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The long string at the stretched horizon and the entropy of large non-extremal black holes

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Highly excited or high-temperature string gases are dominated by long strings. The avatar of this process on the thermal manifold is the singly wound state (thermal scalar) that becomes massless at the critical Hagedorn temperature. This logic can be extended to curved spacetime, and in particular to Rindler space where one finds that the Hagedorn temperature equals the Hawking temperature of the black hole itself. This results in a long random walk surrounding the event horizon localized at string length distance. Long strings also make their appearance when throwing in strings into the black hole as shown by Susskind long ago. The string elongates as it approaches the horizon. We combine and compare these two pictures to discuss how the thermally equilibrated long string gas accounts for the black hole entropy, and provide a qualitative picture for Hawking radiation.

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1. Introduction and Motivation

It is known since the 80's that high energy strings tend to coalesce and form fewer long strings instead. This can be demonstrated from simple entropic arguments. More sophisticated arguments utilize a thermal ensemble of strings, where long string dominance was also demonstrated. An at first sight unrelated context where long strings appear naturally occurs when throwing strings into a black hole. As shown by Susskind [1], strings elongate due to having only a finite detector resolution: the string's internal fluctuations slow down according to asymptotic observers and one effectively 'sees' more of the string.

These two origins of long strings (high excitation and finite detector resolution) can be nicely summarized into a formula for the transverse length of a closed string in d flat spacetime dimensions:

$$\ell \equiv \langle N,\bar{N}| \int_0^{2\pi} d\sigma \sqrt{\left|\sum_i \frac{\partial X^i}{\partial \sigma}\right|^2} |N,\bar{N}\rangle \sim \sqrt{\alpha'}\sqrt{(d-2)\sum_{n=1}^{+\infty} n + N + \bar{N}}, \tag{1.1}$$

the first term requires a cut-off and is insensitive to string excitation. The second part is the high energy contribution: $\ell \sim \sqrt{\alpha' N} \sim E\alpha'$.

As well-known, the thermal manifold carries the same information as the real-time thermal ensemble. From that perspective, the long string regime is contained within the string state that is singly wound around the thermal circle, the thermal scalar. It becomes massless and dominates thermodynamics at high temperature when $\beta \approx \beta_H = 2\sqrt{2}\pi\sqrt{\alpha'}$ (type II), the (inverse) Hagedorn temperature. We apply this perspective of the thermal scalar to Rindler space, the near-horizon approximation to generic black holes, and will find long strings close to horizons.

2. The thermal scalar in Rindler space

The thermal scalar action in Rindler space $ds^2 = a^2\rho^2 dt^2 + d\rho^2 + d\mathbf{x}_\perp^2$ is [2][3][4]

$$S = \int d^{d-1}x\sqrt{G}e^{-2\Phi}\left[G_{ij}\nabla^i\phi\nabla^j\phi^* + \frac{1}{4\pi^2\alpha'^2}\left(\beta^2 G_{00} - \beta_H^2\right)\phi\phi^*\right]. \tag{2.1}$$

At $\beta = \beta_R = 2\pi/a$, the Rindler temperature, the lowest eigenmode of this action becomes a (normalizable) zero-mode concentrated around $\rho \sim \sqrt{\alpha'}$: $\phi_0 \sim \exp\left(-\frac{\rho^2}{2\alpha'}\right)$. This means the Hagedorn temperature in this space equals the Rindler temperature; and the thermal scalar describes the long string as a random walk. It dominates the free energy and the density of states:

$$\beta F = -\int_0^{+\infty} \frac{dE}{E} e^{\frac{2\pi E}{a}} e^{-\beta E}, \qquad \rho(E) = \frac{e^{\beta_R E}}{E}. \tag{2.2}$$

In the microcanonical ensemble, the dominant contribution at very high energy E of the energy density $\varepsilon(\rho)$ and radial pressure $p(\rho)$ of the string gas are given by [5]

$$\varepsilon(\rho) = 2\frac{E}{Aa\alpha'}\left(\frac{2\rho^2}{\alpha'} - 1\right)e^{-\rho^2/\alpha'}, \qquad p(\rho) = 2\frac{E}{Aa\alpha'}e^{-\rho^2/\alpha'}, \tag{2.3}$$

with A the horizon area. For a Rindler observer, this entropic pressure gradient keeps the long strings from falling arbitrarily close towards the horizon. In this way, a stretched horizon of long strings is formed which realizes the picture of Susskind [1].

PoS(EPS-HEP2017)817

3. Entropy of large non-extremal black holes

The above conclusion can be used to derive the Bekenstein-Hawking entropy from long strings [6]. Close to the horizon, a large non-extremal black hole is described by a Rindler metric with acceleration $a = 1/4GM$. If we throw a shell of mass $\delta M \ll M$ from infinity, the shell has a Hagedorn density of states of the form of (2.2): $\rho(\delta M) \sim e^{\beta_{\text{Hawking}}\delta M}/\delta M$, with the Hagedorn temperature equal to the Hawking temperature. This is an equilibrium formula, which can only happen in string theory. In QFT, any finite energy configuration that is thrown into the black hole will perpetually fall in and will never reach thermal equilibrium with the black hole. This is reflected in the fact that in QFT the horizon causes UV divergences in thermal observables. One interpretation of this is that in the $\varepsilon_{UV} \to 0$ limit, only an infinite amount of QFT matter can reach a radial equilibrium profile.

A related observation was made by Susskind: in tortoise coordinates, QFT wavepackets propagate undeformed towards the horizon at tortoise $r^* = -\infty$. This means the packets shrinks radially, due to the unlimited amount of storage space for fields close to the horizon. Strings on the other hand spread longitudinally at precisely such a rate that compensates their forward propagation speed: the string is seen to hover outside the horizon at a distance scale $\sim \ell_s$.

From the above Hagedorn density of states, the shell adds an entropy of $\delta S = \beta_{\text{Hawking}}\delta M$ which integrates to the Bekenstein entropy:

$$\delta S = 8\pi GM\delta M \quad \to \quad S_{BH} = \frac{A}{4G}. \tag{3.1}$$

4. A comment on Hawking radiation

In flat space a highly excited string radiates zero-mass particles as a black body at the Hagedorn temperature [7]. Since the string gas in Rindler space (close to the black hole horizon) is dominated by a single long string with Hagedorn temperature = Hawking temperature, it will radiate as a black body at the Hawking temperature, providing a microscopic mechanism for Hawking radiation.

References

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