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Production of transversely polarized Λ hyperon from unpolarized quark fragmentation in the diquark model

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We study the spin-dependent (naive) T-odd fragmentation function $D_{1T}^{\perp}$ in a spectator model to obtain the knowledge of the process for the transverse polarization of the Λ^0 hyperon produced fragmented from an unpolarized quark. We consider both scalar diquark and vector diquark to obtain the flavor content of Λ fragmentation functions. We determine the values of the model parameters by fitting the unpolarized fragmentation function D_1^{Λ} to the DSV parametrization for D_1^{Λ}. With the model results of these fragmentation functions, we also estimate the transverse polarizations P_T^{Λ} in both semi-inclusive deep inelastic scattering and single inclusive e^+e^- annihilation.

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1. Introduction

The production of a polarized Λ hyperon from unpolarized pp collisions [1, 2] has become a long-standing challenge in high energy physics [3]. In particular, a spin-dependent (naive) T-odd fragmentation function $D_{1T}^{\perp}$ of the Λ hyperon, can be used to describe the transverse polarization of the Λ hyperon production. In this work, we perform the calculation of $D_{1T}^{\perp}$ for light flavors using a spectator diquark model [4, 5]. In order to obtain the flavor content of Λ fragmentation functions, in our calculation, both the scalar diquark and the vector diquark will be considered. With the same model and parameters, we also give the T-odd fragmentation function $D_{1T}^{\perp}$ for up, down and strange quarks, by considering the gluon scattering effect. We also estimate the numerical results of the transverse polarization of the Λ hyperon production both in the semi-inclusive deep inelastic scattering (SIDIS) $\ell p \to \ell' + \Lambda^{\uparrow} + X$ and the single inclusive e^+e^- annihilation (SIA) $e^+, e^- \to \Lambda(\bar{\Lambda}) + X$.

2. Model calculation of unpolarized Λ fragmentation functions

In this section, we calculate the unpolarized TMD fragmentation function $D_1^{\Lambda}(z, k_T)$ in a spectator diquark model [4, 5], which includes both the spin 0 (scalar diquark) and spin 1 (axial-vector diquark) spectator systems [6]. The unpolarized TMD fragmentation function $D_1^{\Lambda}(z, k_T)$ can be obtained from the following trace [5, 7]

$$D_1^{\Lambda}(z, k_T) = \frac{1}{4}\mathrm{Tr}[(\Delta(z, k_T; S_\Lambda) + \Delta(z, k_T; -S_\Lambda))\gamma^-]\,. \tag{2.1}$$

Using spectator model, the expression of the $\Delta(z, k_T; S_\Lambda)$ at tree level can be given as

$$\Delta(z, k_T; S_\Lambda) = \frac{g_D^2}{4(2\pi)^3}\frac{(\not{k} + m_q)(\not{P}_\Lambda + M_\Lambda)(1 + a_D\gamma_5 \not{S}_\Lambda)(\not{k} + m_q)}{(1-z)P_\Lambda^-(k^2 - m_q^2)^2}\,, \tag{2.2}$$

with $k^2 = \frac{z}{(1-z)}\boldsymbol{k}_T^2 + \frac{m_D^2}{(1-z)} + \frac{M_\Lambda^2}{z}$ and $k^- = \frac{P_\Lambda^-}{z}$. The spin factor a_D takes the values $a_s = 1$ and $a_v = -\frac{1}{3}$, and m_q, m_D and M_Λ denote the masses of the parent quark, the spectator diquark and the fragmenting Λ hyperon, respectively.

From Eqs. (2.1) and (2.2), the unpolarized fragmentation function D_1^{Λ} can be simplified as

$$D_1^{(s)}(z, z^2\boldsymbol{k}_T^2) = D_1^{(v)}(z, z^2\boldsymbol{k}_T^2) = \frac{g_D^2}{2(2\pi)^3}\frac{(1-z)[z^2\boldsymbol{k}_T^2 + (M_\Lambda + zm_q)^2]}{z^4(k_T^2 + L^2)^2}\,, \tag{2.3}$$

with $L^2 = \frac{1-z}{z^2}M_\Lambda^2 + m_q^2 + \frac{m_D^2 - m_q^2}{z}$. In Eq. (2.3), $D_1^{(s)}(z, z^2\boldsymbol{k}_T^2)$ and $D_1^{(v)}(z, z^2\boldsymbol{k}_T^2)$ denote the contributions to D_1^{Λ} from the scalar diquark and the axial-vector diquark components, respectively. Under the summation for all polarization states of the axial-vector diquark: $\sum_\lambda \varepsilon_\mu^{*(\lambda)}\varepsilon_\nu^{(\lambda)} = -g_{\mu\nu} + \frac{P_{\Lambda\mu}P_{\Lambda\nu}}{M_\Lambda^2}$, the final results we get for them turn out to be the same.

In this work, we assume a SU(6) spin-flavor symmetry that the fragmentation functions of the Λ hyperon for light flavors between the different quark flavors and diquark types satisfy the following relations [8]:

$$D^{\mathrm{u}\to\Lambda} = D^{\mathrm{d}\to\Lambda} = \frac{1}{4}D^{(s)} + \frac{3}{4}D^{(v)}\,,\;\; D^{\mathrm{s}\to\Lambda} = D^{(s)}\,, \tag{2.4}$$

where u, d and s denote the up, down and strange quarks, respectively. If the mass differences between the up, down and strange quarks are neglected, we can find that the light quarks fragment equals to Λ for the unpolarized fragmentation function D_1^{Λ}, i.e.,

$$D_1^{\mathrm{u}\to\Lambda} = D_1^{\mathrm{d}\to\Lambda} = D_1^{\mathrm{s}\to\Lambda} \equiv D_1^{\Lambda} \,. \tag{2.5}$$

To obtain the integrated fragmentation function $D^{\Lambda}(z)$, we perform the integration over the transverse momentum of the produced hadron $\boldsymbol{P}_T = -z\boldsymbol{k}_T$:

$$D_1^{\Lambda}(z) = \int d^2\boldsymbol{P}_T D_1^{\Lambda}(z,\boldsymbol{P}_T^2) = \pi z^2 \int_0^{\infty} dk_T^2\, D_1^{\Lambda}(z, z^2\boldsymbol{k}_T^2)\,, \tag{2.6}$$

here we choose a Gaussian form factor $g_D \mapsto \frac{g_D}{z} e^{-\frac{k^2}{\sqrt{z}\lambda^2}}$ to regularize the divergence from large values of k_T when a point-like hyperon-quark-diquark coupling is considered. At last, the analytic result for $D_1^{\Lambda}(z)$ can be expressed as

$$\begin{aligned} D_1^{\Lambda}(z) =& \frac{g_s^2}{4(2\pi)^2}\frac{e^{-\frac{2m_q^2}{\Lambda^2}}}{z^4L^2}\left\{ z(1-z)((m_q+M_\Lambda)^2 - m_D^2)\exp\left(\frac{-2zL^2}{(1-z)\Lambda^2}\right) \right. \\ &\left. + \left((1-z)\Lambda^2 - 2((m_q+M_\Lambda)^2-m_D^2)\right)\frac{z^2L^2}{\Lambda^2}\Gamma\left(0, \frac{2zL^2}{(1-z)\Lambda^2}\right)\right\}, \end{aligned} \tag{2.7}$$

where the incomplete gamma function has the form $\Gamma(0,z) \equiv \int_z^{\infty}\frac{e^{-t}}{t}dt$ and λ, m_D, g_s are model parameters.

3. Model calculation of the T-odd fragmentation function $D_{1T}^{\perp}$

In this section, we compute the T-odd TMD fragmentation function $D_{1T}^{\perp}$ which describes the number density of a transversely polarized Λ hyperon fragmented from an unpolarized quark [9]. As is well known, T-odd fragmentation functions are vanishing in tree level calculation, its nonzero contribution comes from loop correction [10].

The TMD fragmentation function $D_{1T}^{\perp}$ can be obtained from the following trace [11]:

$$\frac{\varepsilon_T^{\rho\sigma} k_{T\rho} S_{\Lambda T\sigma}}{M_\Lambda} D_{1T}^{\perp}(z,k_T) = \frac{1}{4}\mathrm{Tr}[(\Delta(z,k_T;S_{\Lambda T}) - \Delta(z,k_T;-S_{\Lambda T}))\gamma^-]\,. \tag{3.1}$$

Integrating over the loop momentum l and using the Cutkosky cutting rules, we can first give the expression for $D_{1T}^{\perp}$ from the scalar diquark component

$$\begin{aligned} D_{1T}^{\perp(s)}(z,k_T^2) =& \frac{\alpha_s g_s^2 C_F}{(2\pi)^4}\frac{e^{\frac{-2k^2}{\Lambda^2}}}{z^2(1-z)}\frac{1}{(k^2-m^2)} \\ &\times\left[D_{1T(a)}^{\perp(s)}(z,k_T^2) + D_{1T(b)}^{\perp(s)}(z,k_T^2) + D_{1T(c)}^{\perp(s)}(z,k_T^2) + D_{1T(d)}^{\perp(s)}(z,k_T^2)\right], \end{aligned} \tag{3.2}$$

where the four terms in the brackets correspond to the contributions from the four diagrams (plus their hermitian conjugates) at one loop level [12]:

$$D_{1T(a)}^{\perp(s)}(z,k_T^2) = \frac{m_q M_\Lambda}{(k^2-m_q^2)}(3 - \frac{m_q^2}{k^2})I_1\,, \tag{3.3}$$

$$D_{1T(b)}^{\perp(s)}(z,k_T^2) = M_\Lambda\left\{m_q(2I_2-\mathscr{A}) - M_\Lambda(\mathscr{B}-2I_2+2\mathscr{A})\right\}, \tag{3.4}$$

$$D_{1T(c)}^{\perp(s)}(z,k_T^2) = 0\,, \tag{3.5}$$

$$D_{1T(d)}^{\perp(s)}(z,k_T^2) = \frac{M_\Lambda}{z}\left\{2(1-z)(m_q\mathscr{C}P_\Lambda^- - M_\Lambda\mathscr{D}P_\Lambda^-) - z(M_\Lambda\mathscr{B}-m_q\mathscr{A})\right\}, \tag{3.6}$$

here $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$ and $\mathscr{D}$ are functions of k^2, m_q, m_D and M_Λ.

Similarly, we can give the expression for $D_{1T}^\perp$ from the axial vector diquark component

$$\begin{aligned} D_{1T}^{\perp(v)}(z,k_T^2) = {}& \frac{2\alpha_s g_s^2 C_F}{(2\pi)^4}\frac{e^{\frac{-2k^2}{\Lambda^2}}}{z^2(1-z)}\frac{1}{M_\Lambda(k^2-m_q^2)} \\ & \times\left[D_{1T(v)}^{\perp(v)}(z,k_T^2) + D_{1T(b)}^{\perp(v)}(z,k_T^2) + D_{1T(c)}^{\perp(v)}(z,k_T^2) + D_{1T(d)}^{\perp(v)}(z,k_T^2)\right], \end{aligned} \tag{3.7}$$

where the four terms in the r.h.s. of Eq. (3.7) can be written as

$$D_{1T(a)}^{\perp(v)}(z,k_T^2) = \frac{-m_q M_\Lambda}{(1-z)(k^2-m_q^2)}\left(1-\frac{m_q^2}{3k^2}\right)I_1\,, \tag{3.8}$$

$$\begin{aligned} D_{1T(b)}^{\perp(v)}(z,k_T^2) = {}& \frac{1}{3(k^2-m_q^2)}\Big\{2M_\Lambda[m_q(I_2-\mathscr{A}) + M_\Lambda(\mathscr{A}-I_2-\mathscr{B})] + k\cdot P_\Lambda(4I_2-6\mathscr{A}) \\ & -(\mathscr{A}k\cdot P_\Lambda + \mathscr{B}P_\Lambda^2) + \frac{3}{2}\left(\frac{k^2-m_q^2}{2k^2}I_1 + (k^2-m_q^2)\mathscr{A}\right)\Big\}, \end{aligned} \tag{3.9}$$

$$D_{1T(c)}^{\perp(v)}(z,k_T^2) = 0\,, \tag{3.10}$$

$$\begin{aligned} D_{1T(d)}^{\perp(v)}(z,k_T^2) = {}& \frac{-1}{3M_\Lambda(k^2-m_q^2)}\Big\{[M_\Lambda\big((k^2-m_q^2)\mathscr{C}P_\Lambda^- + 2M_\Lambda^2\mathscr{D}P_\Lambda^- - 2m_qM_\Lambda\mathscr{C}P_\Lambda^-\big) \\ & + 2k\cdot P_\Lambda(m_q\mathscr{C}P_\Lambda^- - M_\Lambda\mathscr{D}P_\Lambda^-) + z\frac{m_q}{2}I_1 + \frac{(k^2-m_q^2)}{2}(M_\Lambda\mathscr{D}P_\Lambda^- - m_q\mathscr{C}P_\Lambda^-)] \\ & - M_\Lambda(m_qM_\Lambda\mathscr{A} + 2k\cdot P_\Lambda\mathscr{A} + M_\Lambda^2\mathscr{B}) - \frac{2M_\Lambda}{z}(m_qM_\Lambda\mathscr{C}P_\Lambda^- + k\cdot P_\Lambda\mathscr{C}P_\Lambda^-)\Big\}. \end{aligned} \tag{3.11}$$

We assume that the relations in Eq. (2.4) also hold for the fragmentation function $D_{1T}^\perp$:

$$D_{1T}^{\perp u} = D_{1T}^{\perp d} = \frac{1}{4}D_{1T}^{\perp(s)} + \frac{3}{4}D_{1T}^{\perp(v)}\,,\ \ D_{1T}^{\perp s} = D_{1T}^{\perp(s)}\,, \tag{3.12}$$

we can obtain $D_{1T}^\perp$ for light flavors and calculate the half k_T-moment of $D_{1T}^\perp$

$$D_{1T}^{\perp(1/2)}(z) = z^2\int d\boldsymbol{k}_T^2\frac{|\boldsymbol{k}_T|}{2M_\Lambda}D_{1T}^\perp(z,\boldsymbol{k}_T^2)\,. \tag{3.13}$$

After the integration over k_T^2, one can get an approximate expression for the positivity bound in terms of $D_{1T}^{\perp(1/2)}(z)$

$$2D_{1T}^{\perp(1/2)}(z) \le D_1(z)\,. \tag{3.14}$$

Using fitted results of the model parameters $g_s = 1.983$, $m_D = 0.745$GeV and $\lambda = 5.967$GeV, we give the numerical result of the half k_T-moment of the Λ fragmentation function $D_{1T}^\perp$ for light

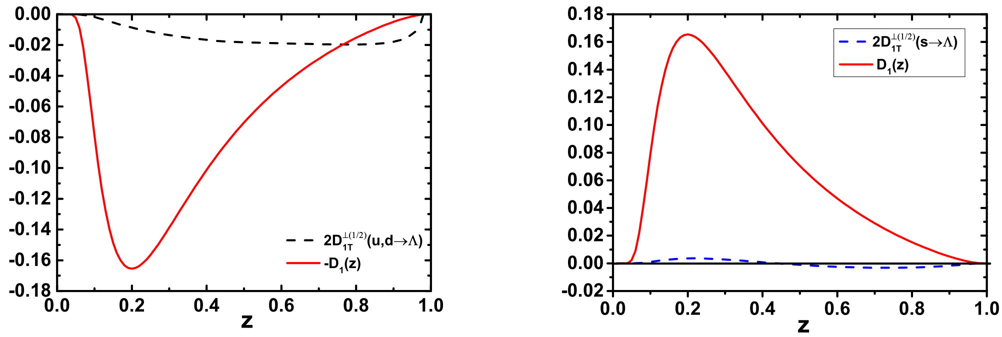

Figure 1: Left panel: the $D_{1T}^{\perp(1/2)}(z)$ (multiplied by 2) (dashed line) and $D_1(z)$ (multiplied by -1) (solid line) of the up and down quark. Right panel: the $D_{1T}^{\perp(1/2)}(z)$ (multiplied by 2) (dashed line) and $D_1(z)$ (solid line) of the strange quark.

flavors at the model scale $Q^2 = 0.23\,\mathrm{GeV}^2$, by choosing the strong coupling constant at the model scale as $\alpha_s(\mu_0^2) = 0.817$. We plotted the numerical results of $D_{1T}^{\perp(1/2)}(z)$ (multiplied by a factor of 2) in Fig. 1. The unpolarized fragmentation function $D_1(z)$ (solid lines) is also plotted as the positivity bound for comparison.

Using the model result for $D_{1T}^{\perp}$ of the Λ hyperon, we predict the transverse Λ polarization P_T^{Λ} in SIDIS and SIA, which is a direct experimental observable. Generally, P_T^{Λ} in high energy processes is defined as

$$P_T^{\Lambda} = \frac{d\Delta\sigma}{d\sigma} = \frac{[d\sigma(S_{\Lambda T}) - d\sigma(-S_{\Lambda T})]}{[d\sigma(S_{\Lambda T}) + d\sigma(-S_{\Lambda T})]}. \tag{3.15}$$

Using the $SU(3)_f$ symmetric unpolarized Λ fragmentation functions, i.e., $D_1^{u\to\Lambda} \equiv D_1^{d\to\Lambda} \equiv D_1^{s\to\Lambda}$, and ignoring the sea quarks $f_{\bar{q}/p}$ and strange quark $f_{s/p}$ contributions to SIDIS, we can obtain an approximate result for P_T^{Λ} in SIDIS:

$$P_T^{\Lambda}\big|_{\mathrm{DIS}} \approx \frac{\Delta D_{\Lambda^\uparrow/\mathrm{u}}}{D_1^{\mathrm{u}\to\Lambda}}. \tag{3.16}$$

Similarly, we can get a simplified form for P_T^{Λ} in SIA:

$$P_T^{\Lambda}\big|_{\mathrm{SIA}} = \frac{5\Delta D_{\Lambda^\uparrow/\mathrm{u}} + \Delta D_{\Lambda^\uparrow/\mathrm{s}}}{5D_1^{\mathrm{u}\to\Lambda} + D_1^{\mathrm{s}\to\Lambda}}. \tag{3.17}$$

In Fig. 2, we present the numerical result for the transverse Λ polarization P_T^{Λ} after averaging over the transverse momentum of the Λ hyperon in both SIDIS (solid line) and SIA (dashed line). The results show that P_T^{Λ} is negative and substantial in the large z region. Its size increases with increasing z in both cases. Our results are consistent with the phenomenological analysis presented in Ref. [9] and with the calculation of Ref. [13]. Furthermore, the shape of P_T^{Λ} in SIA is very similar to that in SIDIS, as a consequence of the up and down quark dominance for $D_{1T}^{\perp}$ in our model. As can be seen from Eqs. (3.16) and (3.17), the difference between P_T^{Λ} in SIA and SIDIS is given by $D_{\Lambda^\uparrow/\mathrm{s}}$. We hope that the difference may provide a test for the strange quark contribution to P_T^{Λ}.

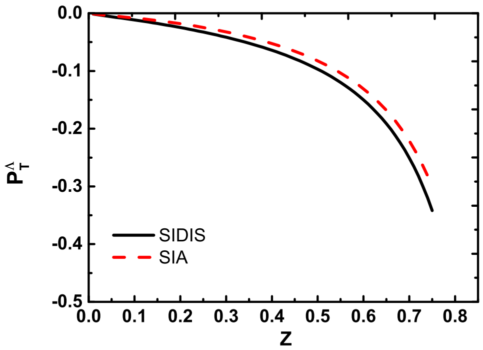

Figure 2: The transverse Λ polarizations, P_T^Λ vs z, averaged over $P_{\Lambda T}$, for SIDIS (solid line) and SIA (dashed line).

4. Discussion and Conclusion

In this work, we computed the T-odd fragmentation function $D_{1T}^\perp$ using the diquark spectator model and studied its contribution to the transversely polarized Λ production in SIDIS and SIA. We included in the calculation both the scalar diquark and the axial-vector diquark. We also applied the relation between the quark flavors and diquark types for fragmentation functions, motivated by the SU(6) symmetric wave functions of the Λ hyperon. The values of the model parameters were determined by fitting the model result of unpolarized fragmentation function $D_1(z)$ with the DSV parametrization for D_1 at the initial scale $\mu_0^2 = 0.23\ \mathrm{GeV}^2$. Using our model result of $D_{1T}^\perp$, we estimated the transverse polarization P_T^Λ in SIDIS and SIA, and found the polarizations are negative and substantial in the large z region.

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