$\gamma \to \rho^0$ impact factor in QCD: light-cone sum rule calculation

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We discuss the $\gamma \to \rho^0$ impact factor, i.e., the transition amplitude of a photon to a neutral vector meson ρ^0, where the transition is mediated by the two t-channel gluons. The impact factor is a building block in the QCD descriptions of high-energy exclusive processes like $\gamma p \to \rho^0 p$, and $\gamma\gamma \to \rho^0\rho^0$, in particular, for the forward production of ρ^0. The impact factor for the longitudinal polarization of the vector meson obeys the QCD factorization, while the factorization is known to break down for the transverse polarization, indicating that the impact factor for the transversely polarized vector meson is dominated by the "non-factorizable" soft contributions. We study the $\gamma \to \rho^0$ impact factor constructing the light-cone QCD sum rules for the corresponding amplitudes, which allow us to estimate the relevant soft contributions in a largely model-independent way, with the use of dispersion relations and quark-hadron duality.

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1. Introduction

In the framework of QCD factorization, the impact factor is expressed as a convolution of the partonic amplitude, $\gamma g \to q\bar{q}g$, in perturbation theory and the quark-antiquark light-cone distribution amplitudes for a vector meson ρ^0. The corresponding factorization formula has been successfully derived for the case of the longitudinal polarization of ρ^0. For the transverse polarization of ρ^0, however, the corresponding formula is associated with the higher twist (twist-three) contributions and the factorization is known to break down due to infrared divergences which manifest themselves as endpoint singularities arising in the convolution integral; the impact factor for the transversely polarized vector meson is dominated by the "non-factorizable" soft contributions.

We study the $\gamma \to \rho^0$ impact factor constructing the light-cone QCD sum rules for the corresponding amplitudes, with the use of dispersion relations and quark-hadron duality. As a result, we are able to obtain the finite result for the impact factor with the transversely polarized ρ^0 meson, which we denote as "ρ_T^0", as well as for ρ_L^0 with the longitudinal polarization. We compare our results with the approach based on the vector meson dominance model associated with the pomeron exchange. As an application, we calculate the cross sections for $\rho^0\rho^0$ production in two-photon collisions, in particular, the cross sections for $\gamma\gamma \to \rho_L^0\rho_T^0$, i.e., the production of the two ρ^0 mesons with different (longitudinal and transverse) polarizations, where each ρ^0 meson is produced in the forward direction of each photon beam. This may be measured in, e.g., Belle II experiment.

2. Impact representation for $\gamma\gamma \to VV$

An example of the high-energy exclusive processes, where the $\gamma \to V$ impact factor (V denotes a neutral vector meson, $V = \rho^0, \omega, \phi, \ldots$) arises as a building block in their QCD descriptions, is $\gamma\gamma \to VV$ as represented in Fig. 1 of [1], and the impact factor is the transition amplitude of a photon to V, with the transition being mediated by the two t-channel gluons. As demonstrated in Appendix A of [1], the leading contribution to the amplitude of $\gamma\gamma \to VV$ for the forward production of the vector mesons at high energy, i.e., at $s \gg -t \gg \Lambda_{\rm QCD}^2$, obeys the "impact representation",

$$\mathfrak{M}_{\gamma\gamma\to VV} = is \int \frac{J_{\gamma V}^{(1)}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) J_{\gamma V}^{(2)}(-\boldsymbol{k}_\perp, -\boldsymbol{\Delta}_\perp)}{\boldsymbol{k}_\perp^2 (\boldsymbol{k}_\perp - \boldsymbol{\Delta}_\perp)^2} \frac{d^2 k_\perp}{(2\pi)^2}\,, \tag{2.1}$$

up to the corrections suppressed by powers of $1/s$, where $s = (q_1+q_2)^2 = 2q_1\cdot q_2$, with q_1, q_2 being the 4-momenta of the two colliding photons ($q_1^2 = q_2^2 = 0$), and $t = \Delta^2 = -(s/2)(1-\cos\theta)$, with Δ_μ and θ being the associated 4-momentum transfer and center-of-mass scattering angle. The kinematics to allow the final-state vector mesons, with $(q_1+\Delta)^2 = m_V^2$ and similarly for $q_2 - \Delta$, implies $t \simeq -\boldsymbol{\Delta}_\perp^2$; then, the propagators of the two t-channel gluons give the denominator of (2.1) for the dominant integration region corresponding to $k_0^2 - k_3^2 \ll \boldsymbol{k}_\perp^2$, and each of the two parts of the $\gamma \to V$ transition, interconnected by those gluons, is given by the impact factor $J_{\gamma V}^{(1)}$ ($J_{\gamma V}^{(2)}$), as

$$J_{\gamma V}^{(1)}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = -i \int_{-\infty}^{\infty} \frac{d\alpha_k}{2\pi} \frac{q_2^\mu q_2^\nu}{s} \mathcal{A}_{\mu\nu}\left(\gamma(q_1)g(k) \to V(q_1+\Delta)g(k-\Delta)\right)\Big|_{\beta_k\to 0}\,, \tag{2.2}$$

using the photon-gluon (γg) scattering amplitude, $\mathcal{A}_{\mu\nu}\left(\gamma(q_1)g(k) \to V(q_1+\Delta)g(k-\Delta)\right)$, producing the vector meson V in the final state; here μ (ν) is the Lorentz index for the initial (final) gluon and the Sudakov variables are introduced for the momentum of the initial gluon as

$k = \alpha_k q_2 + \beta_k q_1 + k_\perp$. As indicated explicitly in (2.2), the leading term in the Taylor expansion with respect to β_k leads to the leading contribution in powers of $1/s$. The integral over α_k of (2.2) is performed by closing the integration contour at the lower half-plane and yields the discontinuity of the integrand. $J^{(2)}_{\gamma V}(-\boldsymbol{k}_\perp, -\boldsymbol{\Delta}_\perp)$ is given by (2.2) with the formal substitutions, $q_1 \leftrightarrow q_2$, $\alpha_k \leftrightarrow \beta_k$, $k \to -k$, and $\Delta \to -\Delta$. The contraction with $q_2^\mu q_2^\nu/s$ in (2.2) and the similar contraction with $q_1^\lambda q_1^\sigma/s$, arising in $J^{(2)}_{\gamma V}$, come from the decomposition of the metric tensor in the numerator of the gluon propagators into the lightlike and orthogonal directions as $g^{\mu\lambda} = (2/s)(q_2^\mu q_1^\lambda + q_2^\lambda q_1^\mu) + g_\perp^{\mu\lambda}$ and making the replacement, $g^{\mu\lambda} g^{\nu\sigma} \to (2/s) q_2^\mu q_1^\lambda (2/s) q_2^\nu q_1^\sigma$, to project onto the leading contribution for large s. We note that (2.2) thus obtained is independent of s, as emphasized in [1].

The averaging for the color indices a, b of the initial and final gluons is assumed using $(\lambda^a/2) \times (\lambda^b/2) \to \delta^{ab}/(2N_c)$ in (2.2), as well as in $J^{(2)}_{\gamma V}$, with λ^a being the corresponding color Gell-Mann matrices for N_c color, such that the two gluons are projected onto the colorless t-channel state, which may correspond to QCD pomeron with the vacuum quantum numbers. As well known, the impact factor (2.2) plays roles also in other high-energy processes, such as $\gamma p \to V p$, producing a vector meson in the forward direction.

3. QCD factorization for $\gamma \to \rho^0$ impact factor

The high-energy γg scattering amplitude $\mathcal{A}_{\mu\nu}(\gamma(q_1) g(k) \to V(q_1 + \Delta) g(k - \Delta))$ of (2.2) receives the contribution of the partonic amplitude, $\gamma g \to q\bar{q} g$, in perturbation theory as expressed in Fig. 2 of [1], and its convolution with the nonperturbative hadronization process of $q\bar{q}$ into V, which is represented by the quark-antiquark light-cone distribution amplitudes (DAs) for the vector meson V, provides the QCD factorization formula for the impact factor (2.2). The corresponding formula, with the final-state ρ^0 meson being longitudinally polarized ("ρ_L^0"), was calculated in [1], and it is straightforward to extend the result for the case with the replacement $\gamma(q_1) \to \gamma^*(q)$ with $q = q_1 - (Q^2/s) q_2$, such that the the initial-state photon has the virtuality $q^2 \equiv -Q^2$ (< 0), as [2, 3],

$$J^{(1)}_{\gamma^* \rho_L^0}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)\Big|_{\rm LO} = \sqrt{4\pi\alpha_{\rm em}}\, 4\pi\alpha_s \frac{\delta^{ab}}{2N_c} \frac{f_\rho}{\sqrt{2}} \int_0^1 du\, \phi_\parallel(u) \frac{2u-1}{2} \left\{ \frac{\boldsymbol{e}_\gamma \cdot \boldsymbol{\Delta}_\perp}{u \boldsymbol{\Delta}_\perp^2 + (1-u) Q^2} \right.$$
$$\left. - \frac{\boldsymbol{e}_\gamma \cdot \boldsymbol{\Delta}_\perp}{(1-u)\boldsymbol{\Delta}_\perp^2 + u Q^2} - \frac{\boldsymbol{e}_\gamma \cdot (u\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp)}{(u\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp)^2 + u(1-u) Q^2} + \frac{\boldsymbol{e}_\gamma \cdot [(1-u)\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp]}{[(1-u)\boldsymbol{\Delta}_\perp - \boldsymbol{k}_\perp]^2 + u(1-u) Q^2} \right\}, \quad (3.1)$$

in the leading order (LO) in QCD perturbation theory, where $e_\gamma^\mu = (0, \boldsymbol{e}_\gamma)$ denotes the polarization vector of $\gamma^*(q)$, which is assumed, here and in the following, to be transversely polarized, and we follow the definition of the decay constant f_ρ and the DA $\phi_\parallel(u)$ for the ρ meson given in [4], with u and $1-u$ denoting the longitudinal momentum fraction carried by the quark and antiquark, respectively. We may also calculate similarly the impact factor with the virtual photon $\gamma^*(q)$ and the transversely polarized ρ_T^0, as $J^{(1)}_{\gamma^* \rho_T^0}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)\big|_{\rm LO}$, whose formula is formally similar to (3.1), but, instead of $\phi_\parallel(u)$, the corresponding DAs for the transversely polarized ρ_T^0 participate, and these are actually the two independent DAs of twist three, $g_\perp^{(v)}(u)$ and $g_\perp^{(a)}(u)$; see [4] for their definitions and properties in QCD. It is worth noting that $\phi_\parallel(u)$ in (3.1) is the twist-two DA for the longitudinally polarized ρ_L^0 and we have another twist-two DA, $\phi_\perp(u)$, for ρ_T^0 [4]. However, because $\phi_\perp(u)$ is of chiral-odd, its contribution to $J^{(1)}_{\gamma^* \rho_T^0}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)\big|_{\rm LO}$ is accompanied by the quark mass m_q, as in [1], and, therefore, is strongly suppressed compared with the above-mentioned twist-three contribution associated with $g_\perp^{(v,a)}(u)$. We also note that the derivative, $\partial g_\perp^{(a)}(u)/\partial u$, arises in $J^{(1)}_{\gamma^*\rho_T^0}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)\big|_{\rm LO}$.

When $Q^2 \to 0$, the formula (3.1) has the well-behaved and finite limit which reproduces the LO impact factor for the real photon, $J^{(1)}_{\gamma\rho_L^0}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)\big|_{\rm LO}$, derived in [1], while we find, $J^{(1)}_{\gamma^*\rho_T^0}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)\big|_{\rm LO} \sim \log(Q^2/\boldsymbol{\Delta}_\perp^2)$, for the ρ_T^0 case; this is due to the end-point behaviors, $g_\perp^{(v)}(u) \sim 1$, $\partial g_\perp^{(a)}(u)/\partial u \sim 1$, for $u \to 0$ and 1, while we have $\phi_\parallel(u) \sim u(1-u)$; see (4.3), (4.4) below. Note that these end-point behaviors of the DAs for the light vector mesons are direct consequences of conformal symmetry in massless QCD [4]. Thus, the factorization formula for $J^{(1)}_{\gamma^*\rho_T^0}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)$ is suffered from infrared (IR) divergence in the real photon limit and, thus, is useful only for the case with the highly-virtual initial-state photon with $Q \gg \Lambda_{\rm QCD}$. This implies that, for $Q \lesssim \Lambda_{\rm QCD}$, the impact factor for the transversely polarized vector meson should receive the "non-factorizable" soft QCD contributions.

4. Light-cone sum rule calculation for $\gamma \to \rho_T^0$ impact factor

Still, the impact factor for small as well as large Q^2 should obey the dispersion relation,

$$J^{(1)}_{\gamma^*\rho_T^0}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) = \int_0^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2} = \frac{a}{Q^2+m_\rho^2} + \int_{m_{\rm th}^2}^\infty dm^2 \frac{\chi(m^2)}{Q^2+m^2}\ , \tag{4.1}$$

with the corresponding spectral weight function $\chi(m^2)$; here, the lowest resonance contribution, which is given as the ρ-meson pole with the residue a, is explicitly shown and the higher resonance contributions are expressed as the continuum integral starting from the threshold, $m_{\rm th}^2$. Those quantities arising in the dispersion relation may be determined by matching with the factorization formula of $J^{(1)}_{\gamma^*\rho_T^0}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)\big|_{\rm LO}$ in the region $Q \gtrsim \Lambda_{\rm QCD}$ where the QCD factorization is applicable; in particular, invoking the quark-hadron duality, the spectral function $\chi(m^2)$ for the continuum contribution in the RHS of (4.1) is taken to be the same as implied by the factorization formula; then, performing the Borel transformation with the parameter M_B to allow an efficient matching, we get

$$a = -\sqrt{4\pi\alpha_{\rm em}}\,4\pi\alpha_s \frac{\delta^{ab}}{2N_c}\frac{f_\rho m_\rho}{\sqrt{2}}\left(\boldsymbol{e}_\gamma\cdot\boldsymbol{e}^*_{\rho T}\right)\left[e^{\frac{m_\rho^2}{M_B^2}}\int_{u_0}^1 du\left(g_\perp^{(v)}(u) - \frac{1}{4}\frac{\partial g_\perp^{(a)}(u)}{\partial u}\right)e^{-\frac{(1-u)\Delta_\perp^2}{uM_B^2}}\right] + \cdots\ , \tag{4.2}$$

with $e^\mu_{\rho T} = (0, \boldsymbol{e}_{\rho T})$ being the polarization vector for the transversely polarized ρ meson. Here, the ellipses stand for the terms with more complicated structure, having the dependence on $\boldsymbol{k}_\perp$ as well as on the three-body ($\bar{q}qg$) DAs for the transversely polarized ρ meson. Those terms are obtained by matching with the factorization formula which takes into account the next-to-leading term in the collinear expansion and the $\bar{q}qg$ three-body Fock states in the vector meson. Those effects corresponding to the ellipses in (4.2) are neglected for simplicity in the following discussion and will be discussed elsewhere. The formula (4.2) corresponds to the light-cone QCD sum rule, where the relevant nonperturbative effects are encoded into the meson DAs, i.e., the light-cone dominated matrix elements; note that the quark-hadron duality also allows us to determine the lower limit of the integral, u_0, in terms of the threshold parameter $m_{\rm th}$ of (4.1). It is worth mentioning that the light-cone sum rules have been successfully applied to the calculation of the soft QCD contributions associated with the real photons in various exclusive processes.

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We use a first few terms in the Gegenbauer expansion of the vector meson DA, as ($\xi \equiv 2u-1$)

$$\phi_{\parallel}(u) \;=\; 6u(1-u)\left(1+\sum_{n=1}^{\infty} b_{2n}C_{2n}^{3/2}(2u-1)\right) \simeq 6u(1-u)\left(1+b_2\frac{3}{2}(5\xi^2-1)\right)\,, \qquad (4.3)$$

in (3.1). Similar expansions are constructed for the twist-three DAs [4], such that all constraints from QCD equations motion and conformal symmetry are obeyed, and we use their first few terms,

$$g_{\perp}^{(v)}(u) \simeq \frac{3}{4}\left(1+\xi^2\right)+b_2\frac{3}{7}\left(3\xi^2-1\right)\,, \quad g_{\perp}^{(a)}(u) \simeq 6u(1-u)\left(1+b_2\frac{1}{4}\left(5\xi^2-1\right)\right)\,, \qquad (4.4)$$

for the DAs arising in (4.2). We use $b_2 = 0.18$; this is the value given at the scale 1 GeV [4], and we neglect the renormalization scale dependence of the DAs in the following calculations. $m_\rho = 0.775$ GeV, and $f_\rho = 0.198$ GeV are used as usual. The strong coupling constant is fixed to the value $\alpha_s = 0.32$, which was used in [1]. The threshold parameter, arising in (4.1) and (4.2), is taken as $m_{\rm th}^2 = 1.5$ GeV^2, as deduced from the QCD sum rules to determine m_ρ and f_ρ, see [4].

Fig. 1(a) shows the quantity in the square brackets "[ ]" in (4.2) as a function of M_B^2, with good stability in the relevant region of auxiliary parameter M_B. Using this result, we evaluate the light-cone sum rule result of (4.1), and this evaluation of (4.1), divided by the factor in front of the square brackets in (4.2), is shown by the gold curve in Fig. 1(b) as a function of Q^2, yielding the finite real-photon limit for $Q^2 = 0$. The blue curve is obtained using the corresponding factorization formula instead of (4.1), (4.2); the blue curve diverges logarithmically as $Q^2 \to 0$. The ρ-meson pole term dominates (4.1) for small $\mathbf{\Delta}_\perp^2$, as controlled by (4.2), exhibiting behaviors like the vecor meson dominance (VMD) model. Our result (4.1), (4.2) provides an *interpolating forumula* between the VMD, which is to be associated with pomeron and is justified phenomenologically for $Q^2 \sim 0$, and the QCD factorization, associated with the two gluons and reliable for $Q^2 \gg \Lambda_{\rm QCD}$.

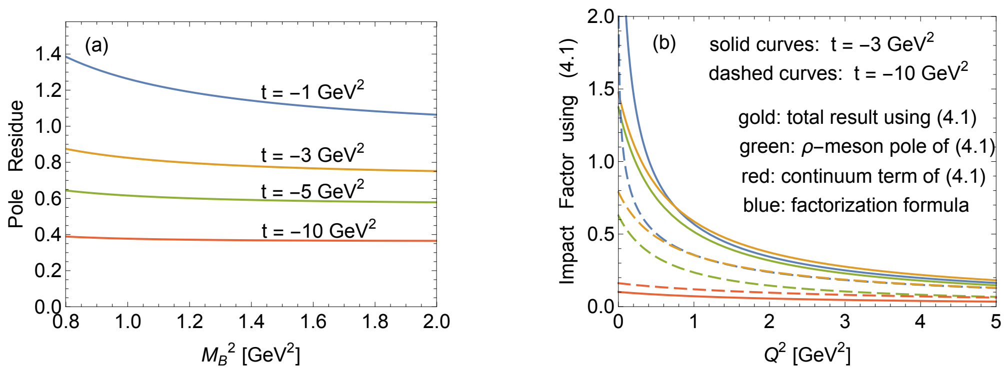

Figure 1: Light-cone sum rule result for (a) the quantity in the square brackets in (4.2) as a function of the Borel parameter M_B^2 and (b) the impact factor (4.1), divided by the prefactor of (4.2), as a function of Q^2.

5. Application to two-photon process $\gamma\gamma \to \rho^0\rho^0$

Substituting the above results into (2.1), we obtain the forward amplitude for $\gamma\gamma \to \rho^0\rho^0$ at high energy, i.e., at $s \gg -t \gg \Lambda_{\rm QCD}^2$, and thus the corresponding cross sections specifying the particular polarizations for each of the produced ρ mesons. For the longitudinally polarized ρ_L^0, we use the impact factor (3.1) based on the LO factorization formula, while, for the transversely polarized ρ_T^0, we use the impact factor (4.1) using (4.2). We give an estimate of the cross sections with $s = 100$ GeV^2 for $\gamma^*\gamma \to \rho_T^0 \rho_L^0$, where γ^* with the virtuality Q^2 is assumed to be transversely polarized and to evolve into ρ_T^0 in the forward direction with $\boldsymbol{\Delta}_\perp^2 = 3$ GeV^2, by the gold curve in Fig. 2(a) as a function of Q^2; the blue curve is obtained by replacing our impact factor (4.1) with the corresponding LO factorization formula $J^{(1)}_{\gamma^*\rho_T^0}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp)|_{\rm LO}$ (compare with the blue curve in Fig. 1(b)). For comparison, we also show, by the dot plotted at $Q^2 = 0$, the $\gamma\gamma \to \rho_L^0 \rho_L^0$ cross section with the two real photons, which is consistent with the results in [1, 2]. We provide an estimate for the $\gamma\gamma \to \rho_T^0 \rho_L^0$ cross section in the real photon limit for the first time, and the result suggests that the cross section for $\gamma\gamma \to \rho_T^0 \rho_L^0$ could be larger than that for $\gamma\gamma \to \rho_L^0 \rho_L^0$. Fig. 2(b) shows the cross section for $\gamma^*\gamma \to \rho_T^0 \rho_L^0$ as a function of $\cos^2\theta$ for various values of Q with $s = 100$ GeV^2. It is worth mentioning that, for $\cos^2\theta \gtrsim 0.4$, the $\gamma\gamma \to \rho_L^0 \rho_L^0$ cross section due to the t-channel two gluon (gg) exchange, corresponding to the dot in Fig. 2(a), is shown to exceed considerably the corresponding cross section due to the t-channel $q\bar{q}$ exchange; see the discussion in [1].

The final states of $\rho_T^0 \rho_L^0$ with different polarizations, as well as the final states with different flavor types, $\rho^0\omega$, $\rho^0\phi$, ..., are allowed only in the gg exchange mechanism, while the final states with the charged vector mesons as well as pions, such as $\rho^+\rho^-$, $\pi^+\pi^-$, are allowed only in the $q\bar{q}$ exchange mechanism. The $\gamma^{(*)}\gamma \to \rho_T^0 \rho_L^0$ may be measured in Belle II experiment. Its improved calculations taking into account the ellipses in (4.2) will be presented elsewhere.

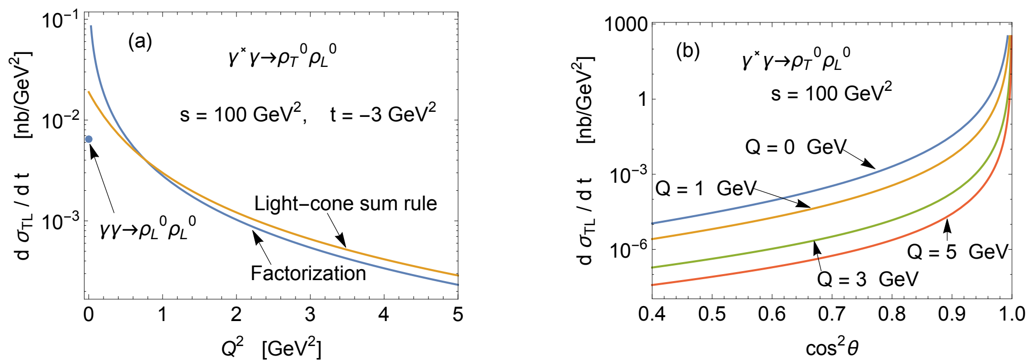

Figure 2: Light-cone sum rule results for the $\gamma^*\gamma \to \rho_T^0 \rho_L^0$ cross section with $s = 100$ GeV^2 as a function of (a) Q^2 and (b) $\cos^2\theta$. In (a), the result using factorization and the $\gamma\gamma \to \rho_L^0 \rho_L^0$ cross section are also shown.

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