

# Combining the large-*N<sub>C</sub>* and low-momentum expansions to describe parity violation in few-nucleon systems

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> Parity-violating (PV) nucleon-nucleon (*NN*) interactions at low energies can be completely characterized by five low energy constants (LECs). The field of hadronic parity violation has long been plagued by uncertainties in these five LECs. Experiments to reliably extract these LECs are difficult as they involve few nucleon systems which give PV asymmetries roughly of the size  $10^{-8}$ to  $10^{-7}$ . Theoretically determining the low energy constants from the fundamental interactions of QCD is arguably more difficult, being hindered by the non-perturbative nature of QCD at low energies. In light of these facts, a theoretically motivated organizing principle for the relative size of the five LECs to guide future experiments would be highly desirable. Such an organizing principle has recently been provided by a large- $N_C$  (number of colors) analysis of QCD, which is the subject of this paper. This large- $N_C$  analysis serves both as a testable prediction from QCD and a possible organizing principle to assess the feasibility of future experiments.

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## 1. Introduction

Parity-violating (PV) nucleon-nucleon (*NN*) interactions or hadronic parity violation in the Standard Model is mediated by the exchange of W and Z bosons and offers a unique probe of QCD. At energies below the Z-pole (~100 GeV) the exchange of Z and W bosons between quarks can be approximated by parity-violating effective four-quark interactions. At even lower energies ( $E < \Lambda_{QCD}$ ) these four-quark interactions give rise to PV interactions between nucleons. Although PV interactions between nucleons are in principle calculable using the four-quark interactions with QCD, this is not currently practical at physical quark masses. Using the four-quark interactions a lattice QCD calculation of the  $\Delta I = 2$  contribution to the PV *NN* interaction has been demonstrated at  $m_{\pi} \sim 800$  MeV [1]. Future efforts will include calculating the finite volume effects and performing the renormalization of the bare matrix element [1]. Lattice calculations have also been performed for the PV pion exchange coupling at  $m_{\pi} \sim 389$  MeV, but did not include disconnected diagrams [2].

The standard approach to hadronic parity violation has been through the so called DDH (Desplanques, Donoghue, and Holstein) model [3], which consists of seven phenomenological PV constants describing the exchange of the lightest scalar and vector mesons between nucleons. DDH provided "best guesses" and reasonable ranges for these unknown constants using quark model techniques and  $SU(6)_W$  symmetry [4]. Other groups have attempted to estimate the size of the DDH coefficients using various theoretical techniques and fits to experiment [5, 6, 7, 8, 9]. Recently PV *NN* interactions have come to be described using effective field theories [10, 11]. EFTs offer a model independent approach to nuclear interactions, based on the underlying symmetries of QCD, with controlled error estimates for theoretical calculations.

At low energies  $(E < m_{\pi}^2/M_N)$  pions can be integrated out as degrees of freedom leaving a theory of *NN* interactions entirely in terms of nucleon fields. This theory known as pionless effective field theory (EFT( $\pi$ )) has been used to great success to describe properties of few-nucleon systems at low energies [12, 13]. At energies where EFT( $\pi$ ) is valid the PV *NN* interaction is characterized by five low energy constants LECs [14, 15]. Many two- and three-nucleon PV observables have been calculated in the EFT( $\pi$ ) framework [11]. At least five complimentary experiments need to be performed to completely characterize the PV *NN* interaction at low energies.

In order to have a clearer path forward both experimentally and theoretically in determining the PV LECs it is desirable to have an organizing principle by which the relative size of these LECs can be compared. Such a scheme is provided by large- $N_C$  in QCD [16, 17], in which the number of colors in QCD can be used as an expansion parameter to investigate the relative size of interactions between nuclei. The large- $N_C$  scaling of the general PV NN interaction was carried out by Phillips, Samart, and Schat (PSS) [18] and used to determine the large- $N_C$  scaling of the DDH coefficients. Building on this work Schindler, Springer, and Vanasse (SSV) [19] found the large- $N_C$  scaling of the five PV LECs in EFT( $\pi$ ) by carefully using Fierz relations which mix terms of different large- $N_C$  scaling. Finally, Gardner, Haxton, and Holstein (GHH) [20] used this large- $N_C$ analysis to analyze available experimental data. The work of all three of these groups as well as a brief review of large- $N_C$  counting for nuclear potentials is discussed below.

## 2. Large-N<sub>C</sub> scaling for nucleon-nucleon interactions

The general NN potential in the center-of-mass (c.m.) frame is given by

$$V(\mathbf{p}_{-},\mathbf{p}_{+}) = \langle \mathbf{p}_{\text{out}}, \boldsymbol{\gamma}; -\mathbf{p}_{\text{out}}, \boldsymbol{\delta} | \hat{H} | \mathbf{p}_{\text{in}}, \boldsymbol{\alpha}; -\mathbf{p}_{\text{in}}, \boldsymbol{\beta} \rangle, \qquad (2.1)$$

where  $\alpha$  and  $\beta$  ( $\gamma$  and  $\delta$ ) represent the incoming (outgoing) nucleon's spin and isospin.  $\hat{H}$  is the Hartree Hamiltonian [21, 22] (exact when  $N_C \rightarrow \infty$  [17])

$$\hat{H} = N_C \sum_{n} \sum_{s,t} v_{stn} \left(\frac{\hat{S}}{N_C}\right)^s \left(\frac{\hat{I}}{N_C}\right)^t \left(\frac{\hat{G}}{N_C}\right)^{n-t-s},$$
(2.2)

constructed from the bosonic quark operators (color has been summed over)

$$\hat{S}^{i} = \hat{q}^{\dagger} \frac{\sigma^{i}}{2} \hat{q}, \quad \hat{I}^{a} = \hat{q}^{\dagger} \frac{\tau^{a}}{2} \hat{q}, \quad \hat{G}^{ia} = \hat{q}^{\dagger} \frac{\sigma^{i} \tau^{a}}{2} \hat{q}.$$
 (2.3)

The coefficients  $v_{stn}$  are functions of momentum that are  $\mathcal{O}(1)$  in large- $N_C$ . In the c.m. frame there are two different combinations of momenta that scale in large- $N_C$  like

$$\mathbf{p}_{-} = \mathbf{p}_{\text{out}} - \mathbf{p}_{in} \sim N_C^0 \quad \mathbf{p}_{+} = \mathbf{p}_{\text{out}} + \mathbf{p}_{in} \sim N_C^{-1}.$$
(2.4)

The momentum  $\mathbf{p}_+$  is associated with relativistic corrections that bring a factor of  $1/M_N$  leading to their large- $N_C$  suppression relative to  $\mathbf{p}_-$ . The operators in the Hartree Hamiltonian have a large- $N_C$  scaling of

$$\langle N'|\hat{S}/N_C|N\rangle \sim \langle N'|\hat{I}/N_C|N\rangle \sim N_C^{-1} \quad , \quad \langle N'|\hat{G}/N_C|N\rangle \sim \langle N'|\hat{\mathbf{1}}/N_C|N\rangle \sim 1.$$
(2.5)

Any term in a *NN* potential can be written in terms of products of these operators and momenta. Knowing the large- $N_C$  scaling of the momenta and operators then allows for the determination of the large- $N_C$  scaling of terms in the *NN* potential. The generic form for the parity conserving non-relativistic *NN* potential is<sup>1</sup>

$$V_{NN} = V_0^0 + V_\sigma^0 \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_0^1 \vec{\tau}_1 \cdot \vec{\tau}_2 + V_\sigma^1 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 + \cdots$$
(2.6)

Matching these onto the operators  $\hat{I}$ ,  $\hat{S}$ , and  $\hat{G}$ , Kaplan and Manohar [21] found the large-N<sub>C</sub> scaling

$$V_0^0 \sim \hat{\mathbf{1}} \sim N_C, \quad V_\sigma^1 \sim \hat{G} \cdot \hat{G} \sim N_C, \quad V_\sigma^0 \sim \hat{S} \cdot \hat{S} \sim N_C^{-1}, \quad V_0^1 \sim \hat{I} \cdot \hat{I} \sim N_C^{-1}, \tag{2.7}$$

for these terms in the PC NN potential.

#### 2.1 Large-N<sub>C</sub> scaling of parity-violating nucleon-nucleon interaction

Using the methods outlined above PSS [18] calculated the large- $N_C$  scaling of terms in the PV NN interaction. They found the  $\mathcal{O}(N_C)$  operators

$$\mathbf{p}_{-} \cdot (\vec{\sigma}_{1} \times \vec{\sigma}_{2}) \vec{\tau}_{1} \cdot \vec{\tau}_{2}$$

$$\mathbf{p}_{-} \cdot (\vec{\sigma}_{1} \times \vec{\sigma}_{2}) \mathscr{I}_{ab} \tau_{1}^{a} \tau_{2}^{b},$$
(2.8)

<sup>&</sup>lt;sup>1</sup>For purposes of demonstrating large- $N_C$  counting only some of the terms for the parity-conserving non-relativistic NN potential are shown in Eq. (2.6). Remaining terms can be seen in Ref. [21].

the  $\mathcal{O}(N_C^0 \sin^2 \theta_W)$  operators, all  $\Delta I = 1$ 

$$\mathbf{p}_{+} \cdot (\vec{\sigma}_{1} \tau_{1}^{3} - \vec{\sigma}_{2} \tau_{2}^{3})$$

$$\mathbf{p}_{-} \cdot (\vec{\sigma}_{1} + \vec{\sigma}_{2})(\vec{\tau}_{1} \times \vec{\tau}_{2})^{3}$$

$$\mathbf{p}_{-} \cdot (\vec{\sigma}_{1} \times \vec{\sigma}_{2})(\vec{\tau}_{1} + \vec{\tau}_{2})^{3}$$

$$\{(\mathbf{p}_{+} \times \mathbf{p}_{-}) \cdot \vec{\sigma}_{1} \mathbf{p}_{-} \cdot \vec{\sigma}_{2} + (\mathbf{p}_{+} \times \mathbf{p}_{-}) \cdot \vec{\sigma}_{2} \mathbf{p}_{-} \cdot \vec{\sigma}_{1}\} (\vec{\tau}_{1} \times \vec{\tau}_{2})^{3},$$
(2.9)

and finally the  $\mathcal{O}(N_C^{-1})$  operators

$$\mathbf{p}_{-} \cdot (\vec{\sigma}_{1} \times \vec{\sigma}_{2})$$
(2.10)  

$$\mathbf{p}_{+}^{2} \mathbf{p}_{-} \cdot (\vec{\sigma}_{1} \times \vec{\sigma}_{2}) \vec{\tau}_{1} \cdot \vec{\tau}_{2}$$
  

$$\mathbf{p}_{+} \cdot (\vec{\sigma}_{1} - \vec{\sigma}_{2})$$
  

$$\mathbf{p}_{+} \cdot (\vec{\sigma}_{1} - \vec{\sigma}_{2}) \vec{\tau}_{1} \cdot \vec{\tau}_{2}$$
  

$$\mathbf{p}_{+} \cdot (\vec{\sigma}_{1} - \vec{\sigma}_{2}) \mathscr{I}_{ab} \tau_{1}^{a} \tau_{2}^{b}$$
  

$$\mathbf{p}_{+}^{2} \mathbf{p}_{-} \cdot (\vec{\sigma}_{1} \times \vec{\sigma}_{2}) \mathscr{I}_{ab} \tau_{1}^{a} \tau_{2}^{b},$$

where  $\mathscr{I}_{ab} = \text{diag}[1, 1, -2]$  gives an isotensor combination of nucleons. Factors of  $\sin^2 \theta_W$  come from the Standard Model Lagrangian for the effective four quark  $\Delta I = 1$  operators [23]. Note, the work of SSV [19] showed that there is also a  $\sin^2 \theta_W$  contribution to the isotensor part of the PV *NN* interactions. PSS did not have this factor on the isotensor term and it is not included in the summary of their results here.

With the large- $N_C$  scaling of the generic PV NN potential, PSS matched their potential to the DDH potential obtaining

$$\begin{aligned} h_{\rho}^{0} &\sim \sqrt{N_{c}} \quad , \quad h_{\rho}^{2} &\sim \sqrt{N_{C}} \\ \frac{h_{\rho}^{1'}}{\sin^{2} \theta_{W}} &\stackrel{<}{\sim} \sqrt{N_{C}} \quad , \quad \frac{h_{\omega}^{1}}{\sin^{2} \theta_{W}} &\sim \sqrt{N_{C}} \\ \frac{h_{\rho}^{1}}{\sin^{2} \theta_{W}} &\stackrel{<}{\sim} \frac{1}{\sqrt{N_{c}}} \quad , \quad \frac{h_{\pi}^{1}}{\sin^{2} \theta_{W}} &\stackrel{<}{\sim} \frac{1}{\sqrt{N_{c}}} \quad , \quad h_{\omega}^{0} &\sim \frac{1}{\sqrt{N_{c}}}, \end{aligned}$$

$$(2.11)$$

for the relative large- $N_C$  scaling of the DDH coefficients. The large- $N_C$  predictions are compared to the DDH "best guesses" and reasonable ranges in Fig. 1, where DDH parameters with a tilde have been rescaled by a factor  $\sin^2 \theta_W$ . Due to the large reasonable ranges it is hard to make any firm conclusions on the validity of large- $N_C$ . However, it is notable that the DDH "best guess" for  $\tilde{h}_{\rho}^1$  and  $h_{\omega}^0$  are smaller than the DDH "best guess" for  $\tilde{h}_{\omega}^1$ ,  $h_{\rho}^2$ , and  $h_{\rho}^0$ . The "best guess" for  $h_{\pi}^1$ is in disagreement with large- $N_C$  expectations but the reasonable range for  $h_{\pi}^1$  is very large. The DDH parameter  $h_{\rho}^{1'}$  is often ignored (including in DDH) as quark model calculations suggest it's small [24]. However, large- $N_C$  gives no reason that  $h_{\rho}^{1'}$  should be any smaller than  $h_{\pi}^1$  or  $h_{\omega}^0$  [18].

### 2.2 Large-N<sub>C</sub> scaling of parity-violating nucleon-nucleon interaction at low energies

Starting from all possible single derivative relativistic PV operators Girlanda [15] showed that there are only five independent non-relativistic single derivative operators by using Fierz relations,



**Figure 1:** Comparison of DDH "best guesses" and ranges to large- $N_C$  predictions. DDH parameters with a tilde have been divided by a factor of  $\sin^2 \theta_W = 0.24$  to make the large- $N_C$  scaling manifest. The DDH "best guesses" for  $\mathcal{O}(\sqrt{N_C})$  ( $\mathcal{O}(1/\sqrt{N_C})$ ) DDH parameters are solid circles (squares) and the error bars give the DDH ranges. The error bands about 1 are  $1/N_C$  and about  $\frac{1}{3}$   $1/N_C^2$ . To normalize the coefficients they have been divided by the average of  $\tilde{h}_{\omega}^1$  and  $h_{\rho}^2$ .

the equations of motion, and a non-relativistic reduction yielding the five distinct terms

$$\begin{aligned} \mathscr{L}_{PV}^{min} = \mathscr{G}_{1}(N^{\dagger}\vec{\sigma}N \cdot N^{\dagger}i \overleftrightarrow{\nabla}N - N^{\dagger}NN^{\dagger}\vec{\sigma} \cdot i \overleftrightarrow{\nabla}N) \qquad (2.12) \\ &- \mathscr{G}_{1}\varepsilon_{ijk}N^{\dagger}\sigma^{i}N\nabla^{j}(N^{\dagger}\sigma^{k}N) \\ &- \mathscr{G}_{2}\varepsilon_{ijk}[N^{\dagger}\tau^{3}\sigma^{i}N\nabla^{j}(N^{\dagger}\sigma^{k}N) + N^{\dagger}\sigma^{i}N\nabla^{j}(N^{\dagger}\tau^{3}\sigma^{k}N)] \\ &- \mathscr{G}_{5}\mathscr{I}_{ab}\varepsilon_{ijk}N^{\dagger}\tau^{a}\sigma^{i}N\nabla^{j}(N^{\dagger}\tau^{b}\sigma^{k}N) \\ &+ \mathscr{G}_{6}\varepsilon_{ab3}\overrightarrow{\nabla}(N^{\dagger}\tau^{a}N) \cdot (N^{\dagger}\tau^{b}\vec{\sigma}N). \end{aligned}$$

Using Fierz rearrangements the Girlanda form of the Lagrangian can be written in a partial wave basis in which the incoming and outgoing angular momentum of the nucleons is manifest [25]

$$\begin{aligned} \mathscr{L}_{PV} &= -\left[\mathscr{C}^{(^{3}S_{1}-^{1}P_{1})}\left(N^{T}\sigma^{2}\vec{\sigma}\tau^{2}N\right)^{\dagger}\cdot\left(N^{T}\sigma^{2}\tau^{2}i\overset{\leftrightarrow}{\nabla}N\right)\right. \\ &+ \mathscr{C}^{(^{1}S_{0}-^{3}P_{0})}_{(\Delta I=0)}\left(N^{T}\sigma^{2}\tau^{2}\vec{\tau}N\right)^{\dagger}\left(N^{T}\sigma^{2}\vec{\sigma}\cdot\tau^{2}\vec{\tau}i\overset{\leftrightarrow}{\nabla}N\right) \\ &+ \mathscr{C}^{(^{1}S_{0}-^{3}P_{0})}_{(\Delta I=1)}\varepsilon_{3ab}\left(N^{T}\sigma^{2}\tau^{2}\tau^{a}N\right)^{\dagger}\left(N^{T}\sigma^{2}\vec{\sigma}\cdot\tau^{2}\tau^{b}\overset{\leftrightarrow}{\nabla}N\right) \\ &+ \mathscr{C}^{(^{1}S_{0}-^{3}P_{0})}_{(\Delta I=2)}\mathscr{I}_{ab}\left(N^{T}\sigma^{2}\tau^{2}\tau^{a}N\right)^{\dagger}\left(N^{T}\sigma^{2}\vec{\sigma}\cdot\tau^{2}\tau^{b}i\overset{\leftrightarrow}{\nabla}N\right) \\ &+ \mathscr{C}^{(^{3}S_{1}-^{3}P_{1})}\varepsilon_{ijk}\left(N^{T}\sigma^{2}\sigma^{i}\tau^{2}N\right)^{\dagger}\left(N^{T}\sigma^{2}\sigma^{k}\tau^{2}\tau^{3}\overset{\leftrightarrow}{\nabla}^{j}N\right)\right] + \mathrm{H.c.}\,.\end{aligned}$$

Naive application of the large- $N_C$  scaling of operators results in incorrect conclusions about the large- $N_C$  scaling of the five PV LECs [19]. The issue arises from the fact that operators related by Fierz identities need not have the same large- $N_C$  scaling as shown by SSV [19]. Since the basis

of five PV LECs comes from Fierz relations on a more general set of operators, the actual large- $N_C$  scaling of operators can be lost. For example, part of the PV potential is given by the terms

$$V = \mathscr{A}_{1}^{+} \mathbf{p}_{+} \cdot (\vec{\sigma}_{1} - \vec{\sigma}_{2}) + \mathscr{A}_{1}^{-} \mathbf{p}_{-} \cdot i(\vec{\sigma}_{1} \times \vec{\sigma}_{2})$$

$$+ \mathscr{A}_{3}^{+} \mathbf{p}_{+} \cdot (\vec{\sigma}_{1} - \vec{\sigma}_{2})\vec{\tau}_{1} \cdot \vec{\tau}_{2} + \mathscr{A}_{3}^{-} \mathbf{p}_{-} \cdot i(\vec{\sigma}_{1} \times \vec{\sigma}_{2})\vec{\tau}_{1} \cdot \vec{\tau}_{2} + \cdots$$

$$(2.14)$$

Matching this potential to the Girlanda Lagrangian Eq. (2.12) gives the relations

$$\mathscr{G}_1 = -\mathscr{A}_1^+ \quad , \quad \tilde{\mathscr{G}}_1 = -\mathscr{A}_1^-.$$
 (2.15)

The large- $N_C$  scaling, given in Eq. (2.10), of the operators  $\mathscr{A}_1^+ \sim N_C^{-1}$  and  $\mathscr{A}_1^- \sim N_C^{-1}$  then gives

$$\mathscr{G}_1 \sim \tilde{\mathscr{G}}_1 \sim N_C^{-1},\tag{2.16}$$

for the large- $N_C$  scaling of the Girlanda LECs. However, using Fierz relations the operators  $\mathscr{A}_3^+$  and  $\mathscr{A}_3^-$  can be related to the operators  $\mathscr{A}_1^+$  and  $\mathscr{A}_-$  leading to the matching

$$\mathscr{G}_{1} = -\mathscr{A}_{1}^{+} + \mathscr{A}_{3}^{+} - 2\mathscr{A}_{3}^{-} \quad , \quad \tilde{\mathscr{G}}_{1} = -\mathscr{A}_{1}^{-} - 2\mathscr{A}_{3}^{+} + \mathscr{A}_{3}^{-}.$$
(2.17)

Matching to Eqs. (2.10) and (2.8) gives the large- $N_C$  scaling of the operators  $\mathscr{A}_3^+ \sim N_C^{-1}$  and  $\mathscr{A}_3^- \sim N_C$ , which gives

$$\mathscr{G}_1 \sim \mathscr{\tilde{G}}_1 \sim N_C \tag{2.18}$$

for the large- $N_C$  scaling of the Girlanda LECs. In addition this large- $N_C$  analysis implies the relation

$$\mathscr{G}_1 = -2\tilde{\mathscr{G}}_1, \tag{2.19}$$

up to  $\mathcal{O}(N_C^{-1})$ . The lesson here is that one should start with the most general set of PV operators and then use Fierz identities to simplify them down to the set of five distinct operators, while keeping track of their dependence on the general set of operators. Then using the large- $N_C$  scaling of the general set of operators the proper large- $N_C$  scaling of the five operators can be properly discerned. This analysis carried out by SSV [19] gives

$$\mathscr{C}^{({}^{3}S_{1}-{}^{1}P_{1})} \sim \mathscr{C}^{({}^{1}S_{0}-{}^{3}P_{0})}_{(\Delta I=0)} \sim N_{C} , \qquad (2.20)$$

$$\mathscr{C}_{(\Delta I=2)}^{(^{1}S_{0}-^{3}P_{0})} \sim N_{C} \sin^{2} \theta_{W} , \qquad (2.21)$$

$$\mathscr{C}_{(\Delta I=1)}^{(^{1}S_{0}-^{^{3}}P_{0})} \sim \mathscr{C}^{(^{3}S_{1}-^{^{3}}P_{1})} \sim N_{C}^{0} \sin^{2} \theta_{W} .$$
(2.22)

for the large- $N_C$  scaling of the LECs in the partial wave basis with the additional relation

$$\mathscr{C}^{({}^{3}S_{1}-{}^{1}P_{1})} = 3\mathscr{C}^{({}^{1}S_{0}-{}^{3}P_{0})}_{(\Delta I=0)},$$
(2.23)

that holds to  $\mathcal{O}(N_C^{-1})$ . Using the fact  $\sin^2 \theta_W \sim 0.24$  is similar to a factor of  $1/N_C$  for  $N_C = 3$  Gardner, Haxton, and Holstein (GHH) defined the large- $N_C$  basis of LECs [20]

$$C_{1}^{(N_{C})} = \frac{1}{4} \mathscr{C}^{(3S_{1}-1P_{1})} + \frac{3}{4} \mathscr{C}^{(1S_{0}-3P_{0})}_{(\Delta I=0)} \qquad \qquad \text{LO}(\mathscr{O}(N_{C}))$$
(2.24)

$$C_{2}^{(N_{C})} = \mathscr{C}_{(\Delta I=2)}^{(^{1}S_{0}-^{3}P_{0})} \qquad \text{LO}(\mathscr{O}(N_{C}))$$
(2.25)

$$C_{3}^{(N_{C})} = \frac{1}{4} \mathscr{C}^{({}^{3}S_{1} - {}^{1}P_{1})} - \frac{3}{4} \mathscr{C}^{({}^{1}S_{0} - {}^{3}P_{0})}_{(\Delta I = 0)} \qquad \text{NNLO}(\mathscr{O}(N_{C}^{-1}))$$
(2.26)

$$C_4^{(N_C)} = \mathscr{C}_{(\Delta I=1)}^{(1S_0 - 3P_0)} , \quad C_5^{(N_C)} = \mathscr{C}^{(3S_1 - 3P_1)}$$
 NNLO( $\mathscr{O}(N_C^0 \sin^2 \theta_W)$ ) (2.27)

in which the relationship in Eq. (2.23) was exploited. The GHH results quoted here do not include the factor  $\sin^2 \theta_W$  on the isotensor term later discovered by SSV. With an additional factor of  $\sin^2 \theta_W$ the LO  $C_2^{(N_C)}$  terms would become NLO in the GHH counting. Using this hierarchy of PV LECs in large- $N_C$ , GHH showed that the experimental data available at the time was in agreement with expectations from large- $N_C$ . However, their analysis did not include the results from the NPDGamma collaboration [26] as they were unavailable at the time. In addition, the GHH analysis relied on use of the so called Rosetta Stone [27, 28] which matches different parametrizations of the PV LECs used in different calculations. However, this should be done with caution as different regulators used in different calculations can introduce spurious regulator dependence into the matching of PV LECs from different calculations [11]

#### **3.** Conclusions

Hadronic parity violation at low energies is described by five distinct LECs. Unfortunately, these LECs are difficult to determine both experimentally and theoretically. Lattice QCD offers an avenue to determine the values of these LECs from the fundamental interactions of QCD [2, 1]. Experimentally only pp scattering [29, 30] and the recent  $\vec{n}p \rightarrow d\gamma$  [26] measurement have given non-zero results for PV asymmetries at low energies for few-body nuclear systems from which the PV LECs can be cleanly extracted. Another low energy "few-nucleon" experiment  $\vec{n} + {}^{3}\text{He} \rightarrow p + {}^{3}\text{H}$  has been carried out at the Spallation Neutron Source at Oak Ridge National Laboratory and its data is currently being analyzed [31, 32]. Given the large amount of effort that goes into these experiments a guiding principle as to which experiments should be carried out would be desirable. A Large- $N_{C}$  analysis from QCD of the relative size of the PV LECs in principle offers such a guiding principle. In addition large- $N_{C}$  offers a testable prediction for the relative sizes of LECs from experiment.

The large- $N_C$  analysis of SSV in the GHH basis shows that two of the five LECs dominate. However, a missing factor of  $\sin^2 \theta_W$  by SSV for the  $\Delta I = 2$  term not taken into account by GHH would mean that only one LEC is dominant in the GHH counting. By using the PV asymmetry of *pp*-scattering [29] at 13.6 MeV lab energies and the bound for the asymmetry of the circular polarization of photons in  $n + p \rightarrow d + \vec{\gamma}$  [33], SSV demonstrated that the  $\Delta I = 2$  and  $\Delta I = 0$  PV *NN* interactions are roughly of the same size despite the  $\sin^2 \theta_W$  suppression of the  $\Delta I = 2$  term [19]. This suggests that  $\sin^2 \theta_W$  determined at the weak matching scale does not lead to significant suppression at hadronic scales [19]. With these considerations large- $N_C$  determines that the five PV LECs are dominated by a linear combination of the  $\Delta I = 0$  PV *NN* interactions and the  $\Delta I = 2$  contribution. The photodisintegration of deuterium with circularly polarized photons is a possible future experiment at an upgraded High Intensity Gamma-Ray Source at the Triangle Universities Nuclear Laboratory [34]. This experiment has the advantage of being sensitive to the isotensor contribution of the PV *NN* interaction and the ideal energy at which it should be run has been assessed in EFT( $\vec{\pi}$ ) [35].

A preliminary updated GHH analysis including the more recent NPDGamma result [26] suggests that the large- $N_C$  scaling is not wholly consistent with available experimental data [36]. In addition a recent three-nucleon EFT( $\pi$ ) calculation using the recent NPDGamma result, the asymmetry from pp scattering [29], and a bound from pd scattering [30] also finds tension between

current experimental data and the predicted large- $N_C$  scaling [37]. This work also shows that a NLO (in EFT(#)) PV three-nucleon force is necessary in contradiction with earlier claims [38]. Thus two- and three-nucleon PV observables can be described to a theoretical accuracy of ~30% with only five two-nucleon LECs. However, to predict ~10% or better for nuclear systems with  $A \ge 3$  a PV three-nucleon force is necessary. This implies that the PV three-nucleon force in  $A \ge 3$  systems is necessary if the N<sup>2</sup>LO in large- $N_C$  LECs are to be extracted from PV observables that contain LO in large- $N_C$  LECs. Clearly further work is needed both experimentally and theoretically to further understand the picture of large- $N_C$  in PV NN interactions.

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