PROCEEDINGS OF SCIENCE

Experimental input for the HLbL contribution to the muon $(g-2)$

Andrzej Kupsc*
Uppsala University
E-mail: Andrzej.Kupsc@physics.uu.se

A summary of the recent experimental results relevant for the hadronic light-by-light contribution to the muon anomalous magnetic moment is given. The focus is on the data related to the neutral pion, η and η' transition form factors.

The 9th International workshop on Chiral Dynamics
17-21 September 2018
Durham, NC, USA

*Speaker.

PoS(CD2018)073

1. Introduction

It is now clear that direct searches for new particles at the high-energy accelerators must be complemented by low-energy high-precision measurements. One very promising observable is the muon anomalous magnetic moment [1]. The persistent deviation of 3–4 standard deviations between Standard Model (SM) predictions [2, 3] and the precision BNL experiment [4] has motivated a new measurement at Fermilab with the 0.14 ppm accuracy [5]. With the new experiment under way it is necessary to improve the SM prediction correspondingly. In order to uncover effects from quantum fluctuations caused by new physics one needs to account for the quantum fluctuations caused by the known particles. The most difficult to determine precisely is the contribution from hadrons. At present the most reliable way to account for the hadronic contributions to $(g-2)$ is a data-driven approach based on dispersion theory. This first-principle approach connects the hadronic quantum fluctuations that enter $(g-2)$ to other observables. The relevant hadronic contributions to $(g-2)$ come from the hadronic vacuum polarization (HVP) and from hadronic light-by-light scattering (HLbL). For the HVP contribution the dispersion theory relates it directly to the total cross section of $e^+e^- \to$hadrons [1].

For the HLbL a much more complicated dispersive framework is needed [6, 7, 8]. The numerically most important contributions to HLbL come from the neutral pion-pole diagram [9, 10], poles of other light pseudoscalar mesons ($P = \eta, \eta'$) and two pion contributions [11]. The necessary input to calculate the pseudoscalar meson pole contribution to $(g-2)$ are transition form factors (TFF) of the corresponding pseudoscalar mesons: Fig. 1(left). In principle the TFFs could be measured experimentally and the status of the recent experiments is presented in the next section. Many of the recent experimental studies of the TFFs were initially motivated by the searches for a dark photon [12]. Recent estimates of the π^0, η and η' poles contribution to HLbL using directly the experimental data on the TFFs are given in Refs. [10, 13, 14]. Alternatively the pseudoscalar meson pole contribution could be obtained from a selected data on hadronic processes using the corresponding dispersive relations. For the π^0 TFF such project has been just finished [15, 16] and for η and η' it is in progress [17]. In the dispersive framework for HLbL [7] the next contribution is from two pseudoscalar processes, where the most important new experimental input comes from two photon production of charged pion pairs.

2. Transition Form Factors

A transition form factor $\mathscr{F}_P(q_1^2, q_2^2)$ is a scalar function of the four-momentum transfer squared of the virtual photons ($q_{1,2}^2$) describing the vertex in Fig. 1(left) and defined as [18, 19]:

$$\mathscr{A}(P \to \gamma^* \gamma^{(*)}) = q_1^\mu \varepsilon_1^\nu q_2^\alpha \varepsilon_2^\beta \varepsilon_{\mu\nu\alpha\beta} \mathscr{F}_P(q_1^2, q_2^2) \tag{2.1}$$

and

$$\frac{m_P^3}{64\pi} |\mathscr{F}_P(0,0)|^2 = \Gamma(P \to \gamma\gamma), \tag{2.2}$$

where $\Gamma(P \to \gamma\gamma)$ is the radiative width of the meson P. The most precise measurements of the π^0 radiative decay width use Primakoff process, photoproduction of pseudoscalar mesons in electric field of nucleus. The value from the PrimEx experiment is $(7.82 \pm 0.14 \pm 0.17)$ eV [20]. A new preliminary result with improved accuracy from PrimExII was presented on this workshop. For the η meson the best result for $\Gamma(\eta \to \gamma\gamma)$ is determined from the cross section of the $e^+e^- \to e^+e^-P$ process at center-of-mass (c.m.) energy of 1 GeV by the KLOE-2 experiment where the final electrons escape the detector at low scattering angles. This cross section is dominated by the contributions from photons with virtualities close to zero, leading to 4.6% accuracy for the η radiative width: $(520 \pm 20 \pm 13)$ eV [21].

In the following a normalized transition form factor:

$$F_P(q_1^2, q_2^2) = \frac{\mathscr{F}_P(q_1^2, q_2^2)}{\mathscr{F}_P(0,0)} \tag{2.3}$$

will be used to study the dependence on the virtualities of the two photons. The photons could be on the mass shell, spacelike or timelike. A quantity often considered in the context of transition form factors is the form factor slope defined as

$$b_P = \left. \frac{\partial \ln F_P(q^2, 0)}{\partial q^2} \right|_{q^2=0} . \tag{2.4}$$

For π^0 it is expressed using a dimensionless parameter: $a_\pi = b_{\pi^0}/m_{\pi^0}^2$. The best experimental result for a_π is from spacelike TFF measurement by CLEO: $a_\pi = (3.26 \pm 0.26 \pm 0.26) \cdot 10^{-2}$ [22]. This value can be compared to $a_\pi = (3.15 \pm 0.09) \cdot 10^{-2}$ from recent dispersive calculations based only on hadronic data [15, 16]. The slope can be generalized to

$$b_P(q_2^2) = \left. \frac{\partial \ln |F_P(q_1^2, q_2^2)|}{\partial q_1^2} \right|_{q_1^2=0} , \tag{2.5}$$

relevant for Dalitz decays of vector mesons like $\phi \to \pi^0 e^+e^-$. Experimental data on the slope parameters for the slopes $b_{\pi^0}(q^2)$ (for $q^2 = m_\omega^2$ and $q^2 = m_\phi^2$) and the $b_\eta(q^2)$ (for $q^2 = 0$ and $q^2 = m_\phi^2$) are shown in Table 1.

An illustration of the π^0 and η TFFs is provided in Fig. 1 based on a simplified hadronic phenomenological model [19]. In principle the necessary information could be provided by precision data on the TFFs for arbitrary pairs of photon virtualities. The separate kinematical regions for TFFs shown in Fig. 1 are probed by the following processes:

1. The space like region with $q_1^2, q_2^2 \le 0$ (virtualities of both photons are zero or spacelike) is probed by Primakoff effect or $e^+e^- \to e^+e^-P$ where two photons fuse to form a pseudoscalar meson. Analyses to extract single off shell TFF ($q_1^2 < 0$ and $q_2^2 = 0$) for π^0 are underway by BESIII with preliminary results shown in Ref. [32] and by KLOE-2 [33]. Recently the BaBar Collaboration has presented recently the first measurement of the doubly off-shell η' TFF [34].

2. $P \to \gamma e^+e^-$ covers $4m_P^2 > q_1^2 > 4m_e^2, q_2^2 = 0$ region. For $\pi^0 \to \gamma e^+e^-$ this process was studied by NA62 [35] – $a_\pi = (3.68 \pm 0.51 \pm 0.25) \cdot 10^{-2}$ and A2 [36] – $a_\pi = (3 \pm 1) \cdot 10^{-2}$. The η meson single Dalitz decays were recently studied by the A2 Collaboration ($\eta \to e^+e^-\gamma$) [26] and the NA60 Collaboration [25] ($\eta \to \mu^+\mu^-\gamma$). The corresponding results for the $b_\eta(0)$ slope parameter are in Table. 1. The first observation of $\eta' \to \gamma e^+e^-$ was reported by BESIII [37].

$b_{\pi^0}(q^2)$ [GeV^{-2}]		
$b_{\pi^0}(m_\omega^2)$	2.4 ± 0.2	Lepton-G [23]
$b_{\pi^0}(m_\omega^2)$	$2.24 \pm 0.06 \pm 0.02$	NA60 [24]
$b_{\pi^0}(m_\omega^2)$	2.223±0.026±0.037	NA60 [25]
$b_{\pi^0}(m_\omega^2)$	1.99±0.21	A2 [26]
$b_\eta(q^2)$ [GeV^{-2}]		
$b_\eta(0)$	1.9 ± 0.4	Lepton-G [27]
$b_\eta(0)$	1.42 ± 0.21	CELLO [22]
$b_\eta(0)$	$1.95 \pm 0.17 \pm 0.05$	NA60 [24]
$b_\eta(0)$	$1.92 \pm 0.35 \pm 0.13$	CB/TAPS [28]
$b_\eta(0)$	1.934±0.067±0.050	NA60 [25]
$b_\eta(0)$	1.97±0.11	A2 [26]
$b_\eta(m_\phi^2)$	3.8 ± 1.8	SND [29]
$b_\eta(m_\phi^2)$	1.17±0.10±0.07	KLOE-2 [30]
$b_{\pi^0}(m_\phi^2)$	2.02±0.11	KLOE-2 [31]

Table 1: Summary of the experimental data on $b_\eta(q^2)$ and $b_{\pi^0}(q^2)$.

3. $P \to 2e^+2e^-$ happens in the hardly visible tiny region between the positive axes and the hyperbola. The process corresponds to the case when the real photons are replaced by two dileptons and one explores the doubly virtual case with both virtualities timelike. In Ref. [38] KLOE-2 has reported first observation of $\eta \to 2e^+2e^-$.

4. $e^+e^- \to P\gamma$ covers $q_1^2 > 4m_P^2, q_2^2 = 0$ region. In principle this process is a part of the hadronic cross section contributing to HVP. In particular $e^+e^- \to \pi^0\gamma$ represents HVP process with the lowest threshold at the c.m. energy of m_{π^0}. However, the cross section is low and therefore the process is mainly of importance for the HLbL contribution. So far the it was studied only in the scan experiments where the collider c.m. energy is tuned to a specific q_1 value to measure the cross section. In the most relevant/important region for the muon $(g-2)$ this is only possible at the Novosibirsk VEPP-2000 collider. The most recent results are from SND detector in the $0.60-1.38$ GeV [39] and $1.075-2$ GeV [40] ranges.

5. $e^+e^- \to \gamma^* \to Pe^+e^-$ corresponds to the colourful regions to the left and right of the hyperbolas. It was studied directly when the c.m. of the reaction was fixed to a narrow vector meson mass such as ω, ϕ or J/ψ resonances. Detailed analysis of the electron-positron invariant mass distribution in the $\phi \to Pe^+e^-$ decay were performed at KLOE-2 [30, 31] and $J/\psi \to Pe^+e^-$ at BESIII [41]. The process can be studied in vector meson decays regardless the production method: in photoproduction by the A2 Collaboration: $\omega \to \pi^0e^+e^-$ [26] and in hadronic processes by the NA60 Collaboration: $\omega \to \mu^+\mu^-\pi^0$ [25]. The π^0 TFF was

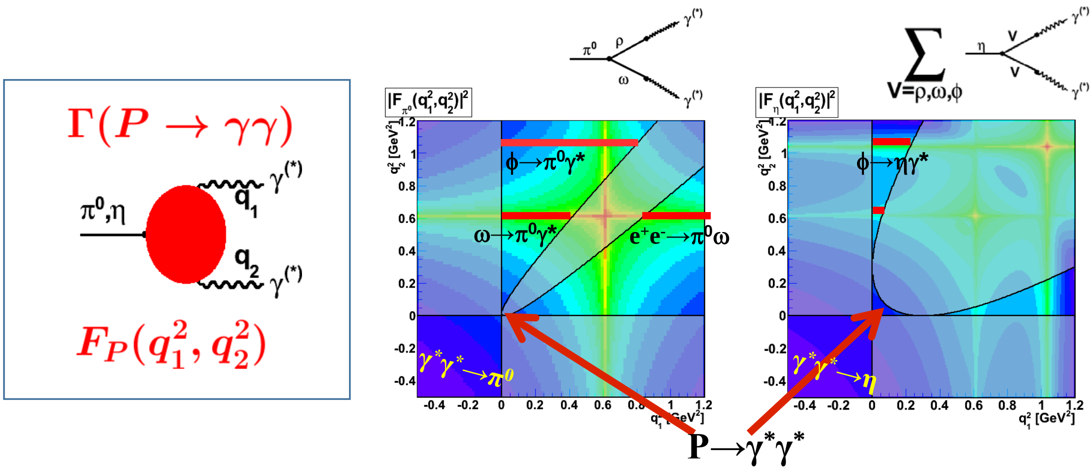

Figure 1: From the left: $P\gamma^*\gamma^*$ vertex – transition form factor; various kinematical regions of the pion and eta TFF as function of the photon virtualities q_1^2, q_2^2 [19]. The ridges are caused by the direct coupling of photons to vector mesons. The greyish regions top left, bottom right and inside of the hyperbola are not experimentally accessible.

extracted for $q_1^2 = m_\omega^2$ and $4m_l^2 < q_2^2 < (m_\omega - m_{\pi^0})^2$ including the $b_{\pi^0}(m_\omega^2)$ values given in Table 1. The directly related process $e^+e^- \to \omega\pi^0$ allows to probe the $(m_\omega + m_{\pi^0})^2 < q_2^2$ region with the most recent results from the SND Collaboration [42].

In addition the decays of the pseudoscalar mesons into lepton-antilepton pair, $P \to l^+l^-$, involve two photon intermediate state. The branching fraction values provide a sensitive test for the TFFs with doubly virtual photons since the process involves a loop where q_1^2 and q_2^2 can take arbitrary values. The decays into the electron-positron pair are additionally suppressed by the (approximate) electron helicity conservation. The puzzle of the observed enhanced $\pi^0 \to e^+e^-$ decay rate [43, 44] is most likely explained by the higher order radiative corrections [45]. The decays $\eta^{(\prime)} \to e^+e^-$ were not yet observed. The best upper limits come from formation experiments $e^+e^- \to \gamma^*\gamma^* \to P$ at VEPP-2000. The 90% C.L. upper limits are $B(\eta \to e^+e^-) < 7 \cdot 10^{-7}$ [46] and $B(\eta' \to e^+e^-) < 5.6 \cdot 10^{-9}$ [47].

3. Other relevant data

Both for the spacelike and timelike region it is unlikely to obtain doubly virtual data with appropriate precision in the near future. The processes are suppressed by powers of α_{em} and by photon propagators that scale inversely with the photon virtualities. An alternative approach to TFFs determination is given by dispersion theory [9, 15, 16]. The π^0 TFF is related to pion vector form factor and to three-pion–photon amplitude. Such contribution could be represented by isovector and isoscalar photons as illustrated in a simplified Vector Meson Dominance diagram in Fig. 1(center). The three-pion–photon amplitude for arbitrary photon virtualities and arbitrary values of the two-pion Mandelstam variables are needed. The dependence on the photon virtuality must be determined from data whereas the dependence on the Mandelstam variables is dominated by pion rescattering and can be predicted by dispersion theory based on the available very precise pion-pion phase shifts [48]. The experimental input for this approach to the pion TFF is given by the studies of the $e^+e^- \to \pi^+\pi^-\pi^0$ reaction cross section and the dynamical variables in this three body process. In addition to the cross section studies in the context of HVP, the analysis of $\omega, \phi \to \pi^+\pi^-\pi^0$ decay dynamics provides a valuable cross check of the dispersive formalism. Until recently, surprisingly little information had been available on the $\omega \to \pi^+\pi^-\pi^0$ Dalitz plot. First observation of a deviation from P-wave phase space consistent with ρ meson contribution was reported by WASA-at-COSY [49]. Recently high statistics result from BESIII was released [50], with accuracy allowing to test dispersive calculations [51, 52].

For η and η' the relation between radiative and Dalitz decays is obtained in a simple, model independent way. The precision experimental data on the radiative decays $\eta^{(\prime)} \to \gamma\pi^+\pi^-$ are used to predict the form factors in in the region relevant to $\eta^{(\prime)} \to \gamma e^+e^-$ [53, 54, 55]. The experimental input for $\eta \to \gamma\pi^+\pi^-$ is provided by WASA-at-COSY [56] and KLOE-2 [57] and for $\eta' \to \gamma\pi^+\pi^-$ by BESIII [58]. To extend the predictions to other kinematic regions of the η and η' TFFs, the data for other processes are needed *e.g.* $e^+e^- \to \eta^{(\prime)}\pi^+\pi^-$ (recent measurements from SND [59] and BaBar [60]) or decays $\eta^{(\prime)} \to \pi^+\pi^-e^+e^-$ (recent study of the η decay at WASA-at-COSY [61]).

For the $\gamma\gamma^{(*)} \to PP$ processes there is recent result from Belle for the $\pi^0\pi^0$ pair production [62]. Two photon production of $\pi^+\pi^-$ pairs is currently analyzed at BESIII [63].

References

[1] F. Jegerlehner and A. Nyffeler, *The Muon g-2*, *Phys. Rept.* **477** (2009) 1.

[2] M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, *Reevaluation of the Hadronic Contributions to the Muon $g-2$ and to $\alpha(MZ)$*, *Eur. Phys. J.* **C71** (2011) 1515, [Erratum: Eur. Phys. J.C72,1874(2012)].

[3] K. Hagiwara, R. Liao, A.D. Martin, D. Nomura and T. Teubner, *$(g-2)_\mu$ and $\alpha(M_Z^2)$ re-evaluated using new precise data*, *J. Phys.* **G38** (2011) 085003.

[4] MUON G-2 collaboration, G.W. Bennett et al., *Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL*, *Phys. Rev.* **D73** (2006) 072003.

[5] MUON G-2 collaboration, A. Chapelain, *The Muon g-2 experiment at Fermilab*, *EPJ Web Conf.* **137** (2017) 08001.

[6] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, *Dispersive approach to hadronic light-by-light scattering*, *JHEP* **09** (2014) 091.

[7] G. Colangelo, M. Hoferichter, B. Kubis, M. Procura and P. Stoffer, *Towards a data-driven analysis of hadronic light-by-light scattering*, *Phys. Lett.* **B738** (2014) 6.

[8] V. Pauk and M. Vanderhaeghen, *Anomalous magnetic moment of the muon in a dispersive approach*, *Phys. Rev.* **D90** (2014) 113012.

[9] M. Hoferichter, B. Kubis, S. Leupold, F. Niecknig and S.P. Schneider, *Dispersive analysis of the pion transition form factor*, *Eur. Phys. J.* **C74** (2014) 3180.

[10] A. Nyffeler, *Precision of a data-driven estimate of hadronic light-by-light scattering in the muon $g-2$: Pseudoscalar-pole contribution*, *Phys. Rev.* **D94** (2016) 053006.

[11] G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, *Dispersion relation for hadronic light-by-light scattering: two-pion contributions*, *JHEP* **04** (2017) 161.

[12] M. Pospelov, *Secluded U(1) below the weak scale*, *Phys. Rev.* **D80** (2009) 095002.

[13] P. Masjuan and P. Sanchez-Puertas, *Pseudoscalar-pole contribution to the* $(g_\mu - 2)$*: a rational approach*, *Phys. Rev.* **D95** (2017) 054026.

[14] H. Czyż, P. Kisza and S. Tracz, *Modeling interactions of photons with pseudoscalar and vector mesons*, *Phys. Rev.* **D97** (2018) 016006.

[15] M. Hoferichter, B.L. Hoid, B. Kubis, S. Leupold and S.P. Schneider, *Pion-pole contribution to hadronic light-by-light scattering in the anomalous magnetic moment of the muon*, *Phys. Rev. Lett.* **121** (2018) 112002.

[16] M. Hoferichter, B.L. Hoid, B. Kubis, S. Leupold and S.P. Schneider, *Dispersion relation for hadronic light-by-light scattering: pion pole*, *JHEP* **10** (2018) 141.

[17] C.W. Xiao, T. Dato, C. Hanhart, B. Kubis, U.G. Meißner and A. Wirzba, *Towards an improved understanding of* $\eta \to \gamma^* \gamma^*$, `arXiv:1509.02194`.

[18] L.G. Landsberg, *Electromagnetic Decays of Light Mesons*, *Phys. Rept.* **128** (1985) 301.

[19] E. Czerwinski, S. Eidelman, C. Hanhart, B. Kubis, A. Kupsc, S. Leupold et al., *MesonNet Workshop on Meson Transition Form Factors*, 2012, `arXiv:1207.6556`.

[20] PrimEx collaboration, I. Larin et al., *A New Measurement of the* π^0 *Radiative Decay Width*, *Phys. Rev. Lett.* **106** (2011) 162303.

[21] KLOE-2 collaboration, D. Babusci et al., *Measurement of* η *meson production in* $\gamma\gamma$ *interactions and* $\Gamma(\eta \to \gamma\gamma)$ *with the KLOE detector*, *JHEP* **01** (2013) 119.

[22] CELLO collaboration, H.J. Behrend et al., *A Measurement of the* π^0*,* η *and* η' *electromagnetic form-factors*, *Z. Phys.* **C49** (1991) 401.

[23] R.I. Dzhelyadin et al., *Study of the Electromagnetic Transition Form-factor in* $\omega \to \pi^0 \mu^+ \mu^-$ *Decay*, *Phys. Lett.* **102B** (1981) 296, [JETP Lett.33,228(1981)].

[24] NA60 collaboration, R. Arnaldi et al., *Study of the electromagnetic transition form-factors in* $\eta \to \mu^+ \mu^- \gamma$ *and* $\omega \to \mu^+ \mu^- \pi^0$ *decays with NA60*, *Phys. Lett.* **B677** (2009) 260.

[25] NA60 collaboration, R. Arnaldi et al., *Precision study of the* $\eta \to \mu^+ \mu^- \gamma$ *and* $\omega \to \mu^+ \mu^- \pi^0$ *electromagnetic transition form-factors and of the* $\rho \to \mu^+ \mu^-$ *line shape in NA60*, *Phys. Lett.* **B757** (2016) 437.

[26] P. Adlarson et al., *Measurement of the* $\omega \to \pi^0 e^+ e^-$ *and* $\eta \to e^+ e^- \gamma$ *Dalitz decays with the A2 setup at MAMI*, *Phys. Rev.* **C95** (2017) 035208.

[27] R.I. Dzhelyadin et al., *Investigation of* η *Meson Electromagnetic Structure in* $\eta \to \mu^+ \mu^- \gamma$ *Decay*, *Phys. Lett.* **94B** (1980) 548, [Yad. Fiz.32,998(1980)].

[28] H. Berghauser et al., *Determination of the eta-transition form factor in the* $\gamma p \to p\eta \to p\gamma e^+ e^-$ *reaction*, *Phys. Lett.* **B701** (2011) 562.

[29] M.N. Achasov et al., *Study of Conversion Decays* $\phi \to \eta e^+ e^-$ *and* $\eta \to \gamma e^+ e^-$ *in the Experiment with SND Detector at the VEPP-2M Collider*, *Phys. Lett.* **B504** (2001) 275.

[30] KLOE-2 collaboration, D. Babusci et al., *Study of the Dalitz decay* $\phi \to \eta e^+ e^-$ *with the KLOE detector*, *Phys. Lett.* **B742** (2015) 1.

[31] KLOE-2 collaboration, A. Anastasi et al., *Measurement of the* $\phi \to \pi^0 e^+ e^-$ *transition form factor with the KLOE detector*, *Phys. Lett.* **B757** (2016) 362.

PoS(CD2018)073

[32] BESIII collaboration, C.F. Redmer, *Measurement of meson transition form factors at BESIII*, in *13th Conference on the Intersections of Particle and Nuclear Physics (CIPANP 2018) Palm Springs, California, USA, May 29-June 3, 2018*, 2018.

[33] KLOE-2 collaboration, S. Giovannella, *KLOE Results on Hadron Physics and Perspectives for KLOE-2*, *EPJ Web Conf.* **166** (2018) 00011.

[34] BABAR collaboration, J.P. Lees et al., *Measurement of the $\gamma^{\star}\gamma^{\star} \to \eta'$ transition form factor*, *Phys. Rev.* **D98** (2018) 112002.

[35] NA62 collaboration, C. Lazzeroni et al., *Measurement of the π^0 electromagnetic transition form factor slope*, *Phys. Lett.* **B768** (2017) 38.

[36] A2 collaboration, P. Adlarson et al., *Measurement of the $\pi^0 \to e^+e^-\gamma$ Dalitz decay at the Mainz Microtron*, *Phys. Rev.* **C95** (2017) 025202.

[37] BESIII collaboration, M. Ablikim et al., *Observation of the Dalitz Decay $\eta' \to \gamma e^+e^-$*, *Phys. Rev.* **D92** (2015) 012001.

[38] KLOE, KLOE-2 collaboration, F. Ambrosino et al., *Observation of the rare $\eta \to e^+e^-e^+e^-$ decay with the KLOE experiment*, *Phys. Lett.* **B702** (2011) 324.

[39] SND collaboration, M.N. Achasov et al., *Study of the reaction $e^+e^- \to \pi^0\gamma$ with the SND detector at the VEPP-2M collider*, *Phys. Rev.* **D93** (2016) 092001.

[40] M.N. Achasov et al., *Measurement of the $e^+e^- \to \pi^0\gamma$ cross section in the energy range 1.075-2 GeV at SND*, *Phys. Rev.* **D98** (2018) 112001.

[41] BESIII collaboration, M. Ablikim et al., *Observation of electromagnetic Dalitz decays $J/\psi \to Pe^+e^-$*, *Phys. Rev.* **D89** (2014) 092008.

[42] M.N. Achasov et al., *Updated measurement of the $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ cross section with the SND detector*, *Phys. Rev.* **D94** (2016) 112001.

[43] KTEV collaboration, E. Abouzaid et al., *Measurement of the Rare Decay $\pi^0 \to e^+e^-$*, *Phys. Rev.* **D75** (2007) 012004.

[44] A.E. Dorokhov and M.A. Ivanov, *Rare decay $\pi^0 \to e^+e^-$: Theory confronts KTeV data*, *Phys. Rev.* **D75** (2007) 114007.

[45] T. Husek, K. Kampf and J. Novotný, *Rare decay $\pi^0 \to e^+e^-$: on corrections beyond the leading order*, *Eur. Phys. J.* **C74** (2014) 3010.

[46] SND collaboration, M.N. Achasov et al., *Search for the process $\mathbf{e^+e^-} \to \eta$*, *Phys. Rev.* **D98** (2018) 052007.

[47] M.N. Achasov et al., *Search for the $\eta' \to e^+e^-$ decay with the SND detector*, *Phys. Rev.* **D91** (2015) 092010.

[48] R. Garcia-Martin, R. Kaminski, J.R. Pelaez, J. Ruiz de Elvira and F.J. Yndurain, *The Pion-pion scattering amplitude. IV: Improved analysis with once subtracted Roy-like equations up to 1100 MeV*, *Phys. Rev.* **D83** (2011) 074004.

[49] WASA-AT-COSY collaboration, P. Adlarson et al., *Measurement of the $\omega \to \pi^+\pi^-\pi^0$ Dalitz plot distribution*, *Phys. Lett.* **B770** (2017) 418.

[50] BESIII collaboration, M. Ablikim et al., *Dalitz Plot Analysis of the Decay $\omega \to \pi^+\pi^-\pi^0$*, *Phys. Rev.* **D98** (2018) 112007.

[51] F. Niecknig, B. Kubis and S.P. Schneider, *Dispersive analysis of $\omega \to 3\pi$ and $\phi \to 3\pi$ decays*, *Eur. Phys. J.* **C72** (2012) 2014 [arXiv:1203.2501].

[52] I.V. Danilkin, C. Fernández-Ramírez, P. Guo, V. Mathieu, D. Schott, M. Shi et al., *Dispersive analysis of $\omega/\phi \to 3\pi, \pi\gamma^*$*, *Phys. Rev.* **D91** (2015) 094029 [arXiv:1409.7708].

[53] F. Stollenwerk, C. Hanhart, A. Kupsc, U.G. Meissner and A. Wirzba, *Model-independent approach to $\eta \to \pi^+\pi^-\gamma$ and $\eta' \to \pi^+\pi^-\gamma$*, *Phys. Lett.* **B707** (2012) 184.

[54] C. Hanhart, A. Kupsc, U.G. Meissner, F. Stollenwerk and A. Wirzba, *Dispersive analysis for $\eta \to \gamma\gamma^*$*, *Eur. Phys. J.* **C73** (2013) 2668, [Erratum: Eur. Phys. J.C75 (2015)242].

[55] C. Hanhart, S. Holz, B. Kubis, A. Kupsc, A. Wirzba and C.W. Xiao, *The branching ratio $\omega \to \pi^+\pi^-$ revisited*, *Eur. Phys. J.* **C77** (2017) 98, [Erratum: Eur. Phys. J.C78(2018)450].

[56] WASA-AT-COSY collaboration, P. Adlarson et al., *Exclusive Measurement of the $\eta \to \pi^+\pi^-\gamma$ Decay*, *Phys. Lett.* **B707** (2012) 243.

[57] KLOE collaboration, D. Babusci et al., *Measurement of $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$ with the KLOE Detector*, *Phys. Lett.* **B718** (2013) 910.

[58] BESIII collaboration, M. Ablikim et al., *Precision Study of $\eta' \to \gamma\pi^+\pi^-$ Decay Dynamics*, *Phys. Rev. Lett.* **120** (2018) 242003.

[59] M.N. Achasov et al., *Measurement of the $e^+e^- \to \eta\pi^+\pi^-$ cross section with the SND detector at the VEPP-2000 collider*, *Phys. Rev.* **D97** (2018) 012008.

[60] BABAR collaboration, J.P. Lees et al., *Study of the process $e^+e^- \to \pi^+\pi^-\eta$ using initial state radiation*, *Phys. Rev.* **D97** (2018) 052007.

[61] P. Adlarson et al., *Measurements of branching ratios for η decays into charged particles*, *Phys. Rev.* **C94** (2016) 065206.

[62] BELLE collaboration, M. Masuda et al., *Study of π^0 pair production in single-tag two-photon collisions*, *Phys. Rev.* **D93** (2016) 032003.

[63] BESIII collaboration, Y. Guo, *Space-like transition form factors from BESIII*, *Nucl. Part. Phys. Proc.* **294-296** (2018) 153.