

Accessing generalized TMDs through double Drell-Yan and double η_Q production processes

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Being the "mother distributions" of all types of two-parton correlation functions, generalized TMDs (GTMDs) have garnered a lot of attention. We address the important question of how to access GTMDs in physical processes. Recently, we have shown that quark GTMDs can in principle be probed through the exclusive pion-nucleon double Drell-Yan process, where the focus was on two particular GTMDs only. We now present new results concerning access to the remaining quark GTMDs in the same process. Moreover, we show that GTMDs for gluons can be explored via exclusive double production of pseudoscalar quarkonia (η_c or η_b) in nucleon-nucleon collisions.

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1. Introduction

Generalized transverse momentum dependent parton distributions (GTMDs) are the most general (two-)parton correlation functions of hadrons [1, 2, 3]. The information encoded in GTMDs goes beyond what one can learn from generalized parton distributions (GPDs) and transverse momentum dependent parton distributions (TMDs), which characterize the 3-D structure of hadrons. Both GPDs and TMDs are (merely) kinematical projections of certain GTMDs.

The interest in GTMDs is not confined to their ‘‘mother distribution’’ character: For a vanishing longitudinal momentum transfer to the hadron, the Fourier transform of a GTMD is a partonic Wigner function that depends on the (average) longitudinal and transverse momentum as well as transverse position of partons, which in principle provides 5-D images of hadrons [4, 5]. Moreover, for quarks and gluons there exists a relation between a specific GTMD — $F_{1,4}$ in the notation of [2] — and the orbital angular momentum (OAM) of partons inside a longitudinally polarized nucleon [6, 7]. Finally, certain GTMDs are related to spin-orbit correlations of the nucleon [6, 8], which have a meaning similar to the ones in the hydrogen atom.

For quite some time it was entirely unclear if/how GTMDs can be measured. Recently, however, it has been argued that gluon GTMDs can be accessed through exclusive hard diffractive di-jet production in DIS [9], which could be studied at a future electron-ion collider. In the meantime, several further works on GTMD observables have appeared [10, 11, 12, 13, 14, 15, 16, 17].

In this contribution we concentrate on two types of processes that are sensitive to GTMDs. In Ref. [15] we have demonstrated that quark GTMDs can be measured via the exclusive double Drell-Yan process. The focus of that work has been on two specific GTMDs. Here we argue that the remaining (chiral-even) quark GTMDs can also be addressed in the same process [18]. We also show that the exclusive double production of pseudoscalar quarkonia in nucleon-nucleon collisions, $N_a N_b \rightarrow \eta_Q \eta_Q N_a N_b$ where η_Q denotes either η_c or η_b , is sensitive to GTMDs of gluons [17].

2. Quark GTMDs and the exclusive double Drell-Yan process

The exclusive double Drell-Yan process in pion-nucleon scattering, $\pi N \rightarrow (\ell_1^- \ell_1^+) (\ell_2^- \ell_2^+) N'$, gives access to leading-twist (chiral-even) GTMDs of quarks in the ERBL region characterized by $-\xi_a \leq x_a \leq \xi_a$, with ξ_a the skewness variable for the nucleon [15]. In total eight (complex-valued) such functions exist, which in Ref. [2] have been denoted by $F_{1,1}^q, \dots, F_{1,4}^q, G_{1,1}^q, \dots, G_{1,4}^q$. While Ref. [15] has mainly concentrated on $F_{1,4}^q$ and $G_{1,1}^q$ — these functions have attracted considerable attention in relation to the nucleon spin structure — we now discuss how the remaining quark GTMDs can be accessed in the same process via suitable polarization observables [18].

2.1 Accessing $F_{1,1}^q$ and $G_{1,4}^q$

We first consider addressing these two GTMDs for a vanishing transverse momentum transfer to the nucleon, $\vec{\Delta}_{a\perp} = 0$. In the case of $F_{1,1}^q$, one can use the following linear combination of polarization observables [18],

$$\begin{aligned} & \frac{1}{4} (\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY}) \Big|_{\vec{\Delta}_{a\perp}=0} \\ &= 2 C^{(-)} \left[F_{1,1}^q(x_a, \vec{k}_{a\perp}) \phi_\pi(x_b, \vec{k}_{b\perp}^2) \right] C^{(-)} \left[F_{1,1}^{q*}(x_a, \vec{p}_{a\perp}) \phi_\pi^*(x_b, \vec{p}_{b\perp}^2) \right]. \end{aligned} \quad (2.1)$$

The indices U (unpolarized), L (longitudinally polarized) and X, Y (transversely polarized) characterize the polarization state of the nucleon, where the first (last) index refers to the incoming (outgoing) nucleon. Note that $\tau_{LL}, \tau_{XX}, \tau_{YY}$ define double-spin asymmetries [15]. In Eq. (2.1), ϕ_π denotes the light-cone wave function of the pion. While the (average) longitudinal quark momentum fractions x_a and x_b for the nucleon and pion, respectively, are fixed by the external kinematics of the process, the transverse quark momenta are integrated over with the constraint $\vec{k}_{a\perp} + \vec{k}_{b\perp} = \Delta\vec{q}_\perp/2$, where $\Delta\vec{q}_\perp = \vec{q}_{1\perp} - \vec{q}_{2\perp}$ is the difference of the transverse momenta of the produced virtual photons [15]. For the precise definition of the convolution integral $C^{(-)}$ for transverse momenta we also refer to [15]. We also mention that we have summed over the photon polarizations. The linear combination $\frac{1}{4}(\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY})$ gives access to $G_{1,4}^q$ [18]. Interestingly, for $\vec{\Delta}_{a\perp} \neq 0$ the aforementioned linear combinations provide two things — sensitivity to $F_{1,1}^q, G_{1,4}^q$ but with richer kinematical dependence, and also access to $F_{1,4}^q$ and $G_{1,1}^q$ [15].

2.2 Accessing other quark GTMDs

For $\vec{\Delta}_{a\perp} = 0$, the following double-spin asymmetry is sensitive to $F_{1,2}^q$ and $G_{1,2}^q$ [18],

$$\begin{aligned} \tau_{XY}|_{\vec{\Delta}_{a\perp}=0} &= \tau_{YX}|_{\vec{\Delta}_{a\perp}=0} \\ &= -4 \frac{(1 - \xi_a^2)^2}{M^2} \Delta q_\perp^1 \Delta q_\perp^2 \left\{ C^{(+)} \left[\frac{\Delta\vec{q}_\perp \cdot \vec{k}_{a\perp}}{\Delta\vec{q}_\perp^2} F_{1,2}^q \phi_\pi \right] C^{(+)} \left[\frac{\Delta\vec{q}_\perp \cdot \vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2} F_{1,2}^{q*} \phi_\pi^* \right] \right. \\ &\quad \left. - C^{(-)} \left[\frac{\Delta\vec{q}_\perp \cdot \vec{k}_{a\perp}}{\Delta\vec{q}_\perp^2} G_{1,2}^q \phi_\pi \right] C^{(-)} \left[\frac{\Delta\vec{q}_\perp \cdot \vec{p}_{a\perp}}{\Delta\vec{q}_\perp^2} G_{1,2}^{q*} \phi_\pi^* \right] \right\}, \quad (2.2) \end{aligned}$$

with M denoting the nucleon mass. For brevity we have omitted the arguments of the GTMDs and the pion wave function. The two terms on the r.h.s. of Eq. (2.2) can be disentangled by exploiting suitable linear polarizations of the photons. Moreover, the observables τ_{XU} and τ_{XL} give us a hold over the imaginary and real parts of $F_{1,2}^q$ and $G_{1,2}^q$, respectively, through interference with $\text{Re} F_{1,1}^q$ and $\text{Re} G_{1,4}^q$ [18]. We recall that $\text{Re} F_{1,1}^q$ and $\text{Re} G_{1,4}^q$ are potentially large since, in the forward limit, they are related to the density of unpolarized and longitudinally polarized quarks [2].

Based on the definitions of $F_{1,3}^q$ and $G_{1,3}^q$ [2, 15] one must have $\vec{\Delta}_{a\perp} \neq 0$ in order to address these two GTMDs. Here we just list one sample observable which is sensitive to $G_{1,3}^q$ [18],

$$\begin{aligned} &\frac{1}{4} \left[\Delta_{a\perp}^1 (\tau_{XU} + \tau_{UX}) + \frac{\xi_a \vec{\Delta}_{a\perp}^2}{2M} (\tau_{LU} + \tau_{UL}) \right] \\ &= -2 \frac{(1 - \xi_a^2)}{M} \varepsilon_\perp^{ij} \Delta q_\perp^i \Delta_{a\perp}^j \text{Im} \left\{ C^{(-)} \left[F_{1,1}^q \phi_\pi \right] C^{(+)} \left[\vec{\beta}_\perp \cdot \vec{p}_{a\perp} F_{1,2}^{q*} \phi_\pi^* \right] \right\} \\ &\quad + 2 \frac{(1 - \xi_a^2)}{M} \frac{\varepsilon_\perp^{ij} \Delta q_\perp^i \Delta_{a\perp}^j}{M^2} \left(\text{Im} \left\{ C^{(-)} \left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} G_{1,1}^q \phi_\pi \right] C^{(-)} \left[\vec{\Delta}_{a\perp} \cdot \vec{p}_{a\perp} G_{1,2}^{q*} \phi_\pi^* \right] \right\} \right. \\ &\quad \left. + \vec{\Delta}_{a\perp}^2 \text{Im} \left\{ C^{(-)} \left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} G_{1,1}^q \phi_\pi \right] C^{(+)} \left[G_{1,3}^{q*} \phi_\pi^* \right] \right\} \right), \quad (2.3) \end{aligned}$$

where the vector $\vec{\beta}_\perp$ is defined through $\Delta\vec{q}_\perp$ and $\vec{\Delta}_{a\perp}$ [15]. Again, using suitable polarization states of the virtual photons allows one to disentangle terms with F-type and G-type GTMDs. Note that, in general, $G_{1,3}^q$ always appears in combination with $G_{1,2}^q$. The same applies to $F_{1,3}^q$ and $F_{1,2}^q$.

3. Gluon GTMDs and exclusive double quarkonium production

3.1 Scattering amplitude

The discussion for the exclusive double production of pseudoscalar quarkonia in nucleon-nucleon collisions has considerable similarities with the double Drell-Yan process. Our presentation here is largely based on the recent work in [17] to which we refer for more details. For this process the large scale, which justifies a perturbative treatment, is given by the quarkonium mass.

A total of eight Feynman graphs contribute to lowest order in the strong coupling constant. They give rise to the scattering amplitude

$$\begin{aligned} \mathcal{T}_{\lambda_a, \lambda'_a; \lambda_b, \lambda'_b} = & -2iA \int d^2\vec{k}_{a\perp} \int d^2\vec{k}_{b\perp} \delta^{(2)}\left(\frac{\Delta\vec{q}_\perp}{2} - \vec{k}_{a\perp} - \vec{k}_{b\perp}\right) \\ & \times \left[W_{\lambda_a, \lambda'_a}^g(x_a, \vec{k}_{a\perp}) W_{\lambda_b, \lambda'_b}^g(x_b, \vec{k}_{b\perp}) + \tilde{W}_{\lambda_a, \lambda'_a}^g(x_a, \vec{k}_{a\perp}) \tilde{W}_{\lambda_b, \lambda'_b}^g(x_b, \vec{k}_{b\perp}) \right], \end{aligned} \quad (3.1)$$

which depends on the helicities of the incoming and outgoing nucleons. The explicit expression for the constant A can be found in Ref. [17]. The quantities W^g and \tilde{W}^g are correlation functions which are defined through the gluon GTMDs $F_{1,1}^g, \dots, F_{1,4}^g$ and $G_{1,1}^g, \dots, G_{1,4}^g$, respectively. Like for the double Drell-Yan process this reaction, to leading order in perturbation theory, provides access to GTMDs in the ERLB region only. According to Eq. (3.1), in general F-type and G-type GTMDs as well as their interference enter in the observables.

3.2 Polarization Observables

When defining polarization observables we focus on the GTMDs of one nucleon (nucleon N_a), while we sum/average over the helicities of the nucleon N_b . In doing so one obtains observables that are analogous to the pion-nucleon double Drell-Yan process. Below we will use $\vec{\Delta}_{b\perp} = 0$ throughout, which simplifies the expressions for the observables. Then one has (again) two independent external transverse momenta only, namely $\Delta\vec{q}_\perp = \vec{q}_{1\perp} - \vec{q}_{2\perp}$ and $\vec{\Delta}_{a\perp} = -(\vec{q}_{1\perp} + \vec{q}_{2\perp})$.

We are specifically interested in gaining access to $F_{1,4}^g$ and $G_{1,1}^g$ because of their relation to the nucleon spin structure. (These functions are the gluonic counterpart of $F_{1,4}^q$ and $G_{1,1}^q$.) In order to address $F_{1,4}^g$ one can consider the following linear combination of polarization observables,

$$\begin{aligned} & \frac{1}{4}(\tau_{UU} + \tau_{LL} - \tau_{XX} - \tau_{YY}) \\ & \approx \frac{(\epsilon_\perp^{ij} \Delta q_\perp^i \Delta a_\perp^j)^2}{M^4} C \left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}^g(x_a, \vec{k}_{a\perp}) F_{1,1}^g(x_b, \vec{k}_{b\perp}) \right] C \left[\vec{\beta}_\perp \cdot \vec{p}_{a\perp} F_{1,4}^{g*}(x_a, \vec{p}_{a\perp}) F_{1,1}^{g*}(x_b, \vec{p}_{b\perp}) \right] \\ & + C \left[G_{1,4}^g(x_a, \vec{k}_{a\perp}) G_{1,4}^g(x_b, \vec{k}_{b\perp}) \right] C \left[G_{1,4}^{g*}(x_a, \vec{p}_{a\perp}) G_{1,4}^{g*}(x_b, \vec{p}_{b\perp}) \right], \end{aligned} \quad (3.2)$$

where ϵ_\perp^{ij} is the 2-dimensional epsilon tensor, and the convolution integral C is defined in Ref [17]. The meaning of $\vec{\beta}_\perp$ is identical to the case of double Drell-Yan. Eq. (3.2) shows an approximated result (see [17] for the full expression). In order to arrive at a simplified expression for the observable we have assumed the following hierarchy of the magnitude of gluon GTMDs: $F_{1,1}^g > G_{1,4}^g \gg$ remaining GTMDs. (Recall also the reasoning in Sect. 2.2.) Unlike the double Drell-Yan process, where the polarization states of the photons can be utilized to disentangle in a model-independent

manner F-type and G-type GTMDs, this is not possible for double η_Q production. This does not work even though the prefactors of the two terms on the r.h.s. of Eq. (3.2) are different. One can readily understand this point by keeping in mind that GTMDs also depend on the variable $\vec{k}_\perp \cdot \vec{\Delta}_\perp$, and therefore also the 2nd term on the r.h.s. of (3.2) depends on the angle φ between $\vec{\Delta}_{a\perp}$ and $\Delta\vec{q}_\perp$. On the other hand, if this dependence is mild, one may be able to separate the two terms in Eq. (3.2) by measuring this observable as function of φ .

For the GTMD $G_{1,1}^g$ the approximated observable corresponding to Eq. (3.2) is

$$\begin{aligned} & \frac{1}{4} (\tau_{UU} + \tau_{LL} + \tau_{XX} + \tau_{YY}) \\ & \approx \frac{\epsilon_\perp^{ij} \Delta_{a\perp}^j}{M} \frac{\epsilon_\perp^{kl} \Delta_{a\perp}^l}{M} C \left[\frac{k_{a\perp}^i}{M} G_{1,1}^g(x_a, \vec{k}_{a\perp}) G_{1,4}^g(x_b, \vec{k}_{b\perp}) \right] C \left[\frac{p_{a\perp}^k}{M} G_{1,1}^{g*}(x_a, \vec{p}_{a\perp}) G_{1,4}^{g*}(x_b, \vec{p}_{b\perp}) \right] \\ & + C \left[F_{1,1}^g(x_a, \vec{k}_{a\perp}) F_{1,1}^g(x_b, \vec{k}_{b\perp}) \right] C \left[F_{1,1}^{g*}(x_a, \vec{p}_{a\perp}) F_{1,1}^{g*}(x_b, \vec{p}_{b\perp}) \right]. \end{aligned} \quad (3.3)$$

While we are interested in the 1st term on the r.h.s. of Eq. (3.3), based on the above discussion about the magnitude of gluon GTMDs we expect that the 2nd term clearly dominates this observable. It is therefore apparently impossible to study $G_{1,1}^g$ through this observable. This situation may change only if one considers polarization of the nucleon N_b as well. In general, one finds that the same problem also exists for other polarization observables that in principle are sensitive to $G_{1,1}^g$.

Like for double Drell-Yan [15], we also consider observables that depend on the interference between $F_{1,4}^g$ (and $G_{1,1}^g$) and other GTMDs that are expected to be large. This situation occurs for

$$\begin{aligned} & \frac{1}{2} (\tau_{UL} + \tau_{LU}) \approx -2 \frac{\epsilon_\perp^{ij} \Delta q_\perp^i \Delta_{a\perp}^j}{M^2} \\ & \times \text{Im} \left\{ C \left[\vec{\beta}_\perp \cdot \vec{k}_{a\perp} F_{1,4}^g(x_a, \vec{k}_{a\perp}) F_{1,1}^g(x_b, \vec{k}_{b\perp}) \right] C \left[F_{1,1}^{g*}(x_a, \vec{p}_{a\perp}) F_{1,1}^{g*}(x_b, \vec{p}_{b\perp}) \right] \right\}, \end{aligned} \quad (3.4)$$

In order to arrive at the approximated expression in Eq. (3.4) we have again made use of the aforementioned hierarchy for gluon GTMDs. Note that $F_{1,4}^g$ is accompanied by three powers of the large GTMD $F_{1,1}^g$. However, in Eq. (3.4) appears the imaginary part of products of GTMDs, which presumably is mostly sensitive to $\text{Im} F_{1,4}^g$. But at present the main interest is in $\text{Re} F_{1,4}^g$ (and $\text{Re} G_{1,1}^g$). To circumvent this problem one can consider $\frac{1}{2} (\tau_{XY} - \tau_{YX})$ instead of the observable in (3.4). The result in this case is identical to the r.h.s. of (3.4), but with an overall minus sign and $\text{Re} \{ \dots \}$ instead of $\text{Im} \{ \dots \}$, which implies that $\text{Re} F_{1,4}^g$ gets multiplied by the large $\text{Re} F_{1,1}^g$.

4. Summary and Outlook

We have proposed the exclusive pion-nucleon double Drell-Yan process and exclusive double production of pseudoscalar quarkonia in nucleon-nucleon collisions to probe GTMDs for quarks and gluons, respectively. To this end, we have performed a leading-order analysis in perturbative QCD, and we have shown that quark/gluon GTMDs can, in principle, be accessed in the ERBL region via suitable (linear combinations of) polarization observables.

For the future it is important to obtain (rough) numerical estimates for the various observables in order to find out if their measurement is feasible. One should also search for processes that allow one to study quark GTMDs in lepton-nucleon scattering and/or that are sensitive to the DGLAP region of GTMDs.

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