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Neutrino pion-production on a nucleon

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Heavy baryon chiral perturbation theory (χPT), where the Δ resonance is included, is used in order to examine the axial charged-current component of the weak interaction process at low neutrino energies. At leading chiral order the Adler theorems, derived using PCAC, are satisfied. At next-to-leading chiral order this effective field theory goes beyond these theorems. I will show that χPT generates deviation from the PCAC predictions, which means that some neutrino-nucleon models that are used in evaluating neutrino nucleus scattering amplitudes, might need modifications.

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1. Introduction

Neutrino beams generated in accelerator-based neutrino experiments have an energy spectrum which are typically in the range of a few hundred MeV to a few GeV. In order to accurately reconstruct the incoming neutrino energy spectrum, to be used in probing the properties of the enigmatic neutrinos, the outgoing muon is measured assuming that quasi-elastic neutrino-nucleus charge-current scattering dominates the process. Nuclear corrections due to two-nucleon correlations and other nuclear effects are incorporated in the theoretical evaluations of the neutrino-nuclear reactions. The relevant neutrino reactions occurring in this energy regime are the purely leptonic charged-current scattering, the neutrino pion-production reactions and deep inelastic scattering. A theoretical understanding of these processes is required [1] in order to accurately reconstruct the neutrino energy-distribution from these nuclear measurements. In the evaluations of these neutrino-nuclear reaction processes, neutrino-nucleon interaction models are the basic theoretical inputs, e.g., see a recent discussion in Ref. [2]. Due to the conservation of the vector current (CVC), the vector component of the weak-interaction models has been tested by electron and photo-nucleon and nuclear reactions and we assume that this vector interaction is well known. However, the neutrino-nucleon axial interaction is less well known and is the focus of this investigation.

In this paper we will discuss the axial component of the neutrino pion-production from nucleons This process is an important background reaction to the purely lepton-nucleon charged-current scattering. The pion-production reactions are manifest at a few hundred MeV where the Δ-degrees of freedom are important. At low neutrino energies below the threshold for Δ production, a low-energy effective field theory (EFT) is applicable. We will use heavy baryon chiral perturbation theory (χPT), which is an EFT respecting the symmetries and the symmetry breaking patterns of QCD. χPT assumes that we have an expansion in a typical momentum p of the order of the pion mass m_π which is small relative to the chiral scale $\Lambda_\chi \approx m_N \approx 1$GeV, i.e., $p/\Lambda_\chi \ll 1$. In χPT we treat the nucleon mass $m_N \ll m_\pi$ as heavy. Furthermore, it is assumed that the leading chiral order (LO) terms dominate and that the next order contributions give only smaller corrections to the LO results. We investigate the axial current structure predicted by χPT and we augment χPT by including the Δ degrees of freedom. At LO χPT satisfies the Adler theorems [3] which were derived using the partial conservation of the axial current (PCAC) expression, see also the review [4]. At next-to-leading chiral order (NLO) we will show that this effective field theory generates deviation from PCAC as expected. It should be noted that for s-wave pion production at threshold, χPT has already been used by Bernard *et al.* [5] to go beyond Adler's soft-pion theorems, and their expressions contain deviations from PCAC. If these NLO contributions are of practical importance, then this implies that some neutrino-nucleon models might need modifications. At next-to-next-to-leading order (NNLO) nucleon vector- and axial-vector form factors are generated in the χPT evaluation, and the Goldberger-Treiman relation acquires a correction term. This is similar to what is observed in χPT approaches to other weak interaction processes at NNLO, e.g., Ref. [6]. A covariant ChPT evaluation of the pion-production process at NNLO has recently been published [7] and the results were presented at this conference and will be summarized at the end of this paper.

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2. The χPT matrix elements

The χPT lagrangian, $\mathscr{L}_\chi$ is written as a sum of terms where the leading order term contributes the dominant amplitude to the process under study. The next order terms of the chiral lagrangian ordinarily give smaller corrections to the LO amplitude. In this note we will evaluate the LO and NLO amplitudes which require only the following terms of the chiral lagrangian,

$$\mathscr{L}_\chi = \mathscr{L}^{(1)}_{\pi N} + \mathscr{L}^{(2)}_{\pi N} + \mathscr{L}^{(2)}_{\pi\pi} + \cdots \,, \tag{2.1}$$

where $\mathscr{L}^{(2)}_{\pi\pi}$ is the lowest order pion lagrangian. The leading order pion-nucleon χPT lagrangian, $\mathscr{L}^{(1)}_{\pi N}$, contains the nucleon velocity chosen to be $v^\mu = (1,\vec{0})$ and the nucleon spin $S^\mu = (0, \frac{1}{2}\vec{\sigma})$,

$$\mathscr{L}^{(1)}_{\pi N} \;=\; N^\dagger (iv\cdot\mathscr{D} + g_A S\cdot u) N \,, \tag{2.2}$$

where g_A is the nucleon axial coupling constant and the expressions for $\mathscr{D}^\mu$ and u^μ can be found in, e.g., Ref. [8]. The NLO correction terms are contained in $\mathscr{L}^{(2)}_{\pi N}$. The explicit expressions for the next order lagrangians, $\mathscr{L}^{(2)}_{\pi N}$ are given in, e.g., Ref. [8]. It is assumed that the next-to-next-to-leading order corrections (NNLO) give smaller corrections than the NLO contributions. We will briefly discuss the importance of the NNLO contributions at the end of this article. We mention that the effective Δ degrees of freedom are an important part in understanding the low-energy neutrino pion productions. We introduced the effective Δ resonance following Hemmert et al. [9], where it is assumed that the nucleon Δ mass difference, $m_\Delta - m_N = \delta_{N\Delta}$ is considered to be of the order of the pion mass, m_π. This differ from the Pascalutsa Phillips treatment of the Δ resonance [10] used in the relativistic ChPT approach of Yao *et al.* [7].

2.1 A short note on the heavy nucleon (and Δ) propagator

In this work the nucleon and Δ propagators are treated differently from the usual HBChPT approach. In the usual HBChPT lagrangian approach the heavy nucleon propagator of momentum $p'_1 + k$ is derived from $\mathscr{L}^{(1)}_{\pi N} = N^\dagger \left\{ v\cdot\partial + \cdots \right\} N$, and is

$$\frac{i}{v\cdot(p'_1+k)} \,. \tag{2.3}$$

The next order lagrangian $\mathscr{L}^{(2)}_{\pi N}$ contains the kinetic Schrödinger operator,

$$\mathscr{L}^{(2)}_{\pi N} \;=\; N^\dagger \left\{ -\frac{1}{2m_N}\mathscr{D}\cdot\mathscr{D} + \frac{1}{2m_N}(v\cdot\mathscr{D})^2 + \cdots \right\} N \,, \tag{2.4}$$

where $\mathscr{D}^\mu = \partial^\mu + \cdots$. The kinetic terms of $\mathscr{L}^{(2)}_{\pi N}$ in Eq. (2.4) generate propagator corrections at second order, i.e., $1/(2m_N)$, as shown by Bernard *et al.* [8, 11]. In the low-energy pion production in two-nucleon collisions [12], it appears to be more effective to treat the nucleon propagator in the following manner, see Refs. [13, 14] for discussions,

$$\frac{i}{v\cdot(p'_1+k) - \frac{(\vec{p}\,'_1+\vec{k})^2}{2m_N}} = \frac{i}{v\cdot k}\left\{ 1 + \frac{\vec{k}^2 + 2\vec{p}\,'_1\cdot\vec{k}}{2m_N(v\cdot k)} + \cdots \right\} . \tag{2.5}$$

The essential argument is that since $(v\cdot p'_1) \approx \frac{(\vec{p}\,'_1)^2}{2m_N}$, one of the leading terms is cancelled by a $1/(2m_N)$ term. We therefore expand the propagator in orders of m_N^{-1} as in Eq. (2.5).

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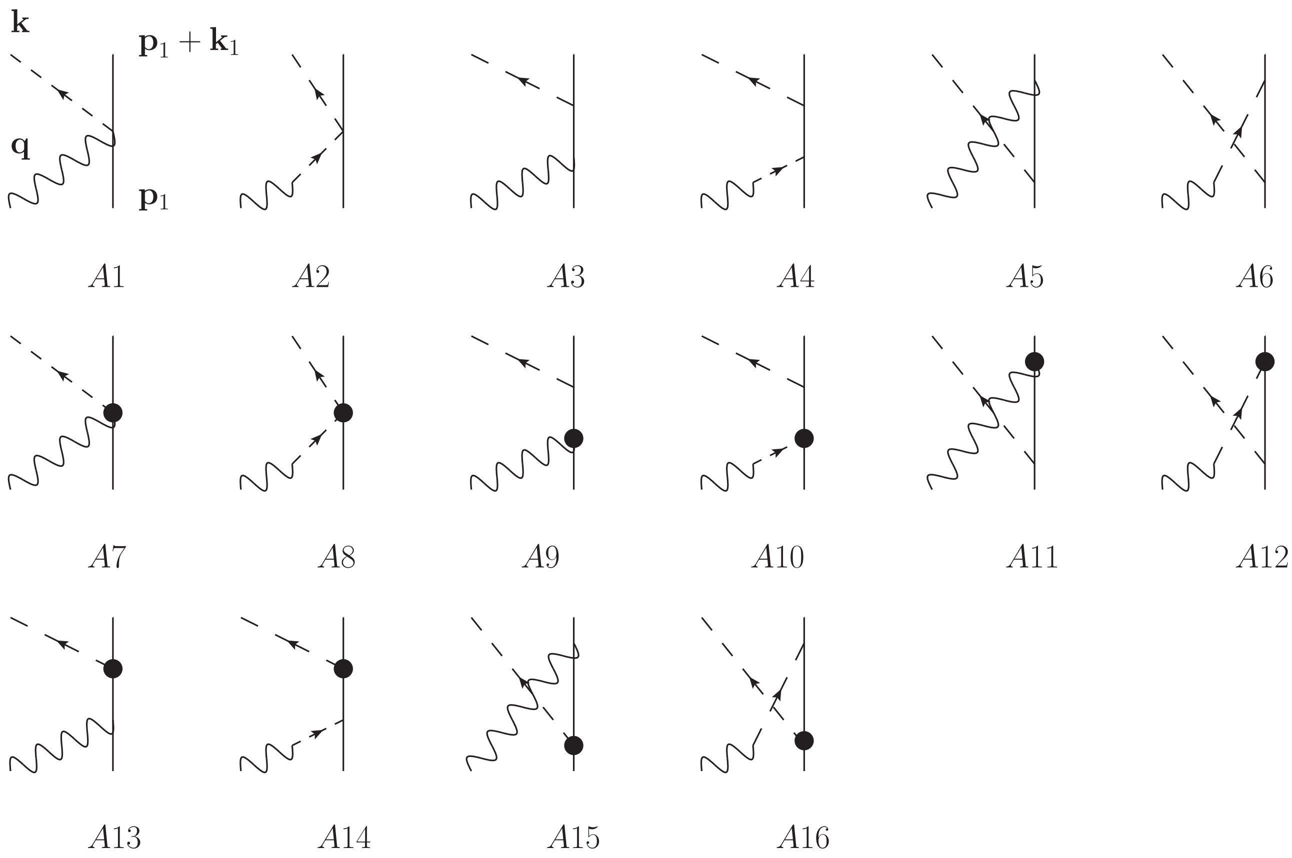

Figure 1: The incoming wavy line is the axial probe with isospin index α and four-momentum q. The incoming axial probe contains the pion-pole as shown explicitly in the diagrams. The dashed line is the outgoing pion of isospin index β and four-momentum k. The solid line is the nucleon line with outgoing momentum $p_1' = p_1 + k_1$. The blobs signify vertices derived from $\mathscr{L}^{(2)}_{\pi N}$. When we evaluate the contributions from the effective Δ, the internal nucleon lines, i.e., nucleon propagators, are replaced by Δ propagators .

2.2 The axial-current contribution to the matrix elements

One remark concerns a leading order diagram, A2, in Fig. 1. The LO Weinberg-Tomozawa (W.-T.) vertex in diagram A2 contains the sum of the energies for the incoming and outgoing pions, $v \cdot (q+k)$, and four-momentum conservation gives, when expanded, the expression

$$v \cdot (q+k) \,=\, v \cdot k + v \cdot p_1' - v \cdot p_1 + v \cdot k = 2\omega_\pi + \frac{\vec{p}_1^{\,\prime 2} - \vec{p}_1^{\,2}}{2m_N} + \cdots . \tag{2.6}$$

Using the first term on the r.h.s. of the last equation (2.6), we confirm that the LO hadronic matrix elements reproduce the soft-pion theorem or PCAC. The expression, which follows below, is derived from the Feynman diagrams in Fig. 1. The nucleon axial-vector probe is indicated by the axial four-vector $\varepsilon_A^\mu = (v \cdot \varepsilon_A, \vec{\varepsilon}_A)$. If PCAC is operative, we expect to find that each axial operator of the matrix elements has the following structure $\varepsilon_A^\mu - \left(\frac{(\varepsilon_A \cdot q)}{q^2 - m_\pi^2}\right) l^\mu$, where l^μ is the μ'th component of a four-vector. As is evident from the expression in Eq. (2.7), we find some terms in the matrix element which do not conform to the expected PCAC-like expression. On closer inspections, these terms all originate from the $1/(2m_N)$ contributions of the LO diagram A2 (see below) and from the NLO diagrams A7 and A8. These are the diagrams where the axial probe produce a pion in a single vertex on the nucleon.

$$
\begin{aligned}
M_A \simeq\, & \varepsilon^{\alpha\beta c}\tau_1^c\left[\frac{(-1)}{2f_\pi}\left(\varepsilon_A\cdot v-\frac{(\varepsilon_A\cdot q)k\cdot v}{q^2-m_\pi^2}\right)+\frac{g_A^2}{2f_\pi\omega_\pi}\left(\vec{\varepsilon}_A\cdot\vec{k}-\frac{(\varepsilon_A\cdot q)\vec{q}\cdot\vec{k}}{q^2-m_\pi^2}\right)\right]-\frac{\delta^{\alpha\beta}g_A^2}{2f_\pi\omega_\pi}\left(\vec{\varepsilon}_A\cdot(\vec{\sigma}_1\times\vec{k})-\frac{(\varepsilon_A\cdot q)\vec{q}\cdot(\vec{\sigma}_1\times\vec{k})}{q^2-m_\pi^2}\right)\\
&+\frac{\varepsilon^{\alpha\beta c}\tau_1^c}{2f_\pi}\left[\frac{(\varepsilon_A\cdot q)}{(q^2-m_\pi^2)}\right]\frac{(\vec{p}_1^{\,\prime 2}-\vec{p}_1^{\,2})}{4m_N}+\\
&+\frac{g_A^2}{4m_N f_\pi(v\cdot k+i\eta)}\left[\vec{\varepsilon}_A-\left(\frac{(\varepsilon_A\cdot q)}{q^2-m_\pi^2}\right)\vec{q}\right]\cdot\Bigg\{\varepsilon^{\alpha\beta c}\tau_1^c\left[\vec{k}\left[\frac{\vec{k}\cdot(\vec{p}_1^{\,\prime}+\vec{p}_1)}{v\cdot k}\right]+i\left(\vec{\sigma}_1\times\vec{k}\right)\left[\frac{\vec{k}\cdot(\vec{k}+\vec{k}_1)}{v\cdot k}\right]\right]\\
&-\delta^{\alpha\beta}\left[\left(\vec{\sigma}_1\times\vec{k}\right)\left[\frac{\vec{k}\cdot(\vec{p}_1^{\,\prime}+\vec{p}_1)}{v\cdot k}\right]-i\vec{k}\left[\frac{\vec{k}\cdot(\vec{k}+\vec{k}_1)}{v\cdot k}\right]\right]\Bigg\}\\
&+(-i)\frac{\delta^{\alpha\beta}}{f_\pi}\left\{\left[\varepsilon_A-\left(\frac{(\varepsilon_A\cdot q)}{q^2-m_\pi^2}\right)q\right]\cdot\left[v\left(2c_2-\frac{g_A^2}{4m_N}\right)(v\cdot k)+k\,2c_3\right]\right.\\
&\left.+\left(\frac{\varepsilon_A\cdot q}{q^2-m_\pi^2}\right)4c_1m_\pi^2\right\}+\frac{\varepsilon^{\alpha\beta c}\tau_1^c}{f_\pi}\left\{\left(\frac{1+\kappa_V}{4m_N}\right)i\vec{\varepsilon}_A\cdot(\vec{\sigma}_1\times\vec{q})+\frac{\vec{\varepsilon}_A\cdot(\vec{p}_1+\vec{p}_1^{\,\prime})}{8m_N}+\right.\\
&\left.+\left[\vec{\varepsilon}_A-\left(\frac{(\varepsilon_A\cdot q)}{q^2-m_\pi^2}\right)(\vec{q}+\vec{k})\right]\cdot\left[\frac{(\vec{p}_1+\vec{p}_1^{\,\prime})}{8m_N}-\left(c_4+\frac{1}{4m_N}\right)i\left(\vec{\sigma}_1\times\vec{k}\right)\right]\right\}\\
&+\frac{(-i)g_A^2}{4m_Nf_\pi}\left(\frac{1}{v\cdot k+i\eta}\right)\left[\varepsilon_A-\frac{(\varepsilon_A\cdot q)q}{q^2-m_\pi^2}\right]\cdot v\left\{\delta^{\alpha\beta}\left[\vec{k}^2+i\vec{\sigma}_1\cdot\left(\vec{k}\times(\vec{p}_1+\vec{p}_1^{\,\prime})\right)\right]+i\varepsilon^{\beta\alpha c}\tau_1^c\left[\vec{k}\cdot(\vec{p}_1+\vec{p}_1^{\,\prime})\right]\right\}\\
&+\frac{(-i)g_A^2}{4m_Nf_\pi}\left(\frac{\omega_\pi}{v\cdot k+i\eta}\right)\left[\vec{\varepsilon}_A-\left(\frac{\varepsilon_A\cdot q}{q^2-m_\pi^2}\right)\vec{q}\right]\cdot\Big\{\delta^{\alpha\beta}\left[(\vec{p}_1^{\,\prime}-\vec{p}_1+\vec{k})+i\left(\vec{\sigma}_1\times(\vec{p}_1+\vec{p}_1^{\,\prime})\right)\right]\\
&+i\varepsilon^{\beta\alpha c}\tau_1^c\left[(\vec{p}_1+\vec{p}_1^{\,\prime})+i\left(\vec{\sigma}_1\times(\vec{p}_1^{\,\prime}-\vec{p}_1+\vec{k})\right)\right]\Big\}
\end{aligned}
\tag{2.7}
$$

In this expressions the low-energy constants (LECs), c_i where $i=1,\cdots,4$, are determined by pion-nucleon scattering data. The first line of Eq. (2.7) contains the LO contributions which exhibit the expected PCAC structure. The second line in Eq. (2.7) contains the m_N^{-1} correction of the W.-T. vertex given in Eq. (2.6) while the third and fourth lines are the m_N^{-1} correction from the nucleon propagators of the LO diagrams in Fig. 1. The main deviations from the expected PCAC-like expressions are the three terms in the sixth line. The most conspicuous one contains the nucleon iso-vector magnetic moment $1+\kappa_V$. In some models of this neutrino reaction, this contribution could be associated with the "heavy" ρ-meson which couple to the nucleon in a t-channel-like exchange, e.g, Ref. [2]. In models for the elementary neutrino - nucleon reaction amplitudes used in neutrino nucleus descriptions, it is frequently assumed that PCAC is valid. However, some of the NLO contributions in Eq. (2.7), which are derived using χPT, reveal that PCAC should be corrected at NLO (and NNLO), corrections which were included in Ref. [5]. Furthermore, we observe that the term with the LEC, c_1, which is connected to the pion-nucleon sigma term, only contributes to the pion-pole amplitude of the axial current.

The tree-diagrams involving an intermediate Δ excitation are included and at leading order (as well as the next order) the Δ contribution to the hadronic matrix element has the structure expected from PCAC, e.g., at the lowest order we obtain

$$
\begin{aligned}
\mathscr{M}_\Delta = & \left((-i)\frac{g^2_{\pi N\Delta}}{f_\pi}\right)\left\{\frac{-4}{9}\delta^{\alpha\beta}\left[\vec{\varepsilon}_A\cdot\vec{k}-\frac{(\varepsilon_A\cdot q)(\vec{q}\cdot\vec{k})}{q^2-m_\pi^2}\right]\left[\frac{2\delta_{N\Delta}}{(v\cdot k)^2-(\delta_{N\Delta})^2}\right]\right. \\
& + \frac{+2i}{9}\varepsilon^{\alpha\beta c}\tau_1^c\left[\vec{\varepsilon}_A\cdot\vec{k}-\frac{(\varepsilon_A\cdot q)(\vec{q}\cdot\vec{k})}{q^2-m_\pi^2}\right]\left[\frac{2(v\cdot k)}{(v\cdot k)^2-(\delta_{N\Delta})^2}\right] \\
& + \frac{2i}{9}\delta^{\alpha\beta}\left[\vec{\varepsilon}_A\cdot(\vec{\sigma}_1\times\vec{k})-\frac{(\varepsilon_A\cdot q)\vec{\sigma}_1\cdot(\vec{k}\times\vec{q})}{q^2-m_\pi^2}\right]\left[\frac{2(v\cdot k)}{(v\cdot k)^2-(\delta_{N\Delta})^2}\right] \\
& \left. + \frac{(+1)}{9}\varepsilon^{\alpha\beta c}\tau_1^c\left[\vec{\varepsilon}_A\cdot(\vec{\sigma}_1\times\vec{k})-\frac{(\varepsilon_A\cdot q)\vec{\sigma}_1\cdot(\vec{k}\times\vec{q})}{q^2-m_\pi^2}\right]\left[\frac{2\delta_{N\Delta}}{(v\cdot k)^2-(\delta_{N\Delta})^2}\right]\right\}
\end{aligned}
\tag{2.8}
$$

3. A short note on the NNLO contributions

The neutrino production of pions at low energy has been evaluated by Yao *et al.* [7] in a covariant ChPT approach. Yao *et al.* included LO, NLO and NNLO contributions of this weak-interaction reaction. As shown in Ref. [7], the NNLO corrections are larger than expected and are very important in order to reproduce the measured low-energy neutrino cross sections. It is expected that the Δ contributions are dominant for neutrino energies of about 1 GeV, and the Δ resonance is included in models which evaluate the neutrino-nuclear reactions, see, e.g., Ref. [15] and later works along these lines [16, 17]. In addition, Ref. [15] included the leading order ChPT contributions. Yao *et al.* showed that at neutrino energies below the Δ-resonance, this resonance is important for the final proton π^+ final state. However, at a neutrino energy of about 400 MeV and for final neutron π^+ or proton π^0 states, their covariant ChPT evaluation showed that the nucleon degrees of freedom are very important, and that the Δ is not as prominent as assumed in many model calculations.

4. Summary

We use the effective low-energy field theory called heavy baryon chiral perturbation theory and reproduce the soft-pion theorems at leading order. We show at next-to-leading order, which goes beyond the soft-pion theorems, that the axial current contribution to the neutrino pion-production reaction on a nucleon gives corrections to PCAC. In other words, our finding does not conform with some model descriptions which assumes PCAC is valid, and which are used in neutrino-nucleus reactions.

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