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# Three dimensional higher spin holography

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We study the three point coupling for two scalars and particle of arbitrary spin in the $AdS_3/CFT_2$ holography from the side of holographic reconstruction approach.

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## 1. Introduction and motivation for higher spin theory

Higher spin (HS) theory is an interesting theory that can thank its recent popularity to its possibility of answering important questions in string theory. It was introduced by Vasilliev [1] and one can consider it as a missing link in the evolution from the field theories of lower spins to superstring theories. It is an important symmetry since it does not result from the symmetry breaking. Due to the fact there is no energy scale that describes it, it can be viewed as a toy model for the fundamental, beyond Planck scale theory [2].

It has encountered a number of "no-go" theorems through history because one cannot find a consistent HS theory on the flat space. The solution came through consideration of the AdS space. In the AdS background there is a solution to an HS gravitational interactions up to cubic order which appear in the action, while if one considers equations of motion, the solution can be found up to all orders.

Using HS theory, one can also learn more about the AdS/CFT duality. Klebanov and Polyakov have conjectured that the HS theory on $AdS_4$ is dual to a 2+1 dimensional O(N) vector model in the large N limit.Analogously, in one dimension lower, $AdS_3/CFT_2$ correspondence has been studied as a correspondence between the Vasiliev theory coupled to pair of complex scalar fields, and t'Hooft limit of the $W_n$ minimal model CFT. Where the coset representation of the latter is

$$\frac{SU(N)_k \oplus SU(N)_1}{SU(N)_{k+1}} \tag{1.1}$$

while the 't Hooft limit is

$$N,k \to \infty, \qquad \lambda = \frac{N}{k+N} \tag{1.2}$$

which is fixed. Duality can be further verified by studying the observables such as partition function, its one loop correction [3], correlation function, global symmetries on the boundary and in the bulk [4], and others. In this talk we focus on the three point correlation function.

Scalar - scalar - higher spin field (00s) correlation function has been computed for the O(N) vector model using the standard field theory methods and compared to the computation using the holographic reconstruction in [5]. The holographic reconstruction method based on computation of the Witten diagrams is general and one can use it to compute the coupling not only for the O(N) vector model, but for the model for which one knows to determine the boundary currents. This means that one can consider analogous procedure for more general $\lambda$. (00s) field three point function for general $\lambda$ has been considered in [6]. Here, we describe the holographic reconstruction method and how to determine the coupling of the (00s) three point function taking into account the result for the (00s) correlation function in the AdS background of Vasiliev's HS theory for any parameter $\lambda$ [6]. The same coupling ought to be obtained from the linearised Vasiliev's equations of motion. In this work, our motivation, beside describing how to obtain (00s) coupling, is to consider whether the holographic reconstruction method can be more convenient for computation of coupling for the higher point functions than using linearised Vasiliev's equations.Earlier, the 00s cubic vertices have been analysed in [7, 8, 9, 10]. The higher point functions in flat space have been considered in [11].

## 2. Holographic reconstruction

Without restriction to particular scaling dimension $\Delta$ the three point function in ambient space formalism can be obtained using standard methods from CFT side, while from the bulk side, one can determine it using Witten diagrams. If one builds a general conformal structure from the three different points, the structure can parametrised using six objects

$$Y_i = \frac{z_i \cdot x_{i,i+1}}{x^2_{i,i+1}} - \frac{z_i \cdot x_{i,i+2}}{x^2_{i,i+2}} \qquad H_i = \frac{1}{x^2_{i+1,i+2}} \left( z_{i+1} \cdot z_{i+2} + \frac{2 z_{i+1} \cdot x_{i+1,i+2} z_{i+2} \cdot x_{i+2,i+1}}{x^2_{i+1,i+2}} \right), \tag{2.1}$$

for $i = 1,2,3$. Conformal symmetry, scale transformations and spin structure determine the general three-point functions obtained from the CFT side

$$\langle J_{s_1}(x_1|z_1) J_{s_2}(x_2|z_2) J_{s_3}(x_3|z_3) \rangle = \sum_{n_i} C^{n_1,n_2,n_3}_{s_1,s_2,s_3} \frac{Y_1^{s_1-n_2-n_3} Y_2^{s_2-n_3-n_1} Y_3^{s_3-n_1-n_2} H_1^{n_1} H_2^{n_2} H_3^{n_3}}{(x^2_{12})^{\frac{\tau_1+\tau_2-\tau_3}{2}} (x^2_{23})^{\frac{\tau_2+\tau_3-\tau_1}{2}} (x^2_{31})^{\frac{\tau_3+\tau_1-\tau_2}{2}}} \tag{2.2}$$

for $\tau_i = \Delta_i - s_i$ twists and coefficients $C^{n_1,n_2,n_3}_{s_1,s_2,s_3}$ which are dependent on the theory. This has for the specific case of the O(N) vector model been determined in [5]. Here, with $J$ we have denoted currents of higher spin $s_i$ for $i = 1,2,3$ while the coordinates $x_i$ are contracted with the auxiliary vectors $z_i$ according to ambient space formalism. $x_{ij}$ denotes the difference of coordinates $x_{ij} = x_i - x_j$. Determining and normalising the currents and choosing the $00s$ case, for $O_0$ scalars and $J_S$ higer spin current, one obtains $\langle O_0 O_0 J_s \rangle$.

To compare this with the bulk side, one needs to consider the on-shell cubic interaction in the ambient framework. The most general cubic vertex in the AdS background in $d+1$ dimensions is [12]

$$\mathscr{V}_{s_1,s_2,s_3} = \sum_{n_i} g^{n_1 n_2 n_3}_{s_1,s_2,s_3} I^{n_1 n_2 n_3}_{s_1 s_2 s_3}(\Phi_i) \tag{2.3}$$

for

$$I^{n_1 n_2 n_3}_{s_1 s_2 s_3}(\phi_i) = \mathscr{Y}_1^{s_1-n_2-n_3} \mathscr{Y}_2^{s_2-n_3-n_1} \mathscr{Y}_3^{s_3-n_1-n_2} \mathscr{H}_1^{n_1} \mathscr{H}_2^{n_2} \mathscr{H}_3^{n_3} \Phi_1(X_1 U_1) \Phi_2(X_2 U_2) \Phi_3(X_3 U_3)|_{X_i = X} \tag{2.4}$$

and

$$\mathscr{Y}_\rangle = \partial_{U_i} \cdot \partial_{X_{i+1}}, \qquad \mathscr{H}_i = \partial_{U_{i+1}} \cdot \partial_{U_{i+2}}. \tag{2.5}$$

These objects, are the bulk quantities that correspond to the six conformal boundary structures (2.1).

Expression (2.3) describes the sum of the couplings $g^{n_1 n_2 n_3}_{s_1,s_2,s_3}$ multiplied with corresponding amplitudes $I^{n_1 n_2 n_3}_{s_1 s_2 s_3}$.

## 3. Determination of coupling

The three point function for the arbitrary $\lambda$ computed from the CFT and AdS side [6] is

$$\langle O_\pm(z_1)\overline{O}_\pm(z_2)J^{(s)}(z_3)\rangle = \langle \mathscr{O}\mathscr{O}J^{(s)}\rangle_{\text{gauge symmetries}} =$$
$$= \frac{(-1)^{s-1}B_\phi}{2\pi^2|z_{12}|^{2(1\pm\lambda)}}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(\pm\lambda)\left(\frac{z_{12}}{z_{13}z_{23}}\right)^2} \tag{3.1}$$
$$= \frac{(-1)^{s-1}}{2\pi}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(1\pm\lambda)}\left(\frac{z_{12}}{z_{13}z_{23}}\right)^s\langle O_\pm(z_1)\overline{O}_\pm(z_2)\rangle. \tag{3.2}$$

It is comparable with the result from the holographic reconstruction when we take into account the normalisation of the currents

$$\langle J^{(s)}(z)J^{(s)}(0)\rangle = \langle JJ\rangle_{\text{gauge symmetries}} = \frac{3}{2^{2s-1}\pi^{5/2}}\frac{\sin(\pi\lambda)}{\lambda(1-\lambda^2)}\frac{\Gamma(s)\Gamma(s-\lambda)\Gamma(s+\lambda)}{\Gamma\left(s-\frac{1}{2}\right)}\frac{1}{z^{2s}}. \tag{3.3}$$

To determine the coupling, one has to take the ratio with the three point function obtained by holographic reconstruction. This result is expected to agree with the coupling studied using the linearised Vasiliev's equations of motion in [13]. There, the coupling has been determined from the linearised equations of motion for the scalar field in the higher spin background field.

## 4. Outline

We have described the procedure for obtaining the coupling of the (00s) three point function for the higher spin theory in $AdS_3/CFT_2$ holography. The talk is based on current work. Analogous computations could be done for the higher point functions from "holographic reconstruction approach" and from "Vasiliev's equations of motion approach". Computations from one approach tend to be more convenient, therefore they are interesting to be determined for the lower correlation functions, so that one could compute them more efficiently for the higher correlation functions in the future.

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