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Low Q^2 Boundary Conditions for DGLAP Equations Dictated by Quantum Statistical Mechanics Functions

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Phenomenology shows the prominent role of quantum statistical mechanics for the description of the parton distribution functions in the nucleon. It provides the low Q^2 boundary conditions for DGLAP equations in terms of Fermi-Dirac and Bose-Einstein functions of the fractional momentum variable x. The successful comparison of the model with HERA fit and NNPDF for the light fermions and gluons distributions is presented. Free model parameters fixed from data with high statistics provide a strong constraint on the information not supplied directly by experiments.

We dedicate this work to the professors Altarelli, Gribov and Lipatov who gave a fundamental contribution to the DGLAP equation, indeed we think that our proposal for the boundary condition might be considered as the completion of that seminal work.

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1. PHENOMENOLOGICAL MOTIVATIONS TO INTRODUCE THE QUANTUM STATISTICAL PARTON MODEL

The flavour asymmetry of light sea quarks $\bar{d}(x) > \bar{u}(x)$ can be understood of the Pauli exclusion principle, as advocated many years ago by Niegawa and Sisiki [1] and by Feynman and Field [2]. This is confirmed by the defect in the Gottfried sum rule [3] [4] and by the larger Drell-Yan [5] production of muon pairs in $p-n$ scattering than in $p-p$ scattering [6]. The correlation between the first moments of the valence partons and the shapes of their x distributions is the one expected for a quantum gas: broader shapes for higher first moments, as it is shown by the approximate relationship [7]

$$\Delta u(x) = u(x) - d(x),$$

which follows from the assumption :

$$2u^{\downarrow}(x) = d(x)$$

and relates the dramatic decrease at high x of the ratio $\frac{F_2^n(x)}{F_2^p(x)}$ [8], which is a consequence of a similar behaviour of the ratio $\frac{d(x)}{u(x)}$, to the shape of $\Delta u(x)$, which gives the main contribution to $xg_1^p(x)$ The decreasing with x of the negative ratio $\frac{\Delta d(x)}{d(x)}$ is also expected within the statistical approach. The x dependance of $xg_1^n(x)$, negative at small x and positive at high x, follows from the different shapes and opposite signs of $\Delta u(x)$ and $\Delta d(x)$ [9] [10].

2. VARIABLE FOR THE STATISTICAL PARTON DISTRIBUTIONS

The usual choice of the energy as the variable appearing in statistical mechanics follows from its appearing in the constraint, which fixes the total energy available.

$$\Sigma n_i \varepsilon_i = E$$

The resulting Boltzmann expression is

$$e^{-\frac{\varepsilon_i}{KT}}$$

The role of Pauli principle suggests to write Fermi-Dirac functions for the quarks in the variable x, which is the one appearing in the moment, Adler [11], Gross-LLewynn Smith [12] and Bjorkeen [13] parton model sum rules for the proton

$$\Sigma_i \int_0^1 x p_i(x) = 1$$

$$\int_0^1 [u(x) - \bar{u}(x)] dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = 1$$

$$\int_0^1 [\Delta u(x) + \Delta\bar{u}(x) - \Delta d(x) - \Delta\bar{d}(x)] = \frac{G_A}{G_V} = 1.26$$

We write the Fermi-Dirac expressions for the valence partons defined by their flavor (u or d) and spin along the proton momentum as:

$$\frac{1}{\exp\left[(x-X_q)/\bar{x}\right]+1}$$

where $\bar{x}$ and X_q play the role of the temperature and the potentials respectively and:

$$q = u^{\uparrow}, d^{\downarrow}, u^{\downarrow}, d^{\uparrow}$$

In order to obey the parton model sum rules the potentials of the valence quarks are expected to be larger than their antiparticles. The defect in the Gottfried sum rule:

$$\bar{d} - \bar{u} > 0$$

implies for the Adler sum rule:

$$u - d < 1$$

The positive (negative) sign of Δu (Δd) implies

$$X_u^+ > X_u^- \text{ and } X_d^- > X_d^+$$

The inequalities $\frac{G_A}{G_V} > u - d$, implies:

$$X_d^- > X_u^-$$

The transverse momenta of the quarks lead to the Melosh-Wigner transformation [14] which relates their helicities to their polarisation along the momentum of the hadron. Its formal expression confirms the group properties of the generator of the transformation, which relates constituent to current quarks:

$$Z = (\vec{W} \times \vec{M})_z$$

with W a $SU(3)$ singlet of the adjoint representation (35) of $SU(6)$ and M a vector with respect to the orbital momentum L.

3. THE TRANSVERSE ENERGY SUM RULE

The transverse distributions have been fixed by a sum rule for the transverse energy [15], defined as the difference between the energy and the momentum. For the initial hadron it is given by

$$P_0 - P_z.$$

At high values of the longitudinal momentum of the hadron P_z its transverse energy can be approximated by:

$$\frac{M^2}{2P_z}$$

while for a massless parton it is given by:

$$\frac{p_T^2}{p_0+p_z} = \frac{p_T^2}{P_z\left(x+\sqrt{x^2+\frac{p_T^2}{P_z^2}}\right)}$$

By multiplying for $2P_z$ we get the sum rule:

$$\Sigma_p \int \frac{2p_T^2}{x+\sqrt{x^2+\frac{p_T^2}{P_z^2}}} f(p,x,p_T^2)dp_T^2 = M^2.$$

By taking P_z the momentum of the initial hadron in the frame of the final hadrons, one gets (neglecting terms in $(xM)^2$):

$$P_z^2 = \frac{Q^2}{4x(1-x)}.$$

This implies the following dependance on x and p_T^2 for the non-diffractive part of $xq(x)$

$$\frac{A'x^{b-1}}{\mu^2 f(x,X_q)g(x,p_T^2)}$$

Where

$$f(x,X_q) = \exp\left[(x-X_q)/\bar{x}\right]+1,$$

$$g(x,p_T^2) = \exp\left(\frac{2P_T^2}{\mu^2\left(x+\sqrt{x^2+\frac{p_T^2}{P_z^2}}\right)} - Y_q^h\right)+1$$

and Y_q^h is the transverse potential. With the transformation $p_T^2 = \frac{2\mu^2\eta}{x+\sqrt{x^2+\frac{p_T^2}{P_z^2}}}$, we obtain the integral in η of

$$\frac{1+\frac{2\eta\mu^2(1-x)}{Q^2}}{e^{(\eta-Y_q^h)}+1},$$

which gives rise to:

$$\ln\left(1+e^{(Y_q^h)}\right) - \frac{2(1-x)\mu^2}{Q^2}\mathrm{Li}_2\left(-e^{(Y_q^h)}\right)$$

that multiplies the longitudinal factor

$$\frac{A'x^b}{e^{(x-X_q^h)/\bar{x}}+1}.$$

The parameter μ^2 will be fixed by the transverse energy sum rule to be 0.200 GeV^2 and is proportional to the denominator of the gaussian form assumed by the transverse distribution for p_T^2 larger than $\mu^2 x Y_q^h$. By requiring equilibrium for the two elementary QCD processes [16], the emission of a gluon by a fermion parton and the conversion of the gluon into a $q\bar{q}$ pair with opposite helicity, one has the important consequence to have a vanishing potential for the gluons of both helicities and opposite values for the potentials for a quark and its antiparticle with opposite helicity. The Bose-Einstein expression for the gluons $xG(x)$ turns into a Planck form $\frac{1}{e^{x/\bar{x}}-1}$ and $\Delta G = 0$. Quark and anti-quark contributions in the e. m. DIS are disentangled thanks to the relation:

$$X_q^h + X_{\bar{q}}^{-h} = 0$$

For the unpolarized distributions the disentangling is obtained also from the obvious conditions

$$u - \bar{u} = 2$$
$$d - \bar{d} = 1$$

On the other hand, for the polarized distributions the equilibrium conditions allow to determine the polarization of the light anti-quarks from the knowledge of the shapes of the valence quark distributions.

4. THE DIFFRACTIVE CONTRIBUTION

At small x, parton distributions are dominated by a diffractive contribution implying an infinite number of partons $q_D(x)$ proportional to $x^{-1.25}$. To reproduce data one had to modify the Fermi-Dirac function with the factor AX_qx^b and add such diffractive term

$$\frac{AX_qx^b}{e^{\frac{(x-X_q)}{\bar{x}}}+1} + \frac{\tilde{A}x^{\tilde{b}}}{e^{\frac{x}{\bar{x}}}+1}$$

X_q is the potential of the parton depending on its flavor and helicity. The diffractive contribution is isoscalar and unpolarized to avoid an infinite contribution to the parton model sum rules (since $\tilde{b} = -0.25$ is negative).

5. THE DESCRIPTION OF THE STATISTICAL PARTON DISTRIBUTIONS AND THE COMPARISON WITH THE HERA AND NNPDF FIT

Some years ago a joint analysis of the DIS data measured in the H1 and ZEUS [17] experiments has been performed to give the unpolarized parton distributions and Jacques Soffer immediately realized the similarity with the statistical distributions. To perform a check for the quantum statistical parton distributions, we determine the parameters introduced [18] in order to reproduce the Hera result for the unpolarized distributions of the light parton fermions, while for the polarized ones we require to reproduce the expressions found in 2002 [19], which have been successful to describe the polarized structure functions $g^{p,d,He^3}(x)$ and the production of $W^{\pm}$ weak bosons [20]. Our results for the parton distributions are described in Figures 1-3 and the parameters are compared with the ones found in 2002 in Table 1.

In the second column of Table 1, we report the values of the parameters obtained by demanding that our expressions reproduce the H1-ZEUS fit. A good test for the statistical parton distributions is provided by the comparison to NNPDF [21], with the parameters fixed by the comparison with HERA. Therefore, the instructive analogy of the three square differences between statistical, NNPDF and Hera divide by the square of NNPDF result integrated on ranges of the x variable for u, d,$\bar{u}$ and $\bar{d}$ for unpolarized distributions, is done in Tables 2-5.

Interestingly enough for u in the range $(0.5, 0.8)$ and for d in the range $(0.2, 0.7)$, the agreement with NNPDF is better than the one of HERA. As long as for the strong disagreement with both NNPDF and HERA for x above 0.8, one should say that at $Q^2 = 4GeV^2$, $M'^2 = M^2 + Q^2(1/x - 1)$

	2002	2015
$\tilde{X}_{u_+}$	0.46188	0.446
$\tilde{X}_{d_-}$	0.30174	0.320
$\tilde{X}_{u_-}$	0.29766	0.297
$\tilde{X}_{d_+}$	0.22775	0.222
b	0.40962	0.43
$\tilde{A}$	0.08318	0.070
$\tilde{b}$	-0.25347	-0.240
$\bar{x}$	0.09907	0.102

Table 1: Parameters value in comparison to 2002 result.

Δx	$\int[\frac{xu_{QSPDF}-xu_{NNPDF}}{xu_{NNPDF}}]^2dx$	$\int[\frac{xu_{HERA}-xu_{NNPDF}}{xu_{NNPDF}}]^2dx$	$\int[\frac{xu_{QSPDF}-xu_{HERA}}{xu_{NNPDF}}]^2dx$
$10^{-5}-0.1$	0.000719015	0.000554071	0.00238961
$0.1-0.2$	0.00104956	0.000026162	0.00117524
$0.2-0.3$	0.000417635	0.0000460843	0.000193605
$0.3-0.4$	0.000043073	0.000009690	0.0000161791
$0.4-0.5$	0.0000013250	0.000058110	0.0000438131
$0.5-0.6$	0.0000128613	0.0000501951	0.0000914226
$0.6-0.7$	0.0000643858	0.000790763	0.00121047
$0.7-0.8$	0.0108695	0.01601150	0.0527942
$0.8-0.9$	318.897	16.1839	478.302

Table 2: Comparison of u quark distribution with NNPDF and HERA

Δx	$\int[\frac{xd_{QSPDF}-xd_{NNPDF}}{xd_{NNPDF}}]^2dx$	$\int[\frac{xd_{HERA}-xd_{NNPDF}}{xd_{NNPDF}}]^2dx$	$\int[\frac{xd_{QSPDF}-xd_{HERA}}{xd_{NNPDF}}]^2dx$
$10^{-5}-0.1$	0.000935016	0.000086975	0.000522174
$0.1-0.2$	0.00023197	0.000122482	0.000264989
$0.2-0.3$	0.000886122	0.00106942	0.000012975
$0.3-0.4$	0.00093035	0.00220476	0.00037315
$0.4-0.5$	0.00011196	0.00343388	0.00384319
$0.5-0.6$	0.00048035	0.00806401	0.0122964
$0.6-0.7$	0.0179844	0.0745385	0.0248437

Table 3: Comparison of d quark distribution with NNPDF and HERA

Δx	$\int[\frac{x\bar{u}_{QSPDF}-x\bar{u}_{NNPDF}}{x\bar{u}_{NNPDF}}]^2dx$	$\int[\frac{x\bar{u}_{HERA}-x\bar{u}_{NNPDF}}{x\bar{u}_{NNPDF}}]^2dx$	$\int[\frac{x\bar{u}_{QSPDF}-x\bar{u}_{HERA}}{x\bar{u}_{NNPDF}}]^2dx$
$10^{-5}-0.1$	0.00403639	0.000211	0.00287137
$0.1-0.2$	0.00508968	0.000418277	0.00736075
$0.2-0.3$	0.00660197	0.0055041	0.0240166
$0.3-0.4$	0.0119411	0.0112131	0.0462101
$0.4-0.5$	0.0222474	0.00533699	0.0475496
$0.5-0.6$	0.0363909	0.00570592	0.0190559
$0.6-0.7$	0.0512114	0.0666935	0.0032279
$0.7-0.8$	0.0629684	0.185068	0.032574
$0.8-0.9$	0.0665441	0.132565	0.0269948

Table 4: Comparison of $\bar{u}$ distribution with NNPDF and HERA

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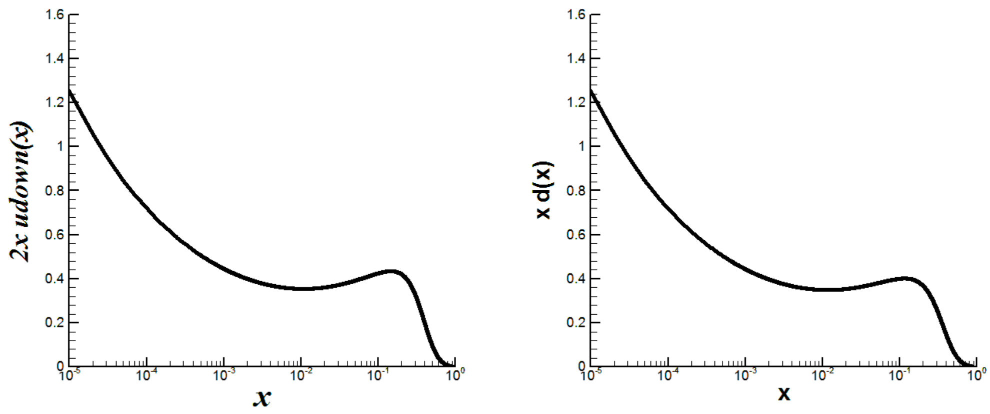

Figure 1: Our results at $Q^2 = 4GeV^2$ for $2xu^{\downarrow}(x)$ and $xd(x)$.

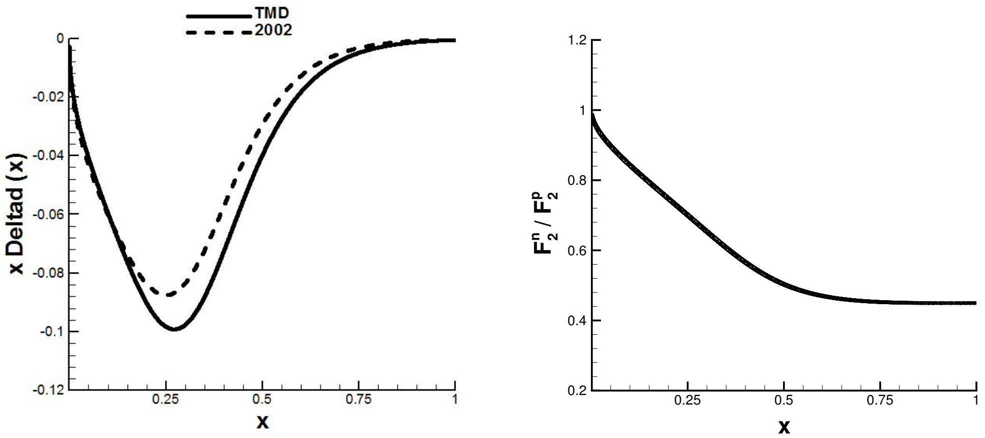

Figure 2: Distribution of $x\Delta d$ in comparison to 2002 and F_2^n/F_2^p at $Q^2 = 4(GeV)^2$.

Δx	$\int [\frac{x\bar{d}_{QSPDF} - x\bar{d}_{NNPDF}}{x\bar{d}_{NNPDF}}]^2 dx$	$\int [\frac{x\bar{d}_{HERA} - x\bar{d}_{NNPDF}}{x\bar{d}_{NNPDF}}]^2 dx$	$\int [\frac{x\bar{d}_{QSPDF} - x\bar{d}_{HERA}}{x\bar{d}_{NNPDF}}]^2 dx$
$10^{-5} - 0.1$	0.00298472	0.00327952	0.000219856
$0.1 - 0.2$	0.00550765	0.00289295	0.000889416
$0.2 - 0.3$	0.00213423	0.0108324	0.0174773
$0.3 - 0.4$	0.0223143	0.359481	0.207696

Table 5: Comparison of $\bar{d}$ distribution with NNPDF and HERA

is less than 1.9, just in the region of the resonances, away from the deep inelastic regime and the large numbers come from the fast decrease at high x induced by the factor $(1-x)^C$. The fact that the limit of ratio $d(x)/u(x)$ at large x supplied by the Boltzmann limit, $e^{-x/\bar{x}}$ is in perfect agreement with the value found by Orwell, Accardi and Melnitchuk [22] is a further point in favour of the hypothesis that the low Q^2 boundary condition for DGLAP equation is fixed by quantum statistical mechanics. For the light sea the isospin and spin asymmetries are automatically predicted in sign and order of magnitude from the QCD equilibrium conditions.

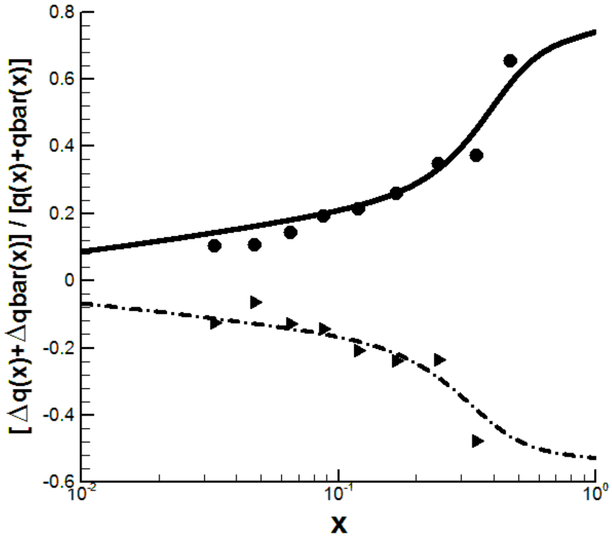

Figure 3: Our results on $\frac{\Delta u(x)+\Delta\bar{u}(x)}{u(x)+\bar{u}(x)}$ (solid) and $\frac{\Delta d(x)+\Delta\bar{d}(x)}{d(x)+\bar{d}(x)}$ (dashed dot) at $Q^2 = 4(GeV)^2$ in comparison to the HERMES data at $Q^2 = 2.5(GeV)^2$.

6. THE COMPARISON WITH THE STANDARD FORM FOR PARTON DISTRIBUTIONS

Despite the fact that $x = 0$ ($Q^2 = 0$) and the neighborhood of $x = 1$ (elastic and resonance production) are not in the domain of DIS, the standard parametrization for parton distribution has the following form:

$$Ax^B(1-x)^C$$

with A, B and C fixed by the comparison with experiment for each parton distribution and a separate analysis for unpolarized and polarized distributions. Sometimes to improve the agreement with data some polynomial factor is introduced. Indeed the diffractive component has a singular power behaviour near $x = 0$, while the valence partons, which dominate the intermediate and the high x regions have a different (more soft) power behaviour at small x, while the positive value of C gives rise to a decrease with x and also to a different weight for the valence partons, 2 (u and d) for the unpolarized distributions and 4 if one considers also the polarized ones.
For the statistical distributions the decrease at high x is naturally explained by the Boltzmann behaviour of the parton distributions $e^{-x/\bar{x}}$ for x larger than the "potential" of each parton.
The variation of the ratios between the different valence parton distributions $\frac{d(x)}{u(x)}$, $\frac{\Delta u(x)}{u(x)}$ and $\frac{\Delta d(x)}{d(x)}$ is concentrated in the range between the lowest (X_d^+) and highest (X_u^+) potential, while in the Boltzmann regime their ratios vary more slowly. This behaviour is the opposite for the standard parametrization, for which the effect of the different exponents for the power $(1-x)^C$ becomes more important as x approaches 1.
The ratio $F_2^n(x)/F_2^p(x)$ at high x depends on the ratio $d(x)/u(x)$ in the same region. The difficulty to obtain the neutron unpolarized structure function at high x is related to the Fermi motion of the

two nucleons in the deuton, which makes very problematic to get it from the ones measured for the proton and for the deuton. So, to get the ratio $d(x)/u(x)$ in that region is not a trivial task. The small statistics and the choice of the standard parametrization give rise to a big uncertainty on that ratio. In the statistical approach the free parameters, from which that ratio depends, the temperature ($\bar{x}$) and the longitudinal and transverse potentials (X_q and Y_q) are fixed in regions, the intermediate x region $(0.22, 0.46)$, where the statistics is large and the systematic errors are small. The perfect agreement of the prediction for $d(1)/u(1) = 0.22$ with the result of the careful analysis by Orwell, Accardi and Melnitchuk is a good confirm for the statistical parton distributions. The form $Ax^B(1-x)^C P(x)$ for the different parton distributions has the disadvantage that the high x behaviour for each distribution is fixed by the exponent C, which comes out different for the different valence quarks with the consequence that the limit $d(x)/u(x)$ for $x \to 1$ comes out 0 or infinity. Indeed in the fit of the joined Hera-Zeus group parameter C is larger for u than for d, while for the sea it is still smaller in such a way that it dominates in that limit. To comply with the experimental behaviour of the ratio $d(x)/u(x)$ they introduce for the parton u the ad-hoc factor $(1+9.7x^2)$.

7. THE GLUON DISTRIBUTION: THE PLANCK FORM

The equilibrium conditions fix the potentials for the gluon to vanish for both helicities, which implies

$$\Delta G(x) = 0 \quad \text{and a Planck form} \quad xG(x) = \frac{A_g x}{e^{x/\bar{x}} - 1}.$$

Where the exponent 1 for the power follows by the idea that the hadron is a black body cavity for the chromomagnetic radiation and A_g is fixed by the sum rule for the longitudinal momentum. Indeed, the fact that HERA data show that $xG(x)$ is growing at small x for $Q^2 = 1.9(GeV^2)$ and decreasing at $Q^2 = 10(GeV^2)$ suggests that the Q^2, where it is stationary, will not be so different from $4(GeV^2)$. In fact $B_H(4(GeV^2)) = 0.0257$. The standard form $Ax^B(1-x)^C$ implies that the decreasing at high x depends on the exponent C and gets faster at increasing x, while the Planck form, as soon as one can neglect the 1 in the denominator, has a more regular behaviour ($e^{-x/\bar{x}}$). Since the gluon distribution in DIS has influence on the logarithmic scaling violation, a method to establish the degree of agreement of the Planck distribution with the experimental information obtained at HERA is to compare at $Q^2 = 4(GeV^2)$:

$$\int_{0.0}^{0.2} xG(x)dx = 0.36 \qquad \text{with} \qquad \int_{0}^{0.2} \frac{A_g x}{e^{x/\bar{x}} - 1} dx = 0.34$$

$$\int_{0.2}^{1} xG(x)dx = 0.05 \qquad \text{with} \qquad \int_{0.2}^{1} \frac{A_g x}{e^{x/\bar{x}} - 1} dx = 0.12$$

The agreement is good for $\int_{0.0}^{0.2} xG(x)dx$ in the range, where most gluons are concentrated, while for x larger than 0.2 HERA gives a faster decrease. Since for the fermion partons the decrease at high x is better described by the statistical distributions, it is legitimate to make the conjecture that the fast decrease at high x advocated by HERA is more a consequence of their parametrization than of the experimental evidence.

Δx	$\int[\frac{xg_{QSPDF}-xg_{NNPDF}}{xg_{NNPDF}}]^2dx$	$\int[\frac{xg_{HERA}-xg_{NNPDF}}{xg_{NNPDF}}]^2dx$	$\int[\frac{xg_{QSPDF}-xg_{HERA}}{xg_{NNPDF}}]^2dx$
$10^{-5}-0.1$	0.00190835	0.000736242	0.00384873
$0.1-0.2$	0.00713221	0.000340814	0.0090403
$0.2-0.3$	0.0173979	0.00838282	0.0490799
$0.3-0.4$	0.0161694	0.0328795	0.0943154
$0.4-0.5$	0.00873125	0.063188	0.118073
$0.5-0.6$	0.0035346	0.0869633	0.125171
$0.6-0.7$	0.00393113	0.0982558	0.140749
$0.7-0.8$	0.120633	0.0882559	0.372319

Table 6: Comparison of gluon distribution with NNPDF and HERA

8. CONCLUSION

The agreement with the Hera distributions with the form dictated by the quantum statistical approach for the fermion parton distributions is an impressive confirm of the validity of the proposal in the 2002 paper with the improved theoretical foundation achieved with the extension of the transverse degrees of freedom and with the consideration of the Melosh-Wigner rotation. The similarity of the values of the parameters with the ones found in the previous work supports the validity of the statistical approach. As long as the p_T dependance in the Boltzmann limit, neglecting the power dependance and with the gaussian approximation for the exponential we get the behaviour $\sqrt(p_T)e^{-2p_T/\mu\sqrt{\bar{x}}}$ with an effective temperature equal to $49MeV$, smaller than the range proposed in the paper by Cleymans, Lykasov, Sorin and Teryaev [23], $120-150MeV$, but the important quantum effect gives rise to a harder p_T distribution. The decrease at high x and the ratios between the different valence partons seem to be better described by the statistical distribution than by the standard distributions. In fact the ratios change more fast in the range $(0.22, 0.46)$ than above 0.46. An attractive feature of the statistical model is that the parameters are fixed by regions of x where there is a large statistics and small systematic errors. Namely, small x for the two parameters associated to the diffractive term, and the intermediate region $(0.22, 0.46)$ for the ones associated to the valence partons. As long as for the gluons, at high x the Planck form is in better agreement with the parametrization independent of NNPDF than the standard parametrization proposed by HERA. A crucial test will be provided by the measurement at high x of $\bar{d}/\bar{u}$ for which a previous experiment gave a weird behaviour abruptly decreasing based on uncertain data. Finally, for the spin and isospin asymmetries of the sea, $\Delta\bar{u}$ and $\Delta\bar{d}$, the Boltzmann behaviour predicted by the statistical model shows a more regular behaviour.

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