{m}) +$$
$$+ G(E,\theta)(\mathbf{P}^{(a)}\mathbf{l})(\mathbf{P}^{(b)}\mathbf{m}) + H(E,\theta)(\mathbf{P}^{(a)}\mathbf{m})(\mathbf{P}^{(b)}\mathbf{l}). \tag{1.2}$$

Here $\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$ are the mutually orthogonal unit vectors, defined as:

$$\mathbf{l} = \mathbf{k}_a/k_a; \quad \mathbf{m} = \frac{\mathbf{l}' - \mathbf{l}(\mathbf{l}'\mathbf{l})}{\sin\theta}; \quad \mathbf{n} = \frac{\mathbf{l}\times\mathbf{l}'}{\sin\theta} \tag{1.3}$$

($\mathbf{l}' = \mathbf{k}_c/k_c$); $\mathbf{k}_a$, $\mathbf{k}_c$ are the respective momenta of the particles a and c, E is the total energy in the c.m. frame and $\theta = \arccos(\mathbf{l}\,\mathbf{l}')$ is the emission angle.

Meantime, for the inverse reaction $c+d \to a+b$ with the unpolarized particles c, d and the fixed polarization vectors of the final particles $\vec{\zeta}^{(a)}$, $\vec{\zeta}^{(b)}$ the effective cross-section takes, due to T invariance and the principle of detailed balance, the following form [1]:

$$\sigma_{c+d\to a+b}(\mathbf{k}_c; \mathbf{k}^{(a)}, \vec{\zeta}^{(a)}, \vec{\zeta}^{(b)}) = \frac{1}{4}\,\widetilde{\sigma}_0(E,\theta)\, L(-\mathbf{k}_a, -\vec{\zeta}^{(a)}, -\vec{\zeta}^{(b)}; -\mathbf{k}_c), \tag{1.4}$$

where

$$\widetilde{\sigma}_0(E,\theta) = \frac{4k_a^2}{k_c^2(2j_c+1)(2j_d+1)}\sigma_0(E,\theta) \tag{1.5}$$

is the cross-section of the inverse reaction, summed over the spin projections of the final particles a, b. Further, the two-particle spin density matrix $\hat{\rho}^{(a,b)}$ for the final particles a, b can also be expressed through the same function L, replacing the polarization vectors by the vector Pauli operators:

$$\hat{\rho}^{(a,b)} = \frac{1}{4}\Big[\hat{I}^{(a)}\otimes\hat{I}^{(b)} + (\mathbf{P}^{(a)}(E,\theta)\,\hat{\vec{\sigma}}^{(a)})\otimes\hat{I}^{(b)} + \hat{I}^{(a)}\otimes(\mathbf{P}^{(b)}(E,\theta)\,\hat{\vec{\sigma}}^{(b)}) +$$
$$+ \sum_{i=1}^{3}\sum_{k=1}^{3} T_{ik}(E,\theta)\,\hat{\sigma}_i^{(a)}\otimes\hat{\sigma}_k^{(b)}\Big] = \frac{1}{4}\hat{L}(-\mathbf{k}_a, -\hat{\vec{\sigma}}^{(a)}, -\hat{\vec{\sigma}}^{(b)}; -\mathbf{k}_c). \tag{1.6}$$

In Eq. (1.6) $\hat{I}^a$, $\hat{I}^{(b)}$ are two-row unit matrices, $\mathbf{P}^{(a)}(E,\theta) = -A(E,\theta)\mathbf{n}$, $\mathbf{P}^{(b)}(E,\theta) = -B(E,\theta)\mathbf{n}$ are the polarization vectors of the particles a and b,

$$T_{ik}(E,\theta) = C(E,\theta)\delta_{ik} + D(E,\theta)l_il_k + F(E,\theta)m_im_k + G(E,\theta)l_im_k + H(E,\theta)m_il_k \tag{1.7}$$

are components of the correlation tensor describing spin correlations in the final two-particle system (a,b). In doing so, all the functions A,B,C,D,F,G,H,σ_0 and the unit vectors **l**, **m**, **n** in Eqs. (1.4) – (1.7) are *the same* as for the direct reaction $a+b\to c+d$.

Thus, due to T invariance, the dependence of the effective cross-section of the direct reaction $a+b\to c+d$ upon the polarizations of initial particles *completely determines* the polarization vectors and spin correlations for the same particles a, b produced in the inverse reaction $c+d\to a+b$ with unpolarized primary particles.

2. Polarization effects in the reaction $p+{}^3He\to\pi^+ + {}^4He$

This reaction belongs to the type $1/2+1/2\to 0+0$ (the proton and ${}^3He$ nucleus have spin 1/2, π^+ and ${}^4He$ have zero spin). Thus, on account of the negative internal parity of the π^+ meson, this reaction can proceed *only from triplet* states of the system $(p,\ {}^3He)$ [2,3] (as follows from the parity and angular momentum conservation).

Let us choose the axis of the total spin quantization z along the vector $\mathbf{l}=\mathbf{k}_p/k_p$. There exist three possible triplet states of the $(p,\ {}^3He)$ system with the spin projections $+1$, -1 and 0 onto z:

$$|+1,\mathbf{l}\rangle = |+1/2,\mathbf{l}\rangle^{(p)}\otimes|+1/2,\mathbf{l}\rangle^{(He)},\quad |-1,\mathbf{l}\rangle = |-1/2,\mathbf{l}\rangle^{(p)}\otimes|-1/2,\mathbf{l}\rangle^{(He)},$$

$$|0,\mathbf{l}\rangle = \frac{1}{\sqrt{2}}\Big(|+1/2,\mathbf{l}\rangle^{(p)}\otimes|-1/2,\mathbf{l}\rangle^{(He)}+|-1/2,\mathbf{l}\rangle^{(p)}\otimes|+1/2,\mathbf{l}\rangle^{(He)}\Big). \quad (2.1)$$

The two-particle spin density matrix for the $(p,\ {}^3He)$ system is:

$$\hat\rho^{(p,He)} = \frac{1}{4}(\hat I^{(p)}+\mathbf{P}^{(p)}\vec\sigma^{(p)})\otimes(\hat I^{(He)}+\mathbf{P}^{(He)}\vec\sigma^{(He)}), \quad (2.2)$$

($\mathbf{P}^{(p)}$ and $\mathbf{P}^{(He)}$ are the independent polarization vectors). Using the technique of helicity amplitudes (the helicity amplitude $R_\lambda(E,\theta)$, $\lambda=\pm1,0$, is the amplitude of the reaction $p+{}^3He\to\pi^+ + {}^4He$ proceeding from the state $|\lambda,\mathbf{l}\rangle$ (2.1)), we may write [1]:

$$\sigma_{p+{}^3He\to\pi^+ + {}^4He} = \langle\psi|\hat\rho^{(p,He)}|\psi\rangle = \sum_\lambda\sum_{\lambda'} R_\lambda(E,\theta)\langle\lambda,\mathbf{l}|\,\hat\rho^{(p,He)}\,|\,\lambda',\mathbf{l}\rangle R^*_{\lambda'}(E,\theta); \quad (2.3)$$

$$|\psi\rangle = \sum_{\lambda=\pm1,0} R^*_\lambda(E,\theta)|\lambda,\mathbf{l}\rangle =$$

$$= R^*_1(E,\theta)\Big(|+1/2,z\rangle^{(p)}\otimes|+1/2,z\rangle^{(He)}-|-1/2,z\rangle^{(p)}\otimes|-1/2,z\rangle^{(He)}\Big)+$$

$$+\frac{1}{\sqrt{2}}R^*_0(E,\theta)\Big(|+1/2,z\rangle^{(p)}\otimes|-1/2,z\rangle^{(He)}+|-1/2,z\rangle^{(p)}\otimes|+1/2,z\rangle^{(He)}\Big) \quad (2.4)$$

is the non-normalized initial two-particle spin state selected by the reaction (due to the parity conservation, $R_{+1}(E,\theta)=-R_{-1}(E,\theta)\equiv R_1(E,\theta)$).

Finally, using Eq. (2.4) and the formula (2.2) for the spin density matrix, we find that the cross-section $\sigma_{p+{}^3He\to\pi^+ + {}^4He}$ (2.3) is described by the general structural formula for $\sigma_{a+b\to c+d}$ (1.1) – (1.2), where the functions σ_0, A, B, C, D, F, G, H are bilinear combinations of the helicity amplitudes R_1, R_0 [1]:

$$\sigma_0(E,\theta) = \frac{1}{4}\langle\psi\mid\psi\rangle = \frac{1}{4}(|R_0(E,\theta)|^2+2|R_1(E,\theta)|^2), \quad (2.5)$$

$$A(E,\theta) = B(E,\theta) = \frac{1}{\sqrt{2}\sigma_0(E,\theta)} \mathrm{Im}(R_1(E,\theta)R_0^*(E,\theta)), \tag{2.6}$$

$$C(E,\theta) = 1, \quad D(E,\theta) = -\frac{|R_0(E,\theta)|^2}{2\sigma_0(E,\theta)}, \quad F(E,\theta) = -\frac{|R_1(E,\theta)|^2}{2\sigma_0(E,\theta)}, \tag{2.7}$$

$$G(E,\theta) = H(E,\theta) = \frac{1}{\sqrt{2}\sigma_0(E,\theta)} \mathrm{Re}(R_1(E,\theta)R_0^*(E,\theta)). \tag{2.8}$$

For the particular cases $\theta = 0$ and $\theta = \pi$, when $R_1(E,\theta) = 0$, the expression for cross-section $\sigma_{p+{}^3He\to\pi^+ + {}^4He}$ takes a considerably simpler form:

$$\sigma_{p+{}^3He\to\pi^+ + {}^4He} = \frac{1}{4}|R_0|^2 \left(1 + \mathbf{P}^{(p)}\mathbf{P}^{(He)} - 2(\mathbf{P}^{(p)}\mathbf{l})(\mathbf{P}^{(He)}\mathbf{l})\right). \tag{2.9}$$

3. Spin effects in the inverse reaction $\pi^+ + {}^4He \to p + {}^3He$

In the reaction $\pi^+ + {}^4He \to p + {}^3He$ the $(p, {}^3He)$ system is produced *in the triplet state only*. This state, normalized to unity, is as follows [1]:

$$|\widetilde{\psi}\rangle = \frac{1}{(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2)^{1/2}} \times$$

$$\times \Big[R_1(E,\theta) \left(|+1/2,\mathbf{l}\rangle^{(p)} \otimes |+1/2,\mathbf{l}\rangle^{(He)} - |-1/2,\mathbf{l}\rangle^{(p)} \otimes |-1/2,\mathbf{l}\rangle^{(He)} \right) +$$

$$+ \frac{1}{\sqrt{2}} R_0(E,\theta) \left(|+1/2,\mathbf{l}\rangle^{(p)} \otimes |-1/2,\mathbf{l}\rangle^{(He)} + |-1/2,\mathbf{l}\rangle^{(p)} \otimes |+1/2,\mathbf{l}\rangle^{(He)} \right) \Big], \tag{3.1}$$

and it is symmetric under the interchange of spin quantum numbers of the proton and ${}^3He$. The state $|\widetilde{\psi}\rangle$ (3.1) is similar in structure to the initial triplet state $|\psi\rangle$ selected by the direct reaction $p + {}^3He \to \pi^+ + {}^4He$ (see Section 2), but differs by complex conjugation of helicity amplitudes.

Basing on the T invariance (see Section 1), we obtain [1]:

1) For the effective cross-section of the reaction $\pi^+ + {}^4He \to p + {}^3He$ in the c.m. frame, summed over the spin projections in the final state:

$$\widetilde{\sigma}_0(E,\theta) = (k_p/k_\pi)^2 (|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2); \tag{3.2}$$

2) For the polarization vectors of the proton and ${}^3He$ in the final system:

$$\mathbf{P}^{(p)}(E,\theta) = \langle\widetilde{\psi}|\,\hat{\vec{\sigma}}^{(p)}\,|\widetilde{\psi}\rangle = \mathbf{P}^{(He)}(E,\theta) = \langle\widetilde{\psi}|\,\hat{\vec{\sigma}}^{(He)}\,|\widetilde{\psi}\rangle =$$

$$= -A(E,\theta)\mathbf{n} = -2\sqrt{2}\,\frac{\mathrm{Im}(R_1(E,\theta)R_0^*(E,\theta))}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}\,\mathbf{n}; \tag{3.3}$$

3) For the correlation tensor of the $(p, {}^3He)$ system:

$$T_{ik}(E,\theta) = \langle\widetilde{\psi}|\,\hat{\sigma}_i^{(p)}\hat{\sigma}_k^{(He)}\,|\widetilde{\psi}\rangle = \delta_{ik} - \frac{2}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2} \times$$

$$\times \Big[|R_0(E,\theta)|^2 l_i l_k + 2|R_1(E,\theta)|^2 m_i m_k - \sqrt{2}\,\mathrm{Re}(R_1(E,\theta)R_0^*(E,\theta))(l_i m_k + m_i l_k) \Big]. \tag{3.4}$$

In all the expressions (3.1) – (3.4), the helicity amplitudes $R_0(E,\theta)$, $R_1(E,\theta)$ and the unit vectors **l**, **m**, **n** are *the same* as for the previously considered direct process $p + {}^3He \to \pi^+ + {}^4He$.

In accordance with Eqs. (3.3), (3.4): 1) the polarization of the ${}^3He$ nucleus along the normal to the reaction plane is identical to that of the proton; 2) the correlation tensor $T_{ik}(E,\theta)$, describing the spin correlations in the $(p,\, {}^3He)$ system, is symmetric. Thus, the spins of the proton and the ${}^3He$ nucleus in the reaction $\pi^+ + {}^4He \to p + {}^3He$ must be *tightly correlated*, which enables one, in principle, to prepare a beam of ${}^3He$ nuclei with controllable polarization without acting directly on these nuclei (see [1] for more details).

4. Violation of the incoherence inequalities for the correlation tensor

As it was established in the paper [4], in the case of incoherent mixtures of factorizable two-particle states of spin-1/2 fermions the following inequalities for the diagonal components of the correlation tensor should be satisfied:

$$|T_{11}+T_{22}+T_{33}| \le 1; \quad |T_{11}+T_{22}| \le 1; \quad |T_{11}+T_{33}| \le 1; \quad |T_{22}+T_{33}| \le 1. \tag{4.1}$$

However, for *non-factorizable* quantum-mechanical superpositions these inequalities may be violated. The triplet state $|\widetilde{\psi}\rangle$ (3.1) of the final system in the reaction $\pi^+ + {}^4He \to p + {}^3He$ represents a characteristic example of such non-factorizable spin states (it is well seen that the state $|\widetilde{\psi}\rangle$ cannot be reduced to the product of one-particle spin states).

Let us calculate the components of the correlation tensor T_{ik} (3.4) for the system $(p,\, {}^3He)$ in the coordinate frame with $z \parallel$ **l**, $x \parallel$ **m**, $y \parallel$ **n**. Finally, we obtain the following expressions :

$$T_{11} = \frac{|R_0(E,\theta)|^2 - 2|R_1(E,\theta)|^2}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}; \quad T_{22} = 1; \quad T_{33} = \frac{2|R_1(E,\theta)|^2 - |R_0(E,\theta)|^2}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2} = -T_{11}; \tag{4.2}$$

$$T_{13} = T_{31} = \frac{2\sqrt{2}}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}\,\mathrm{Re}(R_1 R_0^*); \quad T_{12} = T_{21} = T_{23} = T_{32} = 0; \tag{4.3}$$

(indexes: $1 \to x$, $2 \to y$, $3 \to z$); in doing so, $tr(T) = 1$) .

Thus, as follows from Eq. (4.2), in the reaction $\pi^+ + {}^4He \to p + {}^3He$ *one* of the incoherence inequalities (4.1) for the diagonal components of the correlation tensor is *necessarily* violated, irrespective of the concrete mechanism of generation of the system $(p,\, {}^3He)$. Indeed, if $|R_0|^2 > 2|R_1|^2$, we obtain that $|T_{11}+T_{22}| > 1$; if, on the contrary, $|R_0|^2 < 2|R_1|^2$, then $|T_{22}+T_{33}| > 1$. Meantime, in both the cases the other three incoherence inequalities (4.1) are satisfied.

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