

Implications of flavour anomalies for new physics

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We discuss the observed deviations in $b \rightarrow s\ell^+\ell^-$ processes from the Standard Model predictions and present global fits for both hadronic effects and the New Physics description of these anomalies. We investigate whether the different anomalies can be described by a consistent New Physics effect. We consider all the possible relevant new physics contributions to the semileptonic $b \rightarrow s$ transitions. Moreover, we study the prospects of future LHCb upgrade for establishing New Physics with the theoretically clean observables.

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1. Introduction

The full angular analysis of the $B \rightarrow K^* \mu^+ \mu^-$ observables was presented for the first time by the LHCb collaboration in 2013 with 1 fb^{-1} of data [1]. While most of the results were consistent with the Standard Model (SM) predictions, a few deviations were observed. The largest tension was in the P_5' angular observable with 3.7σ significance in the dilepton invariant mass squared bin $q^2 \in [4.30, 8.68] \text{ GeV}^2$. Less significant tensions were observed in some of the other angular observables such as P_2 . The P_5' tension was later reconfirmed by LHCb with 3 fb^{-1} of data [2], in the finer $[4.0, 6.0]$ and $[6.0, 8.0] \text{ GeV}^2$ bins, with 2.8 and 3.0σ significance, respectively. More recently, the Belle collaboration [3] as well as the ATLAS [4] and CMS collaborations [5] have measured P_5' , although with larger experimental uncertainties compared to LHCb.

As the deviation in P_5' is persisting with more experimental data and with several experimental and analysis methods, it seems unlikely that the statistical fluctuations could be the source of the tensions. Hence, either underestimated theoretical (hadronic) uncertainties or New Physics (NP) effects can be responsible for the observed deviations.

The LHCb measurements with 3 fb^{-1} dataset for other $b \rightarrow s \ell^+ \ell^-$ transitions indicate further deviations with the SM predictions at $2\text{-}4\sigma$ significance level in observables such as $\text{BR}(B_s \rightarrow \phi \mu^+ \mu^-)$ [6], but also in the ratios $R_K \equiv \text{BR}(B \rightarrow K^+ \mu^+ \mu^-) / \text{BR}(B \rightarrow K^+ e^+ e^-)$ [7] and $R_{K^*} \equiv \text{BR}(B \rightarrow K^* \mu^+ \mu^-) / \text{BR}(B \rightarrow K^* e^+ e^-)$ [8]. It is important to note that the $2\text{-}3\sigma$ deviations in the theoretically clean ratios R_K and R_{K^*} cannot be explained by underestimated theoretical (hadronic) uncertainties, but the tensions in all $b \rightarrow s \ell^+ \ell^-$ can be explained with a common NP effect, namely about 25% reduction in the $C_9^{(\mu)}$ Wilson coefficient [9–11] (see also Refs. [12–17]).

Besides the $R_{K^{(*)}}$ ratios which are very precisely predicted in the SM, the other observables suffer from hadronic uncertainties. The standard method for calculating the hadronic effects in the low q^2 region for the exclusive $B \rightarrow K^* \ell^+ \ell^-$ decay is the QCD factorisation framework where an expansion of Λ/m_b is employed. However, higher powers of Λ/m_b remain unknown and so far are only “guesstimated”. The significance of the anomalies to a large extent depends on the precise treatment of these non-factorisable power corrections [10, 18, 19]. In the absence of concrete estimates of the power corrections, we make a statistical comparison between a NP fit and a hadronic power corrections fit to the $B \rightarrow K^* \mu^+ \mu^-$ measurements [20–22]. In addition, we examine whether the various observed tensions indicate a common New Physics scenario and we perform NP fits in the most general case where all the relevant Wilson Coefficients can receive contributions from New Physics. Furthermore, the prospects of the LHCb upgrade for corroborating New Physics is studied.

2. Comparison of hadronic fits and New Physics fits

The $b \rightarrow s \ell^+ \ell^-$ transitions can be described via an effective Hamiltonian which can be separated into a hadronic and a semileptonic part [23]:

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}^{\text{had}} + \mathcal{H}_{\text{eff}}^{\text{sl}}, \quad (2.1)$$

where

$$\begin{aligned}\mathcal{H}_{\text{eff}}^{\text{had}} &= -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1,\dots,6,8} C_i O_i, \\ \mathcal{H}_{\text{eff}}^{\text{sl}} &= -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=7,9,10,S,P,T} (C_i O_i + C'_i O'_i).\end{aligned}\quad (2.2)$$

For the exclusive decays $B \rightarrow K^* \mu^+ \mu^-$ and $B_s \rightarrow \phi \mu^+ \mu^-$, the semileptonic part of the Hamiltonian which accounts for the dominant contribution, can be described by seven independent form factors $\tilde{S}, \tilde{V}_\lambda, \tilde{T}_\lambda$, with helicities $\lambda = \pm 1, 0$. The exclusive $B \rightarrow V \ell^+ \ell^-$ decay, where V is a vector meson can be described in the SM by the following seven helicity amplitudes:

$$\begin{aligned}H_V(\lambda) &= -iN' \left\{ C_9^{\text{eff}} \tilde{V}_\lambda(q^2) + \frac{m_B^2}{q^2} \left[\frac{2\hat{m}_b}{m_B} C_7^{\text{eff}} \tilde{T}_\lambda(q^2) - 16\pi^2 \mathcal{N}_\lambda(q^2) \right] \right\}, \\ H_A(\lambda) &= -iN' C_{10} \tilde{V}_\lambda, \\ H_P &= iN' \left\{ \frac{2m_\ell \hat{m}_b}{q^2} C_{10} \left(1 + \frac{m_s}{m_b} \right) \tilde{S} \right\},\end{aligned}\quad (2.3)$$

where the effective part of C_9^{eff} ($\equiv C_9 + Y(q^2)$) as well as the non-factorisable contribution $\mathcal{N}_\lambda(q^2)$ arise from the hadronic part of the Hamiltonian through the emission of a photon which itself turns into a lepton pair. Due to the vectorial coupling of the photon to the lepton pair, the contributions of $\mathcal{H}_{\text{eff}}^{\text{had}}$ appear in the vectorial helicity amplitude $H_V(\lambda)$. It is due to the similar effect from the short-distance C_9 (and C_7) of $\mathcal{H}_{\text{eff}}^{\text{sl}}$ and the long-distance contribution from $\mathcal{H}_{\text{eff}}^{\text{had}}$ that there is an ambiguity in separating NP effects of the type C_9^{NP} (and C_7^{NP}) from non-factorisable hadronic contributions. The non-factorisable contribution $\mathcal{N}_\lambda(q^2)$ is known at leading order in Λ/m_b from QCDf calculations while higher powers are only partially known [24] and can only be guesstimated. These power corrections are usually assumed to be 10%, 20%, etc. of the leading order non-factorisable contribution. However, instead of making an ansatz on the size of the power corrections they can be parametrised by a polynomial with a number of free parameters which can be fitted to the experimental data. One possible description of the power corrections is given in Ref. [25] which is described through 9 complex parameters. With such an ansatz, the NP contributions can be embedded in the hadronic power corrections and it is possible to make a statistical comparison of a hadronic fit and a NP fit of C_9 (and C_7) to the $B \rightarrow K^* \mu^+ \mu^-$ data.

We perform such fits by means of the MINUIT minimisation tool with theoretical predictions from SuperIso v3.6 [26, 27] and considering CP-averaged $B \rightarrow K^* \mu^+ \mu^-$ observables at $q^2 < 8 \text{ GeV}^2$. For the NP scenarios, we have fitted C_9 (and C_7) which assuming complex Wilson coefficient(s) involves 2 (4) free parameters and for the hadronic power corrections we have fitted 18 free parameters.

The various scenarios can be compared through likelihood ratio tests via Wilks' theorem. Considering the difference in number of parameters between two scenarios and taking $\Delta\chi^2$, the p -values are obtained. The p -values imply the significance of adding parameters to go from one nested scenario to a more general case. From Table 1, it can be seen that adding the hadronic parameters (16 more parameters) compared to the C_9^{NP} scenario does not really improve the fits as the fit is only improved by 0.76σ significance, and the NP explanation remains as a justified option for interpreting the tensions in the angular observables. This is partly due to the rather

	δC_9	$\delta C_7, \delta C_9$	Hadronic
plain SM	$3.7 \times 10^{-5} (4.1\sigma)$	$6.3 \times 10^{-5} (4.0\sigma)$	$6.1 \times 10^{-3} (2.7\sigma)$
δC_9	–	0.13(1.5 σ)	0.45(0.76 σ)
$\delta C_7, \delta C_9$	–	–	0.61(0.52 σ)

Table 1: p -values and significances of adding parameters to go from one scenario to another using Wilks' theorem.

	b.f. value	χ_{\min}^2	Pull _{SM}
ΔC_9	–0.48	18.3	0.3 σ
$\Delta C_9'$	+0.78	18.1	0.6 σ
ΔC_{10}	–1.02	18.2	0.5 σ
$\Delta C_{10}'$	+1.18	17.9	0.7 σ
ΔC_9^μ	–0.35	5.1	3.6 σ
ΔC_9^e	+0.37	3.5	3.9 σ
ΔC_{10}^μ	–1.66	2.7	4.0 σ
ΔC_{10}^e	–0.34	2.2	4.0 σ
	–2.36		
	+0.35		

	b.f. value	χ_{\min}^2	Pull _{SM}
ΔC_9	–0.24	70.5	4.1 σ
$\Delta C_9'$	–0.02	87.4	0.3 σ
ΔC_{10}	–0.02	87.3	0.4 σ
$\Delta C_{10}'$	+0.03	87.0	0.7 σ
ΔC_9^μ	–0.25	68.2	4.4 σ
ΔC_9^e	+0.18	86.2	1.2 σ
ΔC_{10}^μ	–0.05	86.8	0.8 σ
	–2.14		
	+0.14	86.3	1.1 σ

Table 2: Best fit values in the one-operator fits considering only the observables $R_{K^* [0.045, 1.1]}$, $R_{K^* [1.1, 6]}$ and $R_{K [1, 6]}$ in the left, and considering all observables (under the assumption of 10% non-factorisable power corrections) except R_K and R_{K^*} in the right. The ΔC_i in the fits are normalised to their SM values. When two numbers are mentioned for a given ΔC_i , they correspond to two possible minima.

large uncertainties of the fitted parameters when using the current data which results in almost all the parameters to be consistent with zero at 1 σ level. However, if in the future LHCb upgrade – with 300 fb^{–1} data – the current central values remain, then a similar statistical comparison will indicate strong preference for the hadronic explanation with a significance of 34 σ compared to the NP explanation.

3. New Physics fits for different sets of observables

The tensions of the measurements with the SM predictions can be explained in a model-independent way by modified Wilson coefficients ($C_i = C_i^{SM} + \delta C_i$), where δC_i can be due to some NP effects. We perform global fits by means of the calculation and minimisation of the χ^2 in which all the theoretical and experimental correlations are considered. To check whether the various anomalies point towards a consistent NP explanation, we have made the NP fits dividing the observables into two different sets, one with the very clean ratios R_K and R_{K^*} and another set with the other $b \rightarrow s\ell^+\ell^-$ observables, a full list of which can be found in [18].

First we consider the impact of NP in one Wilson coefficient at a time, where all other Wilson coefficients are kept to their SM values. In Table 2 we give SM pulls of the various one-operator hypotheses. We see that NP in C_9^e , C_9^μ , C_{10}^e , or C_{10}^μ are favoured by the $R_{K^{(*)}}$ ratios with a significance of 3.6 – 4.0 σ . NP contributions in primed operators have no significant effect in a better description of the data. In the fit to all $b \rightarrow s\ell\ell$ observables without the ratios, the C_9^μ solutions are favoured

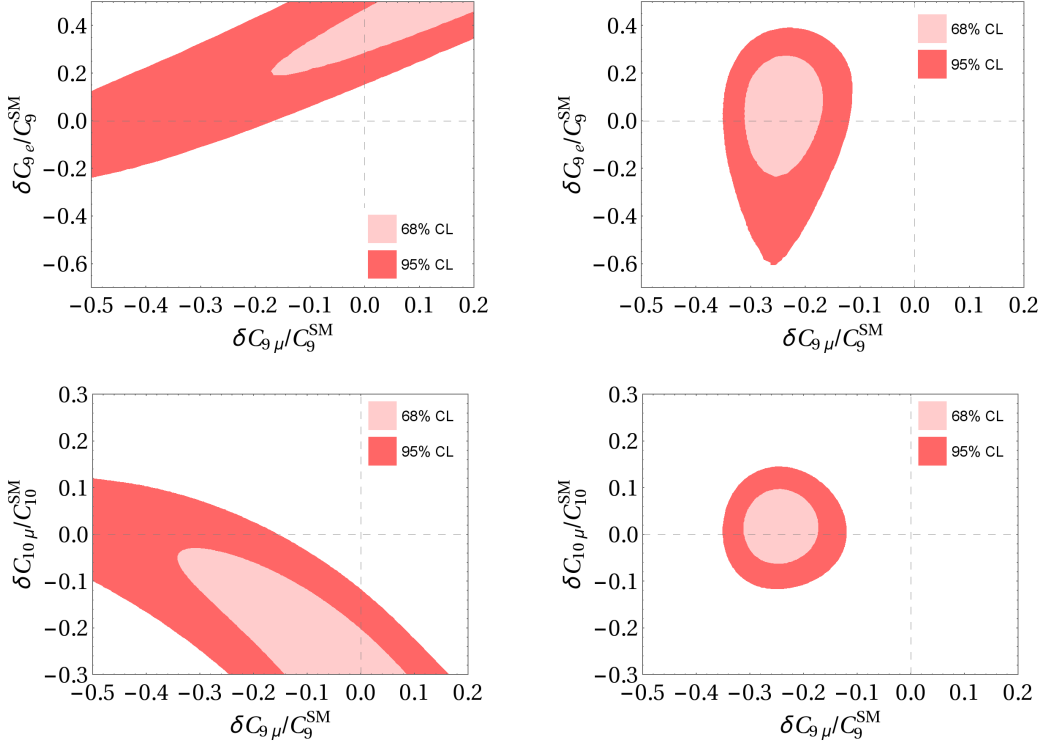


Figure 1: Global fit results with present data, using only R_K and R_{K^*} in the left, and using all observables except R_K and R_{K^*} (under the assumption of 10% non-factorisable power corrections) in the right.

with SM pulls of 3.6 and 4.4σ in the two separate fits respectively but C_9^e is much less favoured. Also, the C_{10} -like solutions do not play a role in the global fit excluding the ratios.

We present in addition fits based on two-operator hypotheses in Figure 1. As can be seen, the two sets of fit namely considering only R_K and R_{K^*} , and considering all observables except R_K and R_{K^*} are compatible at least at the 2σ level.

4. Prospects of future LHCb upgrade

The LHCb detector will be upgraded and is expected to collect a total integrated luminosity of 50 fb^{-1} . A second upgrade at a high-luminosity LHC will allow for a full dataset of up to 300 fb^{-1} . Due to the expected luminosity of 300 fb^{-1} , of 50 fb^{-1} , and in the near future of 12 fb^{-1} the statistical error will be decreased by a factor 10, 4, and 2, respectively.

For the three luminosity cases we consider three upgrade scenarios in which the current central values are assumed to remain and in which the systematic error is either unchanged or reduced by a factor of 2 or 3. In all cases we consider two options regarding the error correlations, namely that the three R_K and R_{K^*} bins/observables have no correlation or 50% correlation between each of the three measurements.

The results for these future scenarios are given in Table 3. Here we show the one-operator NP hypothesis ΔC_9^μ as an exemplary mode. It is obvious from the SM pulls that – within the scenario

ΔC_9^μ	Syst. Pull _{SM}	Syst./2 Pull _{SM}	Syst./3 Pull _{SM}
12 fb ⁻¹	6.1σ (4.3σ)	7.2σ (5.2σ)	7.4σ (5.5σ)
50 fb ⁻¹	8.2σ (5.7σ)	11.6σ (8.7σ)	12.9σ (9.9σ)
300 fb ⁻¹	9.4σ (6.5σ)	15.6σ (12.3σ)	19.5σ (16.1σ)

Table 3: Pull_{SM} for the fit to ΔC_9^μ based on the ratios R_K and R_{K^*} for the LHCb upgrade scenarios with 12, 50 and 300 fb⁻¹ luminosity collected, assuming current central values remain. For each of the upgraded luminosities the systematic error (denoted by ‘‘Syst.’’ in the table) is considered to either remain unchanged or be reduced by a factor of 2 or 3. In each scenario the three R_K and R_{K^*} bins/observables are assumed to have no correlation (50% correlation between each of the three measurements).

in which the central values are assumed to remain – only a small part of the 50 fb⁻¹ is needed to establish NP in the $R_{K^{(*)}}$ ratios even in the pessimistic case that the systematic errors are not reduced by then at all.

In addition the SM pulls for the 6 favoured one-operator NP hypotheses are all very similar in each of the upgrade scenarios. This indicates that also in future scenarios based on much larger data sets there is no differentiation between the NP hypotheses possible. This motivates the search for other ratios in the next section which are sensitive for lepton non-universality *and* serve this purpose.

We also consider the set of $b \rightarrow s\ell\ell$ observables, which is complementary to R_K and R_{K^*} . We again assume that their central values remain. Future prospects are given for two operator fits in Fig. 2. Under this assumption it seems possible that the LHCb collaboration will be able to establish new physics within the angular observables even in the pessimistic case that there will be no theoretical progress on non-factorisable power corrections.

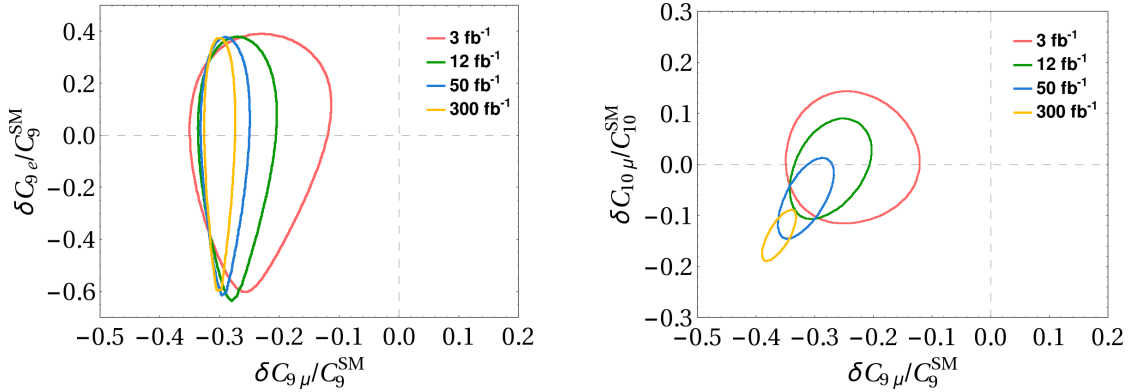


Figure 2: Global fit results for $\delta C_9^e - \delta C_9^\mu$ and $\delta C_9 - \delta C_{10}$, using all $b \rightarrow s\ell\ell$ observables (under the assumption of 10% factorisable power corrections) besides R_K and R_{K^*} are shown with a red solid line (at 2σ level). Future LHCb prospects of the fit (at 2σ level), assuming the current central values remain, are shown with green, blue and yellow (from right to left) lines corresponding to 12, 50 and 300 fb⁻¹ luminosity, respectively, with the 2σ regions shrinking from right towards left.

5. Predictions for other ratios based on the present measurements of R_K and R_{K^*}

Obs.	Predictions assuming 12 fb ⁻¹ luminosity					
	C_9^μ	C_9^e	C_{10}^μ	C_{10}^e	C_{LL}^μ	C_{LL}^e
$R_{F_L}^{[1.1,6.0]}$	[0.785, 0.913]	[0.909, 0.933]	[1.005, 1.042]	[1.001, 1.018]	[0.920, 0.958]	[0.960, 0.966]
$R_{AFB}^{[1.1,6.0]}$	[6.048, 14.819]	[-0.288, -0.153]	[0.816, 0.928]	[0.974, 1.061]	[3.338, 6.312]	[-0.684, -0.256]
$R_{S_3}^{[1.1,6.0]}$	[0.858, 0.904]	[0.795, 0.886]	[0.399, 0.753]	[0.738, 0.832]	[0.586, 0.819]	[0.766, 0.858]
$R_{S_4}^{[1.1,6.0]}$	[0.970, 1.051]	[0.848, 0.926]	[0.344, 0.730]	[0.719, 0.818]	[0.650, 0.841]	[0.780, 0.868]
$R_{S_5}^{[1.1,6.0]}$	[-0.787, 0.394]	[0.603, 0.697]	[0.881, 1.002]	[1.053, 1.146]	[0.425, 0.746]	[0.685, 0.806]
$R_{F_L}^{[15,19]}$	[0.999, 0.999]	[0.998, 0.998]	[0.997, 0.998]	[0.998, 0.998]	[0.998, 0.998]	[0.998, 0.998]
$R_{AFB}^{[15,19]}$	[0.616, 0.927]	[1.002, 1.061]	[0.860, 0.994]	[1.046, 1.131]	[0.992, 0.996]	[0.995, 0.997]
$R_{S_3}^{[15,19]}$	[0.997, 0.998]	[0.998, 0.998]	[0.999, 1.000]	[0.999, 1.000]	[0.999, 1.000]	[0.999, 0.999]
$R_{S_4}^{[15,19]}$	[0.998, 0.999]	[0.998, 0.998]	[0.998, 0.998]	[0.998, 0.999]	[0.998, 0.998]	[0.998, 0.998]
$R_{S_5}^{[15,19]}$	[0.615, 0.927]	[1.002, 1.061]	[0.860, 0.994]	[1.046, 1.131]	[0.991, 0.996]	[0.994, 0.997]
$R_{K^*}^{[15,19]}$	[0.621, 0.803]	[0.577, 0.771]	[0.589, 0.778]	[0.586, 0.770]	[0.585, 0.780]	[0.582, 0.771]
$R_K^{[15,19]}$	[0.597, 0.802]	[0.590, 0.778]	[0.659, 0.818]	[0.632, 0.805]	[0.620, 0.802]	[0.609, 0.791]
$R_\phi^{[1.1,6.0]}$	[0.748, 0.852]	[0.620, 0.805]	[0.578, 0.770]	[0.578, 0.764]	[0.629, 0.800]	[0.600, 0.784]
$R_\phi^{[15,19]}$	[0.623, 0.803]	[0.577, 0.771]	[0.586, 0.776]	[0.583, 0.769]	[0.584, 0.779]	[0.581, 0.770]

Table 4: Predictions of ratios of observables with muons in the final state to electrons in the final state at 95% confidence level, considering one operator fits obtained by assuming the central values of $R_{K^{(*)}}$ with 12 fb⁻¹ luminosity remain the same as the current 3 fb⁻¹ data. The observables $R_{F_L}, R_{AFB}, R_{S_{3,4,5}}$ correspond to ratios of $F_L, A_{FB}, S_{3,4,5}$ of the $B \rightarrow K^* \bar{\ell} \ell$ decay, respectively. The observables $R_{K^{(*)}}$ and R_ϕ correspond to the ratios of the branching fractions of $B \rightarrow K^{(*)} \bar{\ell} \ell$ and $B_s \rightarrow \phi \bar{\ell} \ell$, respectively. The superscripts denote the q^2 bins.

Finally, we make predictions for other ratios within the $b \rightarrow s \ell \ell$ transitions which could test lepton universality. We base our predictions on the measurements of R_K and R_{K^*} – assuming NP in one operator only. We consider the six one-operator hypotheses which were favoured in our fit to the present data, and give the predictions of the ratios for the future 12 fb⁻¹ upgrade in Table 4, assuming the central values of the three observables $R_{K^{(*)}}$ remain at their current values (considering the statistical error is reduced by a factor of 2 and the systematic error remains, while no correlation among the uncertainties is assumed).

From the numbers in the last four rows of Table 4 one can read off that the ratios of decay rates considered in our analysis do not help in differentiating between the six NP models. This feature is expected when one crosschecks the analytical formulae of the decay rates (it can also be directly seen from appendix D of Ref. [18]).

In contrast, the ratios of the angular observables of $B \rightarrow K^* \ell \ell$ in the low- q^2 , namely F_L, A_{FB} , and the three angular observables S_3, S_4, S_5 are able to differentiate between the six new physics options. For example the predictions of the 2σ regions for these observables within the C_9^μ and the C_{10}^μ NP models are not overlapping in any of the cases. The differentiating power will increase significantly with the 12 fb⁻¹ data set of LHCb.

However, the corresponding angular observables in the high- q^2 region have almost no differentiating power. This is expected from the well-known effect that the dependence on the Wilson

coefficients and, thus, also the NP sensitivity, in general, is rather weak for observables in the high- q^2 region.

Some of the angular observables have zero crossings in which case it would be better to use lepton flavour differences instead of ratios. Moreover, an alternative set of observables would be the ratios and/or differences of the well-known P_i observables which are free from form factor dependences to first order.

6. Conclusions

In view of the persisting deviations with the SM predictions in the rare $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ data accumulated by the LHCb experiment during the first run, we address the question of whether these deviations originate from New Physics or from unknown large hadronic power corrections by performing global fits to NP in the Wilson coefficients and to unknown power corrections, and doing a statistical comparison. Our analysis shows that adding the hadronic parameters does not improve the fit compared to the NP fit. Hence, our result is a strong indication that the NP interpretation is still a valid option, even if the situation remains inconclusive.

Assuming New Physics to be responsible for the observed anomalies, we have performed model independent NP fits to different sets of Wilson coefficients separating the very clean observables from the rest. We showed that while the two operator NP fit are consistent at 2σ level for the two different sets of observables, for the one operator fit they give a less coherent picture than often stated where the very clean ratios (in addition to the C_9^ℓ explanation) indicate preference for a scenario with modified C_{10}^ℓ which is not observed for the fit to the rest of the $b \rightarrow s \ell^+ \ell^-$ observables.

Finally, we showed that in the future LHCb upgrade if the central values remain, even with the partial 12 fb^{-1} data, New Physics can be established. Although, in order to identify the preferred New Physics scenario, ratios of further observables which so far have not been measured are needed.

References

- [1] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **111** (2013) 191801 [arXiv:1308.1707 [hep-ex]].
- [2] R. Aaij *et al.* [LHCb Collaboration], JHEP **1602** (2016) 104 [arXiv:1512.04442 [hep-ex]].
- [3] A. Abdesselam *et al.* [Belle Collaboration], arXiv:1604.04042 [hep-ex].
- [4] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2017-023.
- [5] CMS Collaboration [CMS Collaboration], CMS-PAS-BPH-15-008.
- [6] R. Aaij *et al.* [LHCb Collaboration], JHEP **1509** (2015) 179 [arXiv:1506.08777 [hep-ex]].
- [7] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **113** (2014) 151601 [arXiv:1406.6482 [hep-ex]].
- [8] R. Aaij *et al.* [LHCb Collaboration], JHEP **1708** (2017) 055 [arXiv:1705.05802 [hep-ex]].
- [9] T. Hurth, F. Mahmoudi and S. Neshatpour, JHEP **1412** (2014) 053 [arXiv:1410.4545 [hep-ph]].
- [10] T. Hurth and F. Mahmoudi, JHEP **1404** (2014) 097 doi:10.1007/JHEP04(2014)097 [arXiv:1312.5267 [hep-ph]].

- [11] T. Hurth, F. Mahmoudi, D. Martinez Santos and S. Neshatpour, *Phys. Rev. D* **96** (2017) no.9, 095034 [arXiv:1705.06274 [hep-ph]].
- [12] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, *JHEP* **1801** (2018) 093 [arXiv:1704.05340 [hep-ph]].
- [13] W. Altmannshofer, P. Stangl and D. M. Straub, *Phys. Rev. D* **96** (2017) no.5, 055008 [arXiv:1704.05435 [hep-ph]].
- [14] G. D'Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre and A. Urbano, *JHEP* **1709** (2017) 010 [arXiv:1704.05438 [hep-ph]].
- [15] G. Hiller and I. Nisandzic, *Phys. Rev. D* **96** (2017) no.3, 035003 [arXiv:1704.05444 [hep-ph]].
- [16] L. S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X. L. Ren and R. X. Shi, *Phys. Rev. D* **96** (2017) no.9, 093006 [arXiv:1704.05446 [hep-ph]].
- [17] M. Ciuchini, A. M. Coutinho, M. Fedele, E. Franco, A. Paul, L. Silvestrini and M. Valli, *Eur. Phys. J. C* **77** (2017) no.10, 688 [arXiv:1704.05447 [hep-ph]].
- [18] T. Hurth, F. Mahmoudi and S. Neshatpour, *Nucl. Phys. B* **909** (2016) 737 [arXiv:1603.00865 [hep-ph]].
- [19] F. Mahmoudi, T. Hurth and S. Neshatpour, *Nucl. Part. Phys. Proc.* **285-286** (2017) 39 [arXiv:1611.05060 [hep-ph]].
- [20] V. G. Chobanova, T. Hurth, F. Mahmoudi, D. Martinez Santos and S. Neshatpour, *JHEP* **1707** (2017) 025 [arXiv:1702.02234 [hep-ph]].
- [21] S. Neshatpour, V. G. Chobanova, T. Hurth, F. Mahmoudi and D. Martinez Santos, arXiv:1705.10730 [hep-ph].
- [22] A. Arbey, T. Hurth, F. Mahmoudi and S. Neshatpour, arXiv:1806.02791 [hep-ph].
- [23] S. Jäger and J. Martin Camalich, *JHEP* **1305** (2013) 043 [arXiv:1212.2263 [hep-ph]].
- [24] A. Khodjamirian, T. Mannel, A. A. Pivovarov and Y.-M. Wang, *JHEP* **1009** (2010) 089 [arXiv:1006.4945 [hep-ph]].
- [25] M. Ciuchini, M. Fedele, E. Franco, S. Mishima, A. Paul, L. Silvestrini and M. Valli, *JHEP* **1606** (2016) 116 [arXiv:1512.07157 [hep-ph]].
- [26] F. Mahmoudi, *Comput. Phys. Commun.* **178** (2008) 745 [arXiv:0710.2067 [hep-ph]].
- [27] F. Mahmoudi, *Comput. Phys. Commun.* **180** (2009) 1579 [arXiv:0808.3144 [hep-ph]].