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The Higgs boson discovery: recent implications for the Finite Unified Theories and SUSY breaking scale

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$N = 1$ supersymmetric GUTs that can be made finite to all orders are called Finite Unified Theories (FUTs). These theories are based on the principle of the reduction of couplings, which consists of searching for renormalization group invariant (RGI) expressions among parameters holding to all orders. FUTs predicted the top quark mass one and half years before its experimental discovery, while four and a half years before the Higgs boson discovery, a particular FUT predicted the light Higgs boson mass in the range $\sim$121–126 GeV, in striking agreement with the LHC discovery. We review the basic properties of supersymmetric finite theories resulting from reduction of couplings in both their dimensionless and dimensionful sectors. Then, we analyse the SU(5)-based FUT, which is favoured from a phenomenological point of view. This particular FUT leads to a version of the MSSM that is constrained by the finiteness conditions at the unification scale and predicts a relatively heavy spectrum with coloured supersymmetric particles above 2.7 TeV, consistent with the non-observation of those particles at the LHC. The electroweak supersymmetric spectrum starts below 1 TeV, while parts of the allowed spectrum could be accessible at CLIC. The FCC-hh is expected to be able to fully test the predicted spectrum.

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1. Introduction

Within uncertainties, the new particle discovered at the LHC in 2012 [1,2] is compatible with predictions for the Higgs boson of the Standard Model (SM) [3]. Taking the experimental results and the respective uncertainties into account, on the other hand, it occurs that many models beyond the SM can accommodate the data. Furthermore, the hierarchy problem, the neutrino masses, the dark matter, the over twenty free parameters of the SM ask for a more fundamental theory.

Therefore, one of the main aims of this fundamental theory is to relate these free parameters, or, in other words, to reduce the number of these parameters (ideally to one). This reduction is usually based on the introduction of a larger symmetry, rendering the theory more predictive, such as Grand Unified Theories (GUTs) [4–8] and their supersymmetric extensions [9,10]. One example is the minimal $SU(5)$, where the number of gauge couplings is reduced to one due to unification. Data from LEP [11] suggested that a $N = 1$ global Supersymmetry (SUSY) [9, 10] broken in the TeV scale is necessary to be introduced for the unification to be viable. Relations among the Yukawa couplings are also suggested in GUTs. The $SU(5)$, for example, predicts the ratio of tau mass to bottom mass M_τ/M_b [12] in the SM. However, GUTs based on larger gauge groups introduce new degrees of freedom and loose predictivity due to the different ways of breaking the larger symmetry.

One way to reduce further the couplings of a GUT is to relate its Yukawa and gauge sectors (Gauge-Yukawa Unification - GYU) [13–15]. Unfortunately the interesting possibility that $N = 2$ SUSY [16] could play this role is highly limited, since it predicts the existence of light mirror fermions. Phenomenological problems also appear in composite models and superstring theories.

Another approach is to search for all-loop Renormalization Group Invariant (RGI) relations among Yukawa and gauge couplings [17, 18], which hold from the unification scale up to the Planck scale [13–15, 19–24]. With this approach GYU is possible. Remarkably, RGI expressions that guarantee finiteness to all orders in perturbation theory can be found, too, assuming, among others, finiteness at one loop in $N = 1$ gauge theories [25–27].

The above approach needs SUSY as an essential ingredient. Moreover, the SUSY breaking sector has to be treated in a similar way, since it introduces several new parameters in the theory. In fact, the RGI relation searches have been extended to the Soft SUSY Breaking (SSB) sector [24, 28–30] relating parameters of mass dimension one and two.

Reduction of couplings in $N = 1$ SUSY theories has led to very interesting phenomenological developments. In past works, a "universal" set of soft scalar masses was assumed in the soft breaking sector, given that: (1) they are part of constraints that preserve two-loop finiteness [31, 32]; (2) they are two-loop RGI in more general SUSY gauge theories, subject to the condition known as $P = 1/3Q$ [28]; and (3) they appear in dilaton-dominated SUSY breaking superstring scenarios [33–35]. However, phenomenological problems occur, all due to the restrictive nature of the "universality" assumption for the soft scalar masses. For instance: (a) in Finite Unified Theories (FUTs) universality predicts that the LSP is charged, namely the superpartner of the τ lepton $\tilde{\tau}$; (b) the Minimal Supersymmetric Standard Model (MSSM) with universal soft scalar masses is incompatible with radiative electroweak symmetry breaking; and worst of all, (c) the universal condition leads to charge and/or color breaking minima deeper than the standard vacuum [36]. There have been attempts, consequently, to relax this condition without loosing its attractive features.

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First, there is an interesting observation in $N = 1$ GYU theories: there exists an RGI sum rule for the soft scalar masses at lower orders; at one loop for the non-finite case [37] and at two loops for the finite case [38]. The sum rule overcomes the above unpleasant phenomenological consequences. Moreover, the sum rule for the soft scalar masses is RGI to all orders [39] for both the general and the finite case. Finally, the exact β-functions for soft scalar masses in the Novikov–Shifman–Vainshtein–Zakharov (NSVZ) scheme [40–42] for the softly broken SUSY QCD have been obtained [39]. The use of RGI both in the dimensionful and the dimensionless sector together with the sum rule allows the construction of realistic and predictive $N = 1$ all-loop finite $SU(5)$ SUSY models with interesting predictions, as shown in [13, 19, 21, 30, 43–47].

2. Theoretical Basis

Here, we describe the idea of reduction of couplings. Any RGI relation among parameters (which does not depend on the renormalization scale μ explicitly) can be expressed in the implicit form $\Phi(g_1, \cdots, g_A) = \text{const.}$, which should satisfy the partial differential equation (PDE):

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla} \cdot \vec{\beta} \;=\; \sum_{a=1}^{A} \beta_a \frac{\partial \Phi}{\partial g_a} \;=\; 0 \,, \tag{2.1}$$

where β_a is the β-function of g_a. This PDE is equivalent to a set of ordinary differential equations (ODEs), the so-called *reduction equations* (REs) [17, 18, 48]

$$\beta_g \frac{dg_a}{dg} = \beta_a \,,\; a = 1, \cdots, A \,, \tag{2.2}$$

where g and β_g are the primary coupling and its β-function respectively and the counting on a does not include g. Since maximally $(A-1)$ independent RGI "constraints" in the A-dimensional space of parameters can be imposed by the Φ_as, one could in principle express all the parameters in terms of a single parameter g. A closer look at the set of Eq. (2.2), however, reveals that their general solutions contain as many integration constants as the number of equations themselves. That means that, using such integration constants, we have just traded an integration constant for each ordinary renormalized coupling. Consequently, the general solutions cannot be considered as reduced ones. The crucial requirement in the search for RGI relations is to demand power series solutions to the REs,

$$g_a = \sum_n \rho_a^{(n)} \, g^{2n+1} \,, \tag{2.3}$$

which preserve perturbative renormalizability. This ansatz fixes the corresponding integration constant in each of the REs and picks up a special solution out of each general one. Remarkable is the fact that the uniqueness of such power series solutions can be decided already at one-loop level [17, 18, 48]. To illustrate this, we will assume that the β-functions have the form:

$$\begin{aligned}
\beta_a &= \frac{1}{16\pi^2}[\sum_{b,c,d\neq g} \beta_a^{(1)\,bcd} g_b g_c g_d + \sum_{b\neq g} \beta_a^{(1)\,b} g_b g^2] + \cdots \,, \\
\beta_g &= \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots \,,
\end{aligned} \tag{2.4}$$

where $\cdots$ stands for higher-order terms and $\beta_a^{(1)\,bcd}$s are symmetric in b,c,d. We will then assume that the $\rho_a^{(n)}$s with $n \leq r$ are uniquely determined. In order to obtain $\rho_a^{(r+1)}$s, we then insert the power series (2.3) into the REs (2.2), collect terms of $\mathscr{O}(g^{2r+3})$ and find:

$$\sum_{d\neq g} M(r)_a^d \, \rho_d^{(r+1)} = \text{lower-order quantities} \ ,$$

where the r.h.s. is known by assumption and

$$M(r)_a^d = 3 \sum_{b,c\neq g} \beta_a^{(1)\,bcd}\,\rho_b^{(1)}\,\rho_c^{(1)} + \beta_a^{(1)\,d} - (2r+1)\,\beta_g^{(1)}\,\delta_a^d \ , \tag{2.5}$$

$$0 = \sum_{b,c,d\neq g} \beta_a^{(1)\,bcd}\,\rho_b^{(1)}\,\rho_c^{(1)}\,\rho_d^{(1)} + \sum_{d\neq g} \beta_a^{(1)\,d}\,\rho_d^{(1)} - \beta_g^{(1)}\,\rho_a^{(1)} \ . \tag{2.6}$$

Therefore, the $\rho_a^{(n)}$s for all $n > 1$ for a given set of $\rho_a^{(1)}$s can be uniquely determined if $\det M(n)_a^d \neq 0$ for all $n \geq 0$.

As it will be clear later, the various couplings in SUSY theories have the same asymptotic behaviour, so searching for power series solutions of the form (2.3) to the REs (2.2) is justified.

The path to coupling unification described in this section surely is attractive, because the "completely reduced" theory contains only one independent parameter. However, for some models it can be unrealistic. Therefore, it is often to impose fewer RGI constraints. This is the idea of partial reduction [49, 50].

3. Finiteness in *N* = 1 Supersymmetric Gauge Theories

For a chiral, anomaly-free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g, the superpotential is given by

$$W = \frac{1}{2}\, m_{ij}\, \phi_i\, \phi_j + \frac{1}{6}\, C_{ijk}\, \phi_i\, \phi_j\, \phi_k \ , \tag{3.1}$$

where m_{ij} and C_{ijk} are gauge invariant tensors and the matter field ϕ_i transforms according to the irreducible representation R_i of the gauge group G. Then the renormalization constants associated with the superpotential (3.1), assuming that supersymmetry is exact, are:

$$\phi_i^0 = (Z_i^j)^{(1/2)}\, \phi_j \ , \tag{3.2}$$

$$m_{ij}^0 = Z_{ij}^{i'j'}\, m_{i'j'} \ , \tag{3.3}$$

$$C_{ijk}^0 = Z_{ijk}^{i'j'k'}\, C_{i'j'k'} \ . \tag{3.4}$$

By virtue of the $N = 1$ non-renormalization theorem [53–55] there are no mass and cubic-interaction-term infinities, therefore:

$$\begin{aligned} Z_{ijk}^{i'j'k'}\, Z_{i'}^{1/2\,i''}\, Z_{j'}^{1/2\,j''}\, Z_{k'}^{1/2\,k''} &= \delta_{(i}^{i''}\, \delta_j^{j''}\, \delta_{k)}^{k''} \ , \\ Z_{ij}^{i'j'}\, Z_{i'}^{1/2\,i''}\, Z_{j'}^{1/2\,j''} &= \delta_{(i}^{i''}\, \delta_{j)}^{j''} \ . \end{aligned} \tag{3.5}$$

Thus, the only surviving infinities are the wave-function renormalization constants Z_i^j, i.e. one infinity for each field. The one-loop β-function of the gauge coupling g is given by [56]:

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i l(R_i) - 3\,C_2(G)\right], \tag{3.6}$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir operator of the adjoint rep of the gauge group G. The β-functions of C_{ijk}, due to the non-renormalization theorem, are proportional to the anomalous dimensions γ_{ij} of the matter fields ϕ_i:

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl}\,\gamma_k^l + C_{ikl}\,\gamma_j^l + C_{jkl}\,\gamma_i^l\,, \tag{3.7}$$

where at one-loop γ_{ij} is given by [56]:

$$\gamma_j^{i(1)} = \frac{1}{32\pi^2}\left[C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R)\delta_j^i\right], \tag{3.8}$$

where $C^{ijk} = C_{ijk}^*$. Since dimensionful parameters such as masses and couplings of cubic scalar field terms do not influence the asymptotic properties of theories we are interested in, it is sufficient to take into account only the dimensionless SUSY couplings, i.e. g and C_{ijk}. Therefore, we may neglect the existence of dimensionful parameters and assume that C_{ijk} are real (so C_{ijk}^2 are always positive numbers).

One can see from Equations (3.6) and (3.8) that all the one-loop β-functions of the theory vanish if $\beta_g^{(1)}$ and $\gamma_{ij}^{(1)}$ vanish:

$$\sum_i \ell(R_i) = 3C_2(G), \tag{3.9}$$

$$C^{ikl}C_{jkl} = 2\delta_j^i g^2 C_2(R_i). \tag{3.10}$$

The conditions for finiteness for $N = 1$ theories with $SU(N)$ gauge structure are discussed in [57] and the analysis for the anomaly-free and no-charge renormalization requirements for these theories is found in [58]. An interesting result is that conditions (3.9) and (3.10) are also necessary and sufficient for finiteness at the two-loop level [56, 59–62].

In the case that supersymmetry is broken by the inclusion of soft terms, the requirement of finiteness in the one-loop soft breaking terms imposes further constraints [31]. In addition, the set of conditions that are sufficient for one-loop finiteness of the soft terms renders the soft sector two-loop finite as well [32].

The one-loop and two-loop finiteness conditions (3.9) and (3.10) restrict considerably the possible choices of the irreducible representations (irreps) R_i for a group G, as well as Yukawa couplings in the superpotential (3.1). Note that these finiteness conditions cannot be applied to the Minimal Supersymmetric Standard Model (MSSM), since the $U(1)$ gauge group is incompatible with condition (3.9) due to $C_2[U(1)] = 0$. This leads to the expectation that finiteness will be attained at GUT level only, the MSSM being just the corresponding low-energy effective theory.

Another important consequence of one- and two-loop finiteness is that supersymmetry can only be broken softly. Indeed, gauge singlets are unacceptable, F-type spontaneous symmetry breaking terms [63] are incompatible with finiteness, as well as D-type [64] spontaneous breaking, as it requires a $U(1)$ gauge group.

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A natural question to ask is what happens at higher orders. The answer can be found in a theorem [26, 65] that states the necessary and sufficient conditions to achieve all-loop finiteness. Before we discuss the theorem, some introductory remarks are in order. The finiteness conditions impose relations between gauge and Yukawa couplings. However, it is trivial to require such relations that render the couplings mutually dependent at a given renormalization point. What is non-trivial is to guarantee that relations leading to reduction of couplings hold at any renormalization point. As we already know, the necessary and sufficient condition for this to happen is to require that such relations are solutions to the REs

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \tag{3.11}$$

and hold at all orders. It is remarkable that the existence of all-loop power series solutions to (3.11) can be decided at one-loop level, as it was already mentioned.

Let us now turn to the all-order finiteness theorem [26, 65], which states the conditions under which an $N = 1$ SUSY gauge theory can become finite to all orders in the sense of vanishing β-functions, that is of physical scale invariance. It is based on (a) the structure of the supercurrent in $N = 1$ SUSY gauge theory [66–68] and on (b) the non-renormalization properties of $N = 1$ chiral anomalies [25, 26, 65, 69, 70]. Details on the proof can be found in [26, 65] and further discussion in Refs. [25, 27, 69–71].

Theorem: *Consider an $N = 1$ SUSY Yang–Mills theory, with the simple gauge group. If the following conditions are satisfied:*

1. *There is no gauge anomaly.*
2. *The gauge β-function vanishes at one loop:*
$$\beta_g^{(1)} = 0 = \sum_i l(R_i) - 3C_2(G). \tag{3.12}$$
3. *There exist solutions of the form:*
$$C_{ijk} = \rho_{ijk} g, \qquad \rho_{ijk} \in \mathbb{C} \tag{3.13}$$
to the conditions of vanishing one-loop matter fields' anomalous dimensions:
$$\begin{aligned} \gamma_j^{i\,(1)} &= 0 \\ &= \frac{1}{32\pi^2}\,[\,C^{ikl}\,C_{jkl} - 2\,g^2\,C_2(R)\delta_j^i]. \end{aligned} \tag{3.14}$$
4. *These solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β-functions:*
$$\beta_{ijk} = 0. \tag{3.15}$$

Then, each of the solutions (3.13) can be uniquely extended to a formal power series in g, and the associated super Yang–Mills models depend on the single coupling constant g with a β function, which vanishes at all orders.

It is important to note that the requirement of isolated and non-degenerate solutions guarantees the existence of a unique formal power series solution to the reduction equations. Thus, we see that finiteness and reduction of couplings are intimately related. Also, we should note that one cannot extend the validity of a similar theorem in non-SUSY theories.

4. The SSB Sector of Reduced $N = 1$ SUSY and Finite Theories

The method of reducing the dimensionless couplings has been extended [24] to the soft supersymmetry breaking (SSB) dimensionful parameters of $N = 1$ SUSY theories. In addition, it was found [37] that SSB scalar masses in GYU models satisfy an RGI sum rule.

Consider the superpotential given by (3.1) along with the Lagrangian for SSB terms:

$$\begin{aligned}-\mathscr{L}_{\rm SSB} = &\frac{1}{6}h^{ijk}\,\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\,\phi_i\phi_j \\ &+\frac{1}{2}(m^2)^j_i\,\phi^{*i}\phi_j + \frac{1}{2}M\lambda\lambda + \text{h.c.},\end{aligned}$$

where ϕ_i are the scalar parts of chiral superfields Φ_i, λ are the gauginos and M their unified mass.

We assume that the REs admit power series solutions:

$$C^{ijk} = g\sum_n \rho^{ijk}_{(n)} g^{2n}\ . \tag{4.1}$$

In the case of finite theories, we further assume that the gauge group is a simple group and the one-loop gauge β-function vanishes. According to the finiteness theorem Refs. [26, 65] and the assumption given in (4.1), the theory is then all-order finite, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one-loop and two-loop finiteness for h^{ijk} can be achieved by [32]:

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\,g + O(g^5)\ , \tag{4.2}$$

where ... stand for higher order terms.

In order to obtain the two-loop sum rule for soft scalar masses, we will assume that the lowest order coefficients $\rho^{ijk}_{(0)}$ and also $(m^2)^i_j$ satisfy the diagonality relations:

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \quad \text{for all } p \text{ and } q \ \text{ and } \quad (m^2)^i_j = m^2_j\delta^i_j\ . \tag{4.3}$$

Then the following soft scalar-mass sum rule is found [15, 38, 76]:

$$(\,m_i^2 + m_j^2 + m_k^2\,)/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(2)} + O(g^4) \tag{4.4}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$. $\Delta^{(2)}$ is the two-loop correction

$$\Delta^{(2)} = -2\sum_l [(m_l^2/MM^\dagger) - (1/3)]\ T(R_l), \tag{4.5}$$

which vanishes for the universal choice in accordance with the previous findings of Ref. [32].

Using the spurion technique [55, 77–80], one can find the following all-loop relations among β-functions [81–86]:

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right) , \tag{4.6}$$

$$\begin{aligned}\beta_h^{ijk} &= \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} \\ &\quad - 2\gamma^i_{1}{}_l C^{ljk} - 2\gamma^j_{1}{}_l C^{ilk} - 2\gamma^k_{1}{}_l C^{ijl} ,\end{aligned} \tag{4.7}$$

$$(\beta_{m^2})^i{}_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i{}_j , \tag{4.8}$$

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right) , \tag{4.9}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}} , \tag{4.10}$$

where $(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j$, $C_{lmn} = (C^{lmn})^*$, and:

$$\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl} . \tag{4.11}$$

Assuming, following [82], that

$$h^{ijk} = -M(C^{ijk})' \equiv -M\frac{dC^{ijk}(g)}{d\ln g} \tag{4.12}$$

is all-loop RGI and using the all-loop gauge β-function of Novikov et al. [40–42] given by:

$$\beta_g^{\rm NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_l T(R_l)(1-\gamma_l/2) - 3C(G)}{1 - g^2C(G)/8\pi^2}\right] , \tag{4.13}$$

the all-loop RGI sum rule [39] has been found:

$$\begin{aligned} m_i^2 + m_j^2 + m_k^2 = |M|^2\{ &\frac{1}{1 - g^2C(G)/(8\pi^2)}\frac{d\ln C^{ijk}}{d\ln g} + \frac{1}{2}\frac{d^2\ln C^{ijk}}{d(\ln g)^2} \} \\ &+ \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d\ln C^{ijk}}{d\ln g} .\end{aligned} \tag{4.14}$$

Also, the exact β-function for m^2 in the NSVZ scheme has been obtained [39] for the first time:

$$\begin{aligned}\beta_{m_i^2}^{\rm NSVZ} = \Big[&|M|^2\{ \frac{1}{1 - g^2C(G)/(8\pi^2)}\frac{d}{d\ln g} + \frac{1}{2}\frac{d^2}{d(\ln g)^2} \} \\ &+ \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2}\frac{d}{d\ln g} \Big] \gamma_i^{\rm NSVZ} .\end{aligned} \tag{4.15}$$

Surprisingly, the all-loop result (4.14) coincides with the superstring result for the finite case in a certain class of orbifold models [38] if $d\ln C^{ijk}/d\ln g = 1$.

Finally, it is important to emphasize that the sum rule holds always, to the extend that there is reduction of couplings. A consequence from the reduction of dimensionful parameters is that in some cases it is possible to have exact relations among the soft scalar masses and a mass-dimension one parameter (which could be the gaugino unified mass). This option is not phenomenologically viable for the case of Finite Unified Theories, though.

5. A Successful Finite Unified Theory

In this section we will review an all-loop FUT with $SU(5)$ as the gauge group, where the reduction of couplings has been applied to the third fermionic generation. This model was selected on the basis of agreement with known experimental data at the time [44]. It predicted the light Higgs boson mass to be in the range 121–126 GeV four and a half years before the discovery. The particle content of the model we will study, which we denote $SU(5)$-**FUT**, consists of three $(\overline{\mathbf{5}}+\mathbf{10})$ supermultiplets needed for each of the three generations of quarks and leptons, four $(\overline{\mathbf{5}}+\mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the FUT is broken, the theory is no longer finite and we are left with the MSSM [14, 19–23].

A predictive GYU all-loop finite $SU(5)$ model, in addition to the above-mentioned requirements, should have the following properties:

1. One-loop anomalous dimensions are diagonal: $\gamma_i^{(1)\,j} \propto \delta_i^j$.

2. Three fermion generations in the irreducible representations $\overline{\mathbf{5}}_i, \mathbf{10}_i$ $(i=1,2,3)$, which obviously should not couple to the adjoint $\mathbf{24}$.

3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\overline{\mathbf{5}}+\mathbf{5}$, which couple to the third fermionic generation.

After the method of reduction of couplings is applied, the symmetry is enhanced, leading to the superpotential [38, 90]:

$$
\begin{aligned}
W = \sum_{i=1}^{3} [\, \frac{1}{2} g_i^u \,\mathbf{10}_i \mathbf{10}_i H_i + g_i^d \,\mathbf{10}_i \overline{\mathbf{5}}_i \overline{H}_i \,] + g_{23}^u \,\mathbf{10}_2 \mathbf{10}_3 H_4 \\
+ g_{23}^d \,\mathbf{10}_2 \overline{\mathbf{5}}_3 \overline{H}_4 + g_{32}^d \,\mathbf{10}_3 \overline{\mathbf{5}}_2 \overline{H}_4 + g_2^f H_2 \mathbf{24} \overline{H}_2 + g_3^f H_3 \mathbf{24} \overline{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3 \,.
\end{aligned} \tag{5.1}
$$

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give:

$$
\begin{aligned}
&(g_1^u)^2 = \frac{8}{5}\, g^2 \,,\ (g_1^d)^2 = \frac{6}{5}\, g^2 \,,\ (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5}\, g^2 \,, \\
&(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5}\, g^2 \,,\ (g_{23}^u)^2 = \frac{4}{5}\, g^2 \,,\ (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5}\, g^2 \,, \\
&(g^\lambda)^2 = \frac{15}{7} g^2 \,,\ (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2}\, g^2 \,,\ (g_1^f)^2 = 0 \,,\ (g_4^f)^2 = 0 \,,
\end{aligned} \tag{5.2}
$$

and from the sum rule we obtain:

$$
m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2 \,,\ m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} \,,\ m_{\overline{\mathbf{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3} \,. \tag{5.3}
$$

One can observe that we have only two free parameters ($m_{\mathbf{10}}$ and M) for the dimensionful sector.

As it was mentioned already, after the $SU(5)$ gauge symmetry breaking we assume we have the MSSM, i.e. only two Higgs doublets. This can be done by introducing appropriate mass terms that allow for a rotation of the Higgs sector [19, 23, 91–93] in such a way, that only one pair of Higgs doublets, coupled mostly to the third generation, remains light and acquires vevs. To avoid fast proton decay the usual fine tuning to achieve doublet-triplet splitting is performed, although the mechanism is not identical to the one of the minimal $SU(5)$, since we now have an extended Higgs sector.

Thus, after the GUT gauge symmetry is broken, we are left with the MSSM with the boundary conditions for the third family given by the finiteness conditions. The other two families are not restricted.

6. Phenomenological Constraints

In our phenomenological analysis we consider four types of flavour constraints, in which supersymmetry is known to have significant impact. Specifically, we consider the flavour observables $\mathrm{BR}(b \to s\gamma)$, $\mathrm{BR}(B_s \to \mu^+\mu^-)$, $\mathrm{BR}(B_u \to \tau\nu)$ and ΔM_{B_s}.We have not used the latest experimental and theoretical values here, but this has a minor impact on our results. The uncertainties below are the linear combination of the experimental error and twice the theoretical uncertainty in the MSSM.

For the branching ratio $\mathrm{BR}(b \to s\gamma)$ we take the value from the Heavy Flavor Averaging Group (HFAG) [94, 95]:

$$\frac{\mathrm{BR}(b \to s\gamma)^{\mathrm{exp}}}{\mathrm{BR}(b \to s\gamma)^{\mathrm{SM}}} = 1.089 \pm 0.27 \; . \tag{6.1}$$

For the branching ratio $\mathrm{BR}(B_s \to \mu^+\mu^-)$ we use a combination of CMS and LHCb data [96–100]:

$$\mathrm{BR}(B_s \to \mu^+\mu^-) = (2.9 \pm 1.4) \times 10^{-9} \; . \tag{6.2}$$

For the B_u decay to $\tau\nu$ we use the limit [95, 101, 102]:

$$\frac{\mathrm{BR}(B_u \to \tau\nu)^{\mathrm{exp}}}{\mathrm{BR}(B_u \to \tau\nu)^{\mathrm{SM}}} = 1.39 \pm 0.69 \; , \tag{6.3}$$

while for ΔM_{B_s} we use [103, 104]:

$$\frac{\Delta M_{B_s}^{\mathrm{exp}}}{\Delta M_{B_s}^{\mathrm{SM}}} = 0.97 \pm 0.2 \tag{6.4}$$

Since it is well known that the lightest neutralino, being the Lightest SUSY Particle (LSP), is an excellent candidate for CDM [105], one can in principle demand that the lightest neutralino is indeed the LSP and parameters leading to a different LSP could be discarded. The current bound, favoured by a joint analysis of WMAP/Planck and other astrophysical and cosmological data, is at $2\,\sigma$ level given by [106, 107]

$$\Omega_{\mathrm{CDM}} h^2 = 0.1120 \pm 0.0112 \; . \tag{6.5}$$

Since the quartic couplings in the Higgs potential are given by the SM gauge couplings, the lightest Higgs boson mass is not a free parameter, but rather predicted in terms of other parameters. Higher-order corrections are crucial for a precise prediction of M_h; see Refs. [108–110] for reviews.

The discovery of a Higgs-like particle at ATLAS and CMS in July 2012 [1,2] can be interpreted as the discovery of the light $\mathcal{CP}$-even Higgs boson of the MSSM Higgs spectrum [111–113]. The experimental average for the (SM) Higgs boson mass is [114]

$$M_H^{\mathrm{exp}} = 125.1 \pm 0.3 \text{ GeV} \tag{6.6}$$

and, adding a 3 (2) GeV theory uncertainty [115–117] for the Higgs mass calculation in the MSSM, we have an allowing range:

$$M_h = 125.1 \pm 3.1\ (2.1)\ \text{GeV}\ . \tag{6.7}$$

We used the `FeynHiggs` [115, 117, 118] code (Version 2.14.0 beta) to predict the light Higgs mass. The evaluation of the Higgs masses with `FeynHiggs` is based on the combination of a fixed order diagrammatic calculation and a resummation of the (sub)leading logarithmic contributions at all orders of perturbation theory. This combination ensures a reliable evaluation of M_h also for large supersymmetry scales. Refinements in the combination of the fixed order log resummed calculation have been included w.r.t. previous versions [117]. They resulted in a more precise M_h evaluation for high supersymmetric mass scales and also in a downward shift of M_h at the level of $\mathscr{O}(2\ \text{GeV})$ for large SUSY masses.

7. Numerical Analysis

Since the gauge symmetry is broken below M_{GUT}, the finiteness conditions do not restrict the renormalization properties at low energies. Thus, all that remains are boundary conditions on the gauge and Yukawa couplings (5.2), the $h = -MC$ (4.2) relation and the soft scalar-mass sum rules at M_{GUT}.

In Figure 1 we show the $SU(5)$-**FUT** predictions for m_t and $m_b(M_Z)$ as a function of the unified gaugino mass M for the cases $\mu < 0$ and $\mu > 0$. We use the experimental value of the top quark pole mass as in [102].

$$m_t^{\text{exp}} = (173.2 \pm 0.9)\ \text{GeV}\ . \tag{7.1}$$

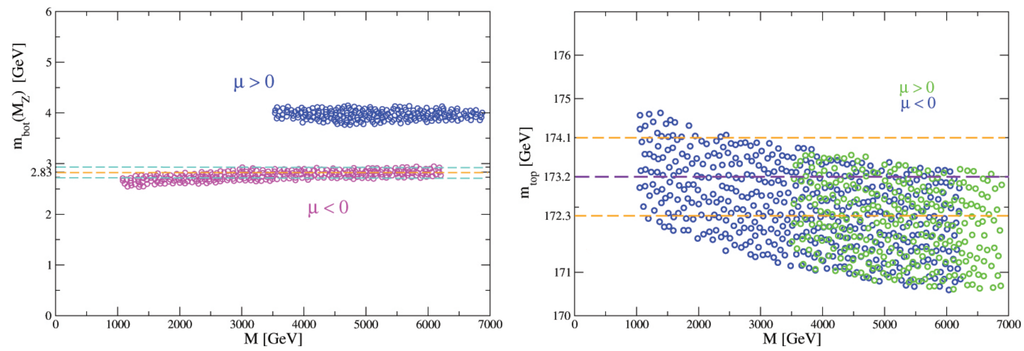


Figure 1: *The bottom quark mass at the Z boson scale (left) and top quark pole mass (**right**) as a function of M for both signs of μ.*

We did not include the latest LHC/Tevatron data leading to $m_t^{\text{exp}} = (173.34 \pm 0.76)$ GeV [119], which would have a negligible impact on our analysis.

The bottom mass is calculated at M_Z, in order to avoid uncertainties that come from running down to the pole mass. The leading supersymmetric radiative corrections to the bottom and tau masses have been taken into account [120]. For the bottom mass we use at M_Z [102]:

$$m_b(M_Z) = (2.83 \pm 0.10)\ \text{GeV}. \tag{7.2}$$

The experimental bounds on the $m_b(M_Z)$ and the m_t mass clearly single out $\mu < 0$ as the only solution compatible with these constraints.

The prediction for M_h of $SU(5)$-**FUT** with $\mu < 0$ is shown in Figure 2 in a range for the unified gaugino mass 0.5 TeV $\lesssim M \lesssim$ 9 TeV. The green points satisfy the B-physics constraints as well. We should note here that these predictions are subject to a theory uncertainty of 3 (2) GeV [115]. Older analyses, including in particular less refined evaluations of the light Higgs mass, are given in Refs. [45, 121, 122].

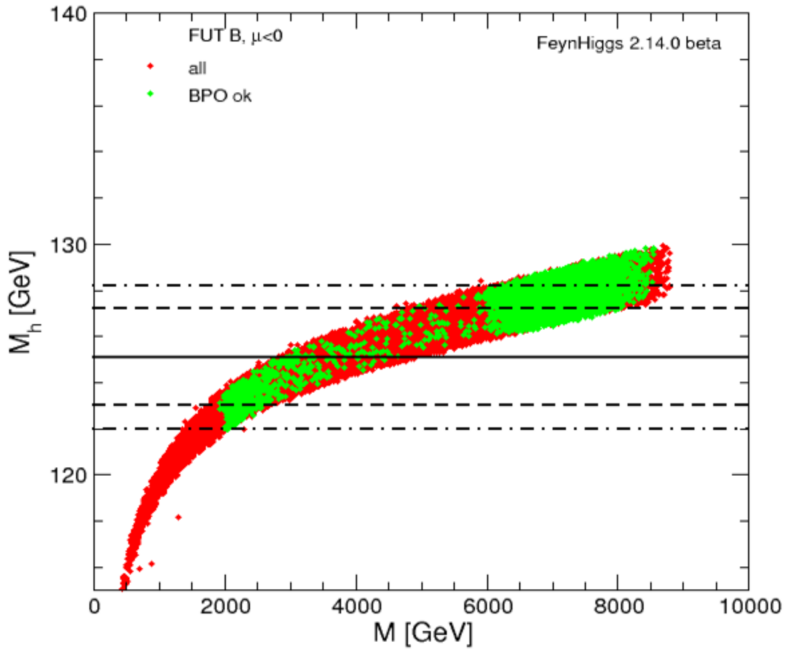
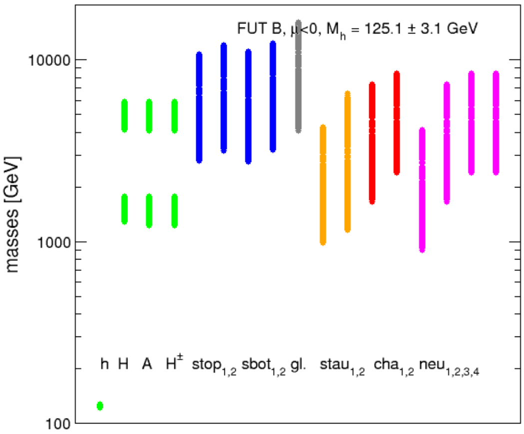


Figure 2: *The lightest Higgs boson mass, M_h, as a function of M for the choice $\mu < 0$. The green points are the ones that satisfy the B-physics constraints.*

The allowed values of the lightest Higgs boson mass limit the allowed supersymmetric masses' values, as it can be seen in Figure 3. In the left (right) plot we impose $M_h = 125.1 \pm 3.1$ (2.1) GeV. In particular, very heavy coloured SUSY particles are favoured (nearly independent of the M_h uncertainty), in agreement with the non-observation of those particles at the LHC [123]. Overall, the allowed coloured supersymmetric masses will remain unobservable at the (HL-)LHC, the ILC or CLIC. The lower part of the electroweak spectrum could be accessible at CLIC with $\sqrt{s} = 3$ TeV The coloured spectrum would be accessible, however, at the FCC-hh [124], as could the full heavy Higgs spectrum.

In Table 1 two example spectra of $SU(5)$-**FUT** (with $\mu < 0$) are shown, which span the mass range of the parameter space that is in agreement with the B-physics observables and the lightest Higgs boson mass measurement. We show the lightest and the heaviest spectrum (based on $m_{\tilde{\chi}_1^0}$) for $\delta M_h = 2.1$ and $\delta M_h = 3.1$. The Higgs boson masses are denoted as M_h, M_H, M_A and $M_{H^\pm}$. $m_{\tilde{t}_{1,2}}$, $m_{\tilde{t}_{1,2}}$, $m_{\tilde{g}}$ and $m_{\tilde{\tau}_{1,2}}$, are the scalar top, bottom, gluino and tau masses, respectively. $m_{\tilde{\chi}^\pm_{1,2}}$ and $m_{\tilde{\chi}^0_{1,2,3,4}}$ stand for chargino and neutralino masses, respectively.



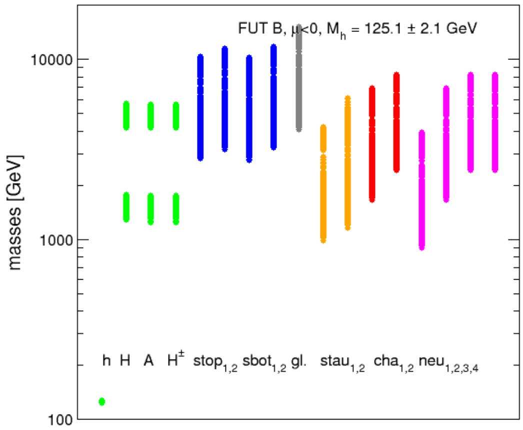


Figure 3: *The (left,right) plots show the spectrum of the $SU(5)$-**FUT** (with $\mu < 0$) model after imposing the constraint $M_h = 125.1 \pm 3.1(2.1)$ GeV. The light (green) points are the various Higgs boson masses; the dark (blue) points following are the two scalar top and bottom masses; the gray ones are the gluino masses; then come the scalar tau masses in orange (light gray); the darker (red) points to the right are the two chargino masses; followed by the lighter shaded (pink) points indicating the neutralino masses.*

No point of $SU(5)$-**FUT** (with $\mu < 0$) fulfills the bound of Eq. (6.5) (we have used the code `MicroMegas` [125–127]). Consequently, a mechanism is needed in our model to reduce the CDM abundance in the early universe. This issue could be related to the problem of neutrino masses. These mass cannot be generated naturally within the FUT we are examining, although a non-zero value for neutrino masses has clearly been established [102]. However, $SU(5)$-**FUT** (with $\mu < 0$) can be, in principle, extended by introducing bilinear R-parity violating terms (that preserve finiteness) and introduce neutrino masses [128–130]. R-parity violation [131] would have a small impact on the above collider phenomenology (apart from the fact that sypersymmmetry search strategies could not rely on a ‘missing energy’ signature), but remove the CDM bound of Eq. (6.5) completely. Other mechanisms, not involving R-parity violation and keeping the ‘missing energy’ signature, that could be invoked if the amount of CDM appears to be too large, concern the cosmology of the early universe. For example, “thermal inflation” [132] or “late time entropy injection” [133] can bring the CDM density into agreement with WMAP measurements.

8. Conclusions

The MSSM is, indeed, a very attractive candidate for describing physics beyond the SM. The serious problem of the many free parameters of the SM is proliferated, however, in the MSSM. Assuming a GUT, following the the idea that a particle physics theory should be more symmetric at higher scales, seems to fit the MSSM. Still, the unification scenario is yet unable to further reduce the large number of free parameters.

In order to reduce these free parameters, an approach was proposed Refs. [17, 18] based on the possible existence of RGI expressions among parameters. FUTs seem to be a very promising field for application of this new approach, while in this case the discovery of such RGI relations above the unification scale ensures all-order finiteness.

$\delta M_h = 2.1$	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	123.1	1533	1528	1527	2800	3161	2745	3219	4077
heaviest	127.2	4765	4737	4726	10328	11569	10243	11808	15268
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	983	1163	1650	2414	900	1650	2410	2414	45
heaviest	4070	5141	6927	8237	3920	6927	8235	8237	46

$\delta M_h = 3.1$	M_h	M_H	M_A	$M_{H^\pm}$	$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$m_{\tilde{g}}$
lightest	122.8	1497	1491	1490	2795	3153	2747	3211	4070
heaviest	127.9	4147	4113	4103	10734	12049	11077	12296	16046
	$m_{\tilde{\tau}_1}$	$m_{\tilde{\tau}_2}$	$m_{\tilde{\chi}_1^\pm}$	$m_{\tilde{\chi}_2^\pm}$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{\chi}_2^0}$	$m_{\tilde{\chi}_3^0}$	$m_{\tilde{\chi}_4^0}$	$\tan\beta$
lightest	1001	1172	1647	2399	899	647	2395	2399	44
heaviest	4039	6085	7300	8409	4136	7300	8406	8409	45

Table 1: *Two example spectra of the $SU(5)$-**FUT** (with $\mu < 0$) . All masses are in GeV and rounded to 1 (0.1) GeV (for the light Higgs mass).*

One can see that the predictions of $SU(5)$-**FUT** are impressive. But one could also add some comments on the theoretical side. The developments on treating the problem of divergencies include string and non-commutative theories, as well as $N = 4$ supersymmetric theories [134, 135], $N = 8$ supergravity [136–140] and the AdS/CFT correspondence [141]. It is interesting that the $N = 1$ FUT discussed here includes ideas that have survived phenomenological and theoretical tests, as well as the ultraviolet divergence problem and solves it in a minimal way.

In our analysis of $SU(5)$-**FUT** [142] we included restrictions of third generation quark masses and B-physics observables and it proved consistent with all the phenomenological constraints. Compared to our previous analyses [45, 46, 121, 122, 143, 144], the improved evaluation of M_h prefers a heavier (Higgs) spectrum and thus allows only a heavy supersymmetric spectrum. The coloured spectrum easily escapes (HL-)LHC searches, but can likely be tested at the FCC-hh. However, the lower part of the EW spectrum could be observable at CLIC.

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