

# Determination of the strong coupling constant $\alpha_S(M_Z)$ in next-to-next-to-leading order QCD using H1 jet cross section measurements

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**Radek Žlebčik\***

on behalf of the **H1** collaboration

DESY

E-mail: [radek.zlebcik@desy.de](mailto:radek.zlebcik@desy.de)

The strong coupling constant  $\alpha_s$  is determined from inclusive jet and dijet cross sections in neutral-current deep-inelastic ep scattering (DIS) measured at HERA by the H1 collaboration using next-to-next-to-leading order (NNLO) QCD predictions. The dependence of the NNLO predictions and of the resulting value of  $\alpha_s(m_Z)$  at the Z-boson mass  $m_Z$  are studied as a function of the choice of the renormalisation and factorisation scales. Using inclusive jet and dijet data together, the strong coupling constant is determined to be  $\alpha_s(m_Z) = 0.1157(20)_{\text{exp}}(29)_{\text{th}}$ . Complementary,  $\alpha_s(m_Z)$  is determined together with parton distribution functions of the proton (PDFs) from jet and inclusive DIS data measured by the H1 experiment. The value  $\alpha_s(m_Z) = 0.1142(28)_{\text{tot}}$  obtained is consistent with the determination from jet data alone. The impact of the jet data on the PDFs is studied. The running of the strong coupling is tested at different values of the renormalisation scale and the results are found to be in agreement with expectations.

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\*Speaker.

## 1. Introduction

The strong coupling constant is one of the least known parameters of the Standard Model of particle physics (SM). Its knowledge is crucial for precision measurements, consistency tests of the SM and searches for physics beyond the SM. It has been determined in a large variety of processes and using different techniques [1]. Jet production in the Breit frame in neutral-current deep-inelastic  $ep$  scattering (NC DIS) is directly sensitive to the strong coupling and has a clean experimental signature with sizable cross sections. It is thus ideally suited for the precision determination of the strong coupling constant  $\alpha_s(m_Z)$  at the  $Z$ -boson mass  $m_Z$ .

Cross section predictions for inclusive jet and dijet production in NC DIS are obtained within the framework of perturbative QCD (pQCD), where for the past 25 years only next-to-leading order (NLO) calculations have been available [2]. The next-to-next-to-leading order (NNLO) predictions for jet production in DIS [3] and hadron-hadron collisions [4, 5] have become available recently. The theoretical uncertainties of the NNLO predictions are substantially reduced compared to those of the NLO predictions and can compete with the precision of current experimental data.

Measurements of inclusive jet and dijet cross sections in NC DIS have been performed at HERA by the H1 collaboration during different data taking periods and for different centre-of-mass energies. See Refs. [6, 7] for the most recent H1 measurements. In general, the predictions in pQCD provide a good description of these data.

There are several determinations of  $\alpha_s(m_Z)$  from jet cross section in DIS at NLO, starting with the pioneer extraction from 1994 [8] up to the most recent one from 2016 [7]. Since 2000, the data were better under control and the theoretical uncertainty from the scale variation became the main limitation of these analyses. Only recently, an  $\alpha_s$  determination was performed using inclusive jet cross sections, where NLO calculations have been supplemented with contributions beyond NLO in the threshold resummation formalism, and a moderate reduction of the scale uncertainty was achieved [9].

The presence of a proton in the initial state in lepton-hadron or hadron-hadron collisions complicates the determination of  $\alpha_s(m_Z)$  and therefore  $\alpha_s(m_Z)$  is often determined together with parton distribution functions of the proton (PDFs). Such simultaneous determinations of  $\alpha_s(m_Z)$  and PDFs were performed using jet cross sections in DIS [10] or jet cross sections at either the LHC or Tevatron [11]. However, the absence of full NNLO corrections for jet production cross sections limited the theoretical precision of these approaches.

For the first time, we present the determination of  $\alpha_s(m_Z)$  making use of the recent calculations of jet production at NNLO [3]. These calculations are also used for the determination of PDFs. For more details, see Ref. [12].

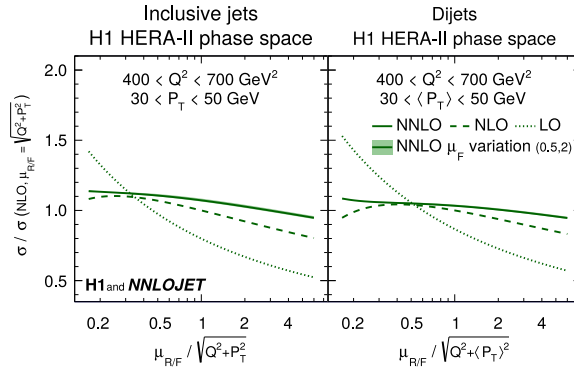
## 2. NNLO calculations

The calculations of the DIS jet cross section  $\sigma_i$  are based on the collinear factorisation theorem:

$$\sigma_i = \sum_{k=g,q,\bar{q}} \int dx f_k(x, \mu_F) \hat{\sigma}_{i,k}(x, \mu_R, \mu_F) c_{\text{had},i}, \quad (2.1)$$

where  $f_k$  are the parton distribution functions (PDFs) which depend on non-perturbative dynamics and must be determined from the experiment whereas the sub-process cross section  $\hat{\sigma}_{i,k}$  is calcula-

ble within pQCD. Factors  $c_{\text{had},i}$  correct for the hadronisation and electro-weak effects. The partonic cross sections can be directly calculated at NNLO QCD accuracy using the NNLOJET program [3], however it is very computationally intense. Therefore, to speed up the calculation, the cross section  $\sigma_i$  is evaluated by fastNLO framework [13] which allows for fast evaluation of  $\sigma_i$  for various PDFs,  $\alpha_s(m_Z)$  and QCD scale choices using the basis elements which are calculated by NNLOJET only once. By default the renormalisation and factorisation scales are considered to be both equal to  $\sqrt{Q^2 + p_T^2}$ , where  $p_T$  is the mean transverse momenta of two leading jets in case that the dijet cross sections are calculated, or it denotes the transverse momentum of the leading jet in case of the inclusive cross sections. The effect of the scale choice on cross section is shown on Fig. 1, where it is clearly visible that with increasing order of the perturbative theory the distribution become flatter and the scale uncertainty at NNLO is about half of the NLO one. Furthermore, the plots shows this size of the higher-order contributions.



**Figure 1:** The impact of the QCD scale  $\mu = \mu_R = \mu_F$  variation on jet DIS cross section at LO, NLO and NNLO. The effect of the  $\mu_F$  variation with respect to the  $\mu_R$  is for NNLO depicted by the colored band.

### 3. Fitting of $\alpha_s$

To achieve maximal sensitivity, both the inclusive jets and dijets data samples are fitted. They cover the  $Q^2$  region between  $5.5 < Q^2 < 15000 \text{ GeV}^2$  and the jet transverse momenta start from  $\sim 5 \text{ GeV}$ . For the dijet sample, to avoid the infrared unsafe region, an asymmetric cut for leading and sub-leading jet is applied. Data samples from the HERA-I data taking period with  $\sqrt{s} = 300 \text{ GeV}$  and  $320 \text{ GeV}$ , and from the HERA-II data taking period with  $\sqrt{s} = 320 \text{ GeV}$  are included. The detailed description of the data samples is given in [12].

In the fits, the  $\alpha_s$ -dependence of both the partonic cross sections and the PDFs is considered in (2.1). It was observed that the PDFs determined at the scale  $\mu = 20 \text{ GeV}$  are nearly independent on the assumption on  $\alpha_s(m_Z)$  made in the PDF fit, as there are many data points which constrain the partons densities at such scale. Consequently, the PDFs at this scale are used in the present fit of DIS jet data. By default the NNPDF 3.1 set, determined assuming  $\alpha_s(m_Z) = 0.118$  is used. The PDFs values at lower or higher scales are obtained by DGLAP evolution at NNLO using the zero-mass variable flavour number scheme (ZM-VFNS) with  $\alpha_s(m_Z)$  in the evolution chosen to be consistent with  $\alpha_s(m_Z)$  in the partonic cross section.

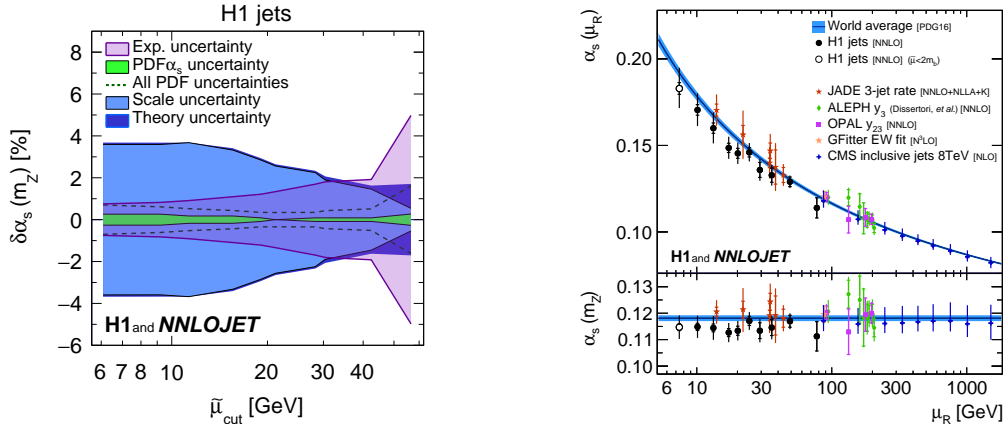
The resulting  $\alpha_s(m_Z)$  value is:

$$\alpha_s(m_Z) = 0.1157 (20)_{\text{exp}} (6)_{\text{had}} (3)_{\text{PDF}} (2)_{\text{PDF}\alpha_s} (3)_{\text{PDFset}} (27)_{\text{scale}}, \quad (3.1)$$

where the main theoretical uncertainty comes from the QCD scale variation which is defined by varying both scales,  $\mu_R$  and  $\mu_F$ , by a factor of two. The uncertainties related to the PDF selection (5 distinct PDF sets were tested), to the  $\alpha_s(m_Z)$  value of the used PDF set and to the intrinsic uncertainties of NNPDF 3.1 are all much smaller and are comparable in size with the error related to the modeling of the hadronisation effects.

It is worth to notice, that the scale uncertainty is higher at lower scales, whereas the experimental uncertainties behave in the opposite way, i.e. are largest at high scales due to declining statistical precision. This is demonstrated in Fig. 2 where the experimental uncertainty and the theoretical uncertainties of  $\alpha_s(m_Z)$  are shown as a function of  $\mu_{\text{cut}}$  parameter which specifies that only the data points with  $\sqrt{Q^2 + p_T^2} > \mu_{\text{cut}}$  are fitted. To have a trade-off between the sizes of the experimental and the scale uncertainties, a cutoff  $\mu_{\text{cut}} = 28 \text{ GeV}$  was used in (3.1).

In addition, the strong coupling constant was independently fitted in several distinct scale-regions to measure the  $\alpha_s$  running (Fig. 2). The H1 data points covering scales between 7 and 80 GeV are compared with LEP and LHC data points.



**Figure 2:** The uncertainty of the fitted  $\alpha_s(m_Z)$  value as a function of the cut-off parameter  $\mu_{\text{cut}}$  (left) and the dependence of the strong coupling constant  $\alpha_s$  on scale  $\mu_R$  (right).

#### 4. Simultaneous PDF and $\alpha_s$ fit

Alternatively  $\alpha_s(m_Z)$  is extracted simultaneously with the PDFs. Here, the PDFs are parametrized at the starting scale  $\mu_0 = 1.38 \text{ GeV}$  and evolved to higher scales by using NNLO DGLAP evolution. The fitting procedure is similar to the approach used in H1PDF2012 [14] or HERAPDF2.0 [10]. In addition to the inclusive NC and CC data sets used in H1PDF2012, jet data sets normalised to the inclusive NC cross sections are included [7, 15, 6]. For the inclusive DIS predictions both scales  $\mu_{R,F}$  are set to be equal to  $Q$  and for the jet observables the same scale as in the previous section is used. The uncertainties are obtained by varying the scale by factor of 0.5 or 2 simultaneously in all calculations.

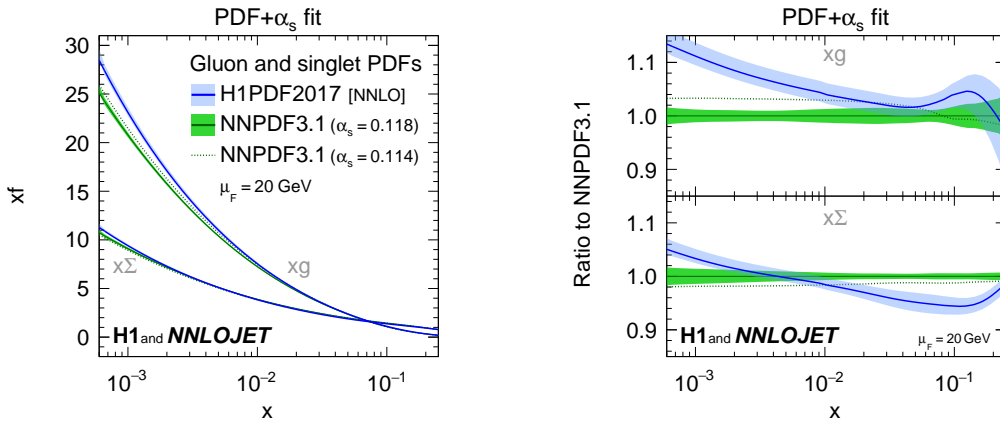
To exclude the region where the pQCD cannot be reliably applied, the photon virtuality  $Q^2$  is restricted to  $Q^2 > 10\text{GeV}^2$  and for the jet cross sections only data points with  $\mu_{\text{cut}} > 2m_b$  are included. Consequently, most of the data points are within  $x = 0.1 - 0.5$ .

The fit yields  $\chi^2/\text{ndf} = 1539.7/(1529 - 13)$ , confirming good agreement between the predictions and the data. The resulting PDFs are able to describe 141 jet data points and the inclusive DIS data simultaneously. The obtained  $\alpha_s(m_Z)$  value

$$\alpha_s(m_Z) = 0.1142 (11)_{\text{exp,had,PDF}} (2)_{\text{mod}} (2)_{\text{par}} (26)_{\text{scale}} \quad (4.1)$$

is consistent with the result in section 3. The uncertainty is dominated by the QCD scale which is much higher than the other uncertainties, e.g. of the parametrisation.

The obtained PDFs, denoted as H1PDF2017, are shown in Fig. 3. It can be seen that at scale



**Figure 3:** The gluon and quark-singlet component of the H1PDF2017 obtained from the simultaneous fit of the inclusive and jet data in DIS. The PDFs are compared to NNPDF 3.1.

20 GeV the PDFs are comparable to NNPDF 3.1 for higher  $x$ , while the gluon components differ for smaller values of  $x$ . This difference cannot be explained by the different  $\alpha_s(m_Z)$  value in the PDF evolution in both fits but can be related to the different choices of data sets used in the fits.

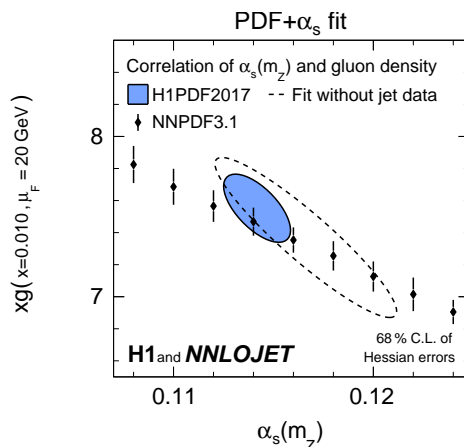
It is worth noting, that there is a correlation between the gluon density and the  $\alpha_s$  value. Figure 4 demonstrates that inclusion of the jet data into the fit significantly improves precision of both these quantities and reduces their correlation.

## 5. Conclusion

The new NNLO pQCD calculations for jet production cross sections in neutral-current DIS are exploited for a determination of the strong coupling constant  $\alpha_s(m_Z)$  using inclusive jet and dijet cross section measurements published by the H1 collaboration. Two methods are explored to determine the value of  $\alpha_s(m_Z)$ .

In the first approach H1 inclusive jet and dijet data are analysed and the external PDFs are used. The strong coupling constant is determined to be  $\alpha_s(m_Z) = 0.1157 (20)_{\text{exp}} (29)_{\text{th}}$ , where the jet data are restricted to scales  $\mu_{\text{cut}} > 28\text{ GeV}$ .

In the second approach the combined determination of parton densities and  $\alpha_s(m_Z)$  is performed. In this fit the H1 inclusive data and the jet data normalised to the inclusive neutral-current



**Figure 4:** The correlation between the  $\alpha_s(m_Z)$  and the gluon component of the PDFs for  $x = 0.01$  and  $\mu_F = 20 \text{ GeV}$  for fit with/without jet data included. As a reference, the NNPDF 3.1 data points are further displayed, although in their determination the value of  $\alpha_s(m_Z)$  was fixed.

cross section are analysed. The obtained strong coupling value  $\alpha_s(m_Z) = 0.1142(28)_{\text{tot}}$  is compatible the value obtained by the other approach and both these values are compatible with the world average. We showed, that inclusion of the jet data reduced the uncertainties of the resulting PDFs. In particular, it better constrains the gluon component which is strongly correlated with the strong coupling constant when using inclusive DIS data alone.

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