

## Parton shower matching and merging

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With Run-II of the LHC starting its final year of data-taking, high precision theoretical predictions through Monte-Carlo event generators whose uncertainties match that of the recorded data are of highest importance. This talk summarises the progress of the field and highlight a number of recent developments.

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## 1. Introduction

Modern Monte-Carlo event generators like PYTHIA [1], HERWIG [2, 3] or SHERPA [4, 5, 6, 7, 8, 9, 10, 11] are the workhorses for the physics analyses and measurements at the LHC. In many cases, the PYTHIA and HERWIG generators (or their older predecessors) receive input from parton level tools which compute the hard core production matrix elements either at LO (ALPGEN [12] or MADGRAPH5\_aMC@NLO [13]), NLO (MADGRAPH5\_aMC@NLO or POWHEG [14]) or even NNLO (POWHEG-MiNLO [15, 16, 17, 18] or GENEVA [19]), which are matched to the parton shower. The following contribution thus highlights a few important improvements that become available in recent years, some of the forming the standard for the experiments now.

## 2. Matching next-to-leading order matrix elements to parton showers

In a first step, the matching of NLO matrix elements to parton showers, which is known for over a decade now, is briefly reviewed. In the literature, there exist various different ansatzes: POWHEG [20, 21] and MC@NLO [22, 23], and various variants thereof [24, 25, 26, 27, 28, 29, 30, 31]. They are available for all processes of interest and, thus, for the minimum baseline standard in LHC physics analyses. Their general aim is to keep both the NLO accuracy in an expansion of the cross section in  $\alpha_s$  at the same time as the full logarithmic accuracy of the parton shower resummation.

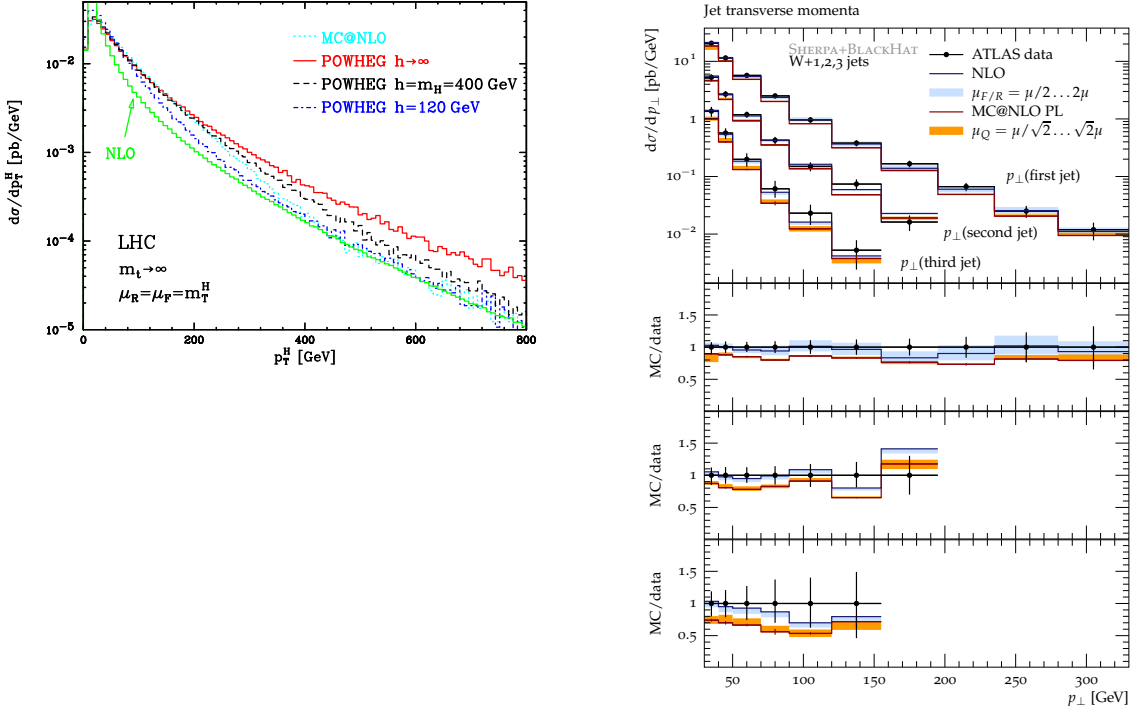
$$\langle O \rangle^{\text{NLOPS}} = \int d\Phi_B \bar{B}(\Phi_B) \widetilde{\text{PS}}_B(\mu_Q^2, O) + \int d\Phi_R H(\Phi_R) \text{PS}_R(t_R, O) \quad (2.1)$$

with the familiar  $\bar{B}$  function defined as  $\bar{B} = B + V + I_K$ , and the hard remainder  $H = R - D_K$ . The matched parton shower  $\widetilde{\text{PS}}_B(\mu_Q^2, O)$  is defined through  $\widetilde{\text{PS}}_B(\mu_Q^2, O) = \Delta_{D_K}(\mu_Q^2, t_c) O(\Phi_B) + \int dt' \frac{D_K}{B} \Delta_{D_K}(\mu^2, t') \text{PS}_R(t', O)$  with a continuing standard parton shower  $\text{PS}_n(t, O)$  operating on the  $n$ -parton configuration with starting scale  $t$ . The splitting kernels of the matched shower are the  $D_K$  and the  $D_K$  are their integrated version. The resummation region is limited by  $\mu_Q$ . The various approaches now differ in their choices of  $D_K$  and  $\mu_Q$ , they are detailed in Tab. 1. While MC@NLO retains the parton shower's splitting function and resummation region definition, POWHEG uses the partitioned real emission matrix element as resummation kernel and fills the entire available phase space. Both cases can lead to artifacts when large NLO corrections are present as, upon expansion of the matched parton shower emission, the emission spectrum is enhanced by a factor of  $\bar{B}/B$  in the resummation region. In POWHEG, these effects can be mitigated using the `hfact`-treatment [32]. Its results are shown in Fig. 1.

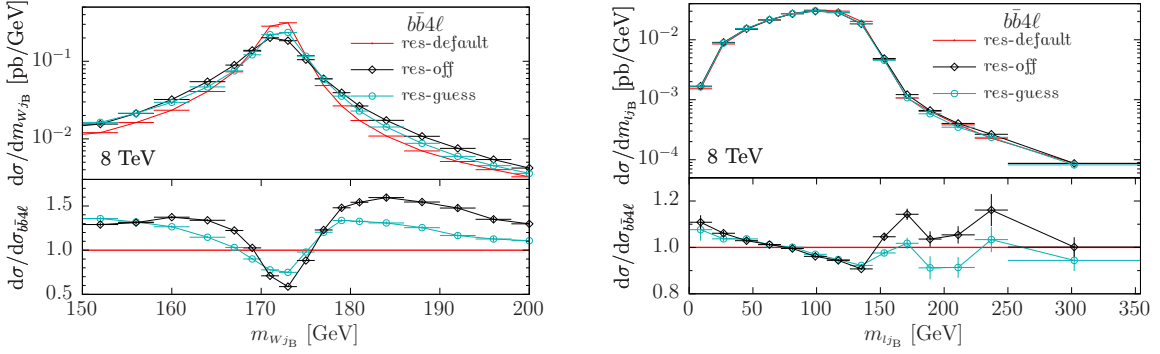
A major recent development for processes with internal resonances is discussed thereafter has been published in [33, 34, 35, 36]. Therein, the inherent subtractions and momentum maps have

	MC@NLO	POWHEG
$D_K$	$B \cdot \tilde{K}_{\text{PS}}$	R
$\mu_Q^2$	$\mu_F^2$	$S_{\text{had}}$

**Table 1:** Choices of resummation kernels and the size of the resummation region in POWHEG- and MC@NLO-type matching algorithms.



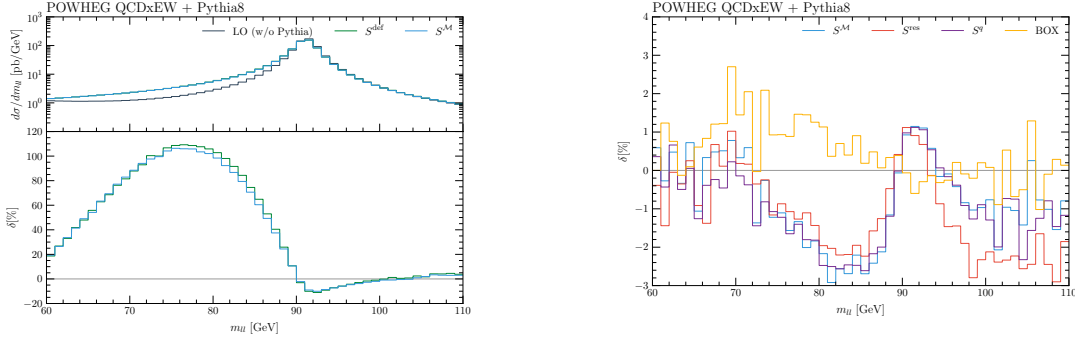
**Figure 1: Left:** Comparison of the  $p_T$  spectrum of the Higgs boson ( $m_h = 400$  GeV) in gluon fusion with MC@NLO and POWHEG, with and without the `hfact` treatment. Figure taken from [32]. **Right:** Parton shower matched results for  $W + n$  jets production using an MC@NLO-like techniques generated by SHERPA. Figure taken from [27].



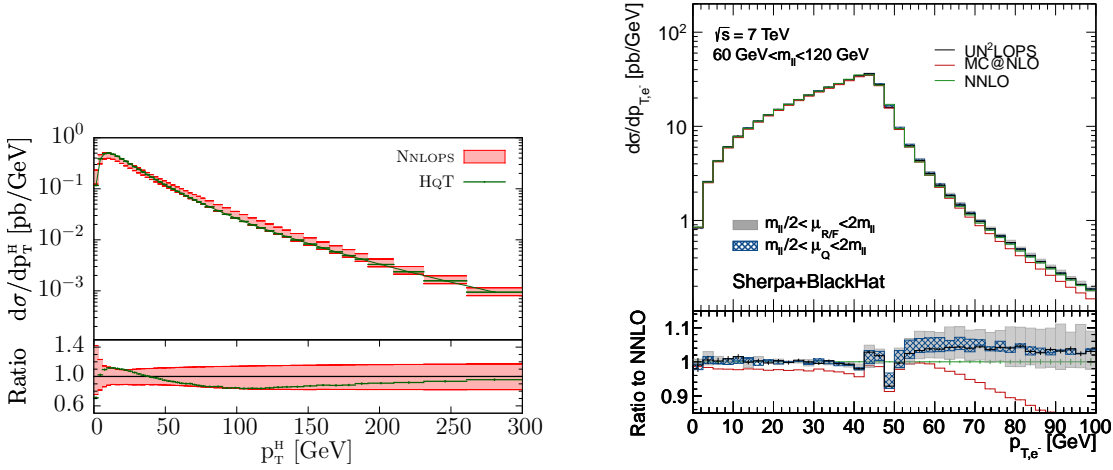
**Figure 2:** Size of the distortion of resonance shape in  $t\bar{t}$  production if not treated properly. Figure taken from [35].

been changed such that the matched results preserves the shape of the internal resonance and the matching to the parton shower does not introduce distortions. Fig. 2 shows that for important observables these distortions can amount to more than 50%, if the matching is unaware of the internal resonance.

Finally, a small class of NLO EW corrections has also been matched to a QED parton shower [37, 38, 39]. Here, an interleaving with the NLO QCD corrections and a resonance aware matching is essential. Selected results are shown in Fig. 3.



**Figure 3:** NLO QCD+EW parton shower matched calculation for  $W$  production. The corrections crucially depend on the resonance awareness of the matching algorithm. Figure taken from [39].



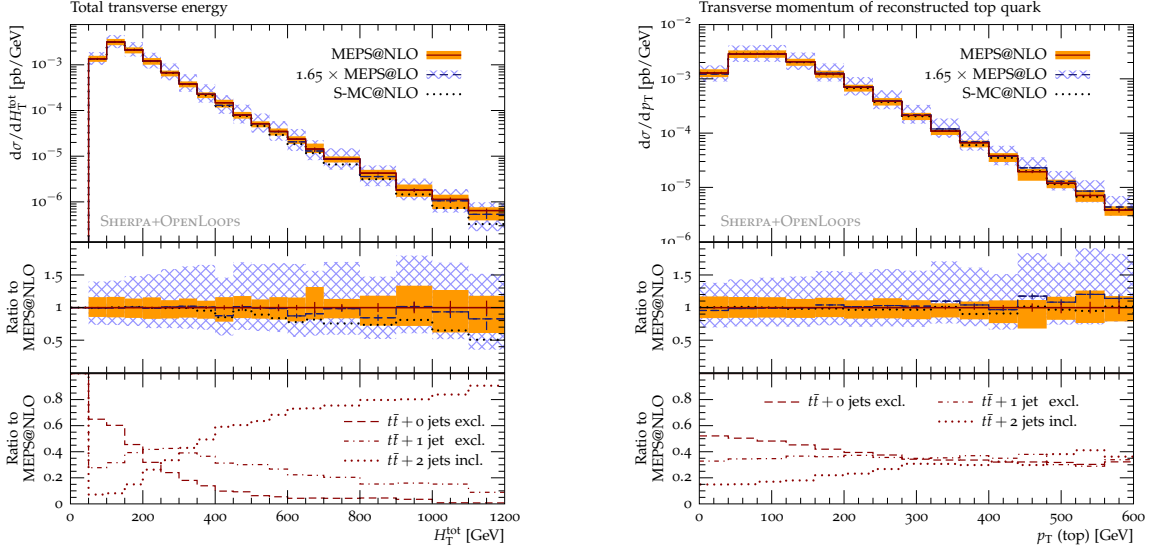
**Figure 4:** NNLOPS matched results for the Higgs boson  $p_T$  in gluon fusion with MiNLO (left) and the  $p_T$  of the electron in Drell-Yan. Figures taken from [15] and [41].

### 3. Matching next-to-next-to-leading order matrix elements to parton showers

Recently, also NNLO computations have been matched to parton showers. The available implementations, however, are currently limited to singlet production where the logarithmic accuracy of the parton showers is sufficient for this task. Three unique formulations exist: MiNLO [40, 15, 16, 17, 18], UN<sup>2</sup>LOPS [41, 42] and GENEVA [19], employing very different matching algorithms. Selected results are shown in Fig. 4.

### 4. Multijet merging at next-to-leading order accuracy

While LOPS, NLOPS and NNLOPS describe observables dominated by topologies of a single jet multiplicity to the given accuracy, a large and important class of observables at LHC receives significant contributions from multiple jet multiplicities. Examples are  $H_T$  and  $p_T$  spectra as well as azimuthal separations. Here, the accuracy of the above described calculations rapidly deteriorates.



**Figure 5:** Total transverse energy and top quark transverse momentum described with MEPS@LO and MEPS@NLO with reduced theoretical uncertainty. Figures taken from [62].

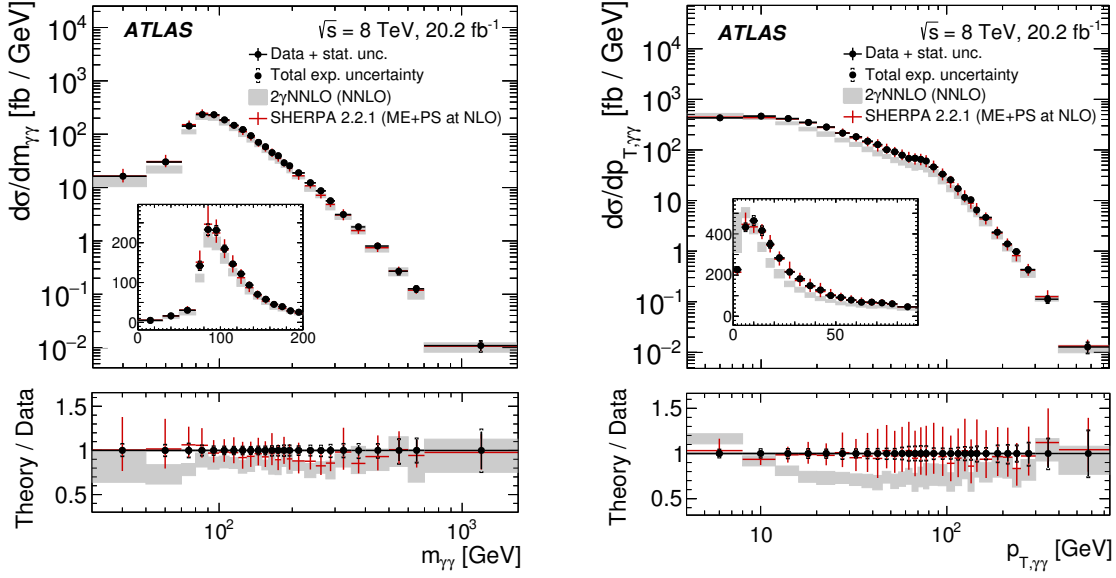
Multijet merging techniques were introduced to consistently combine calculations from various successive jet multiplicities at the highest available precision. At the same time, multijet merged event samples provide the LHC experiments with the largest freedom of projecting these samples onto as many observables as possible without the loss of accuracy. Available multijet merging methods, be it at LO or NLO accuracy, fall into two type of algorithms: CKKW-type and MLM-type.

Both algorithms define a resolution criterion  $Q_{cut}$ , which separates  $n$ -jet production from  $(n+1)$ -jet production. In this way the procedure can be iterated, adding jet multiplicities as long as it computationally feasible. While the MLM-type algorithms merge either jet multiplicities described at LO (MLM [43]) or NLO (FxFx [44]) only, the CKKW-type algorithms can merge successively either LO matrix elements to LO matrix elements (MEPS@LO [45, 46, 47, 48, 49, 50, 51, 52]), NLO matrix elements to LO matrix elements (MENLOPS [53, 54, 55, 56, 36]), or NLO matrix elements to NLO matrix elements (MEPS@NLO [57, 55, 52, 58, 51, 59, 60, 61, 62, 64, 63]). Example results are shown in Figs. 5 and 6.

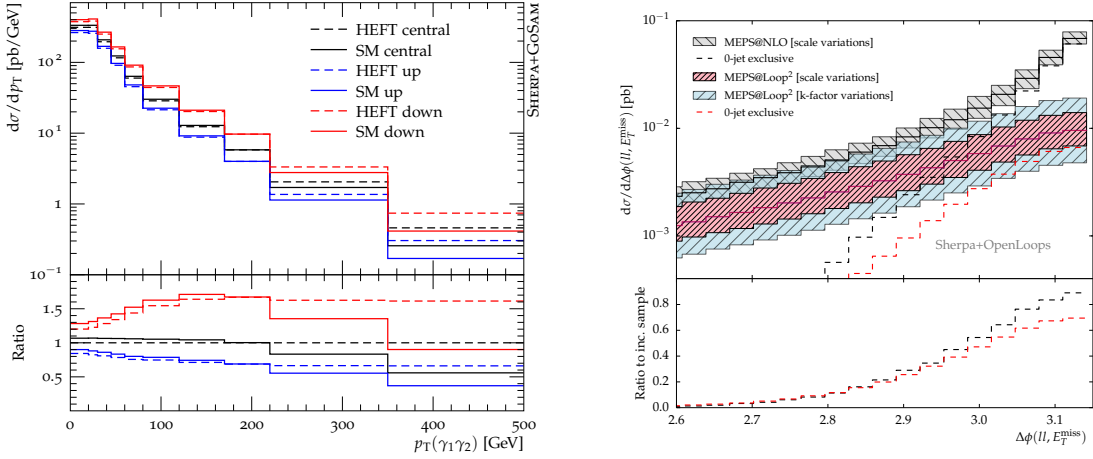
Further, loop-induced process form an important class of processes measured at the LHC. Their theoretical description is complicated that the LO calculation already involves a loop computation. Nonetheless, additional jet activity is also prevalent in this class and a multijet merged calculation is desirable. Therefore, two different ansatzes have been persued. In cases where an effective theory which integrates out the loop exists, the calculation is performed in that effective theory (which has only LO complexity) and then reweighted to include the corrections due to the exact loop dynamics [66, 67]. In this way, a NLO merging in the effective theory including Born level loop corrections are feasible.

On the other hand, at LO accuracy, direct loop-induced calculations can be merged using a technique dubbed MEPS@LOOP<sup>2</sup> [69, 72]. Fig. 7 shows example results.

Finally, approximate NLO EW corrections were incorporated in NLO multijet merging meth-



**Figure 6:** Diphoton invariant mass and pair transverse momentum in photon pair production with MEPS@NLO compared to ATLAS data. Figures taken from [65].



**Figure 7:** Left: Top mass corrections in an NLO multijet merged calculation of the diphoton  $p_T$  spectrum in Higgs production in gluon fusion. Right: NLO  $q\bar{q}$ - and LO loop-induced  $g$ -induced contribution to  $Zh$  associated production. Figure taken from [72].

ods in [33].

## 5. Conclusions

All processes of relevance to LHC analyses are available at least at NLO accuracy. Key processes like  $W$ ,  $Z$  and  $h$  production are even known to NNLO accuracy. To also merge non-colour-singlet process at this accuracy to parton showers, the logarithmic accuracy of the latter need to be improved first [73, 74, 75, 76].

On another, almost orthogonal direction, the recent progress in the automation of NLO EW corrections [77, 78, 33, 79, 80, 81, 82, 83, 36] should be universally matched to parton showers and included in the standard event simulation. Current, approximate methods [33] are only targeted at large- $p_T$  physics. At the same time, developments in including EW effects into parton showers [84, 85, 86, 87, 88] are not only relevant for future colliders at energies up to 100 TeV, but also offer a way of extending multijet merging to include jet emissions through hadronically decaying vector boson production as well.

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