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Large Field Inflation/Quintessence and the Refined Swampland Distance Conjecture

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Attempts to construct string derived effective field theory models realizing large field inflation are plagued by control issues. Targeted at a broader audience, in this article we review recent progress in isolating the underlying conceptual reasons for this failure. Special emphasis is given to models of axion monodromy inflation and their relation to the Swampland Distance Conjecture. This discriminates effective actions that admit a UV completion, the landscape, from those that do not, the swampland. Since they are conceptually very similar, we also comment on implied challenges for axionic quintessence models.

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1. Introduction

It is widely accepted that string theory features a landscape of consistent solutions to its equations of motion. Historically, such a landscape has emerged in two disguises. First, by studying and partially classifying string compactifications to four dimensions with N=1 supersymmetry. Examples are covariant lattice constructions [1], Gepner type constructions [2, 3], Calabi-Yau compactifications with line-bundles [4, 5] or intersecting D6-brane models [6, 7]. It became clear that instead of a small set of such four-dimensional solutions, there exist a whole plethora of them. For instance, in case of the covariant lattice construction estimates of the order $O(10^{1500})$ were given.

Each of these models comes with a number of still massless scalar and pseudo-scalar fields, so that they do not correspond to isolated minima, but live in a higher dimensional moduli space. The pseudo-scalar fields are also called axions and being CP-odd and obeying a perturbative shift symmetry, they couple very differently than the scalars in the effective theory. The latter are also called saxions and they are dangerous for the low-energy phenomenology as they can give rise to (yet undetected) fifth forces and spoil the history of the early universe. Indeed, if the mass of a scalar is below $\sim 30\,\text{TeV}$, its gravitational decay will lead to a reheating temperature below $T_{\text{BBN}} \sim 1\,\text{MeV}$, so that the moduli would decay after nucleosynthesis. This spoils the thermal history of the early universe and is called the cosmological moduli problem. On the contrary, some of the axions can stay very light with masses in the sub-meV regime and contribute to dark matter.

Therefore, eventually the scalar moduli should better be absent or should be stabilized by some mechanism, respectively. There are essentially two working proposals how this could happen. Either non-perturbative effects generate an exponentially suppressed potential for the moduli or they receive already a tree-level potential by turning on more general background fields, so called fluxes. Concerning the latter mechanism, it was shown that type IIB orientifolds compactified on Calabi-Yau threefolds show a discretuum of Minkowski minima [8] in the complex structure and the axio-dilaton moduli. Here, except for the Kähler moduli, generally all minima are isolated and simple estimates gave the (in-)famous number of 10^{500} vacua. This is the second way a landscape arose in the history of string theory and it gave rise to the idea of a multiverse that possibly ameliorates some fine tuning problems, like the problem of the smallness of the cosmological constant in our universe.

In view of this exponentially large number it was questioned whether string theory is able to make any concrete predictions at all, as it appears that almost everything goes. However, that this is not the case is clear at least for each type of concrete string construction. For instance, there are strong tadpole constraints to hold that give an upper bound on the rank for the gauge group. Moreover, the general experience is that whenever one hunts for a concrete model with certain properties, the stringy constraints are severe and finding a model it is not trivial at all. For instance, isolating a supersymmetric Madrid-type intersecting D6-brane model with three generations in the Standard Model gauge group and massless hypercharge was challenging and only one in a billion models satisfied these rough discrete constraints [6]. After all, effective four-dimensional models that can be realized in string theory are guaranteed to have a consistent UV completion.

In the last years the opposite question was under scrutiny, namely whether string theory can in principle be falsified. For that purpose, one takes the concrete model building obstacles seriously and tries to formulate well motivated conjectures that distinguish effective field theories with a UV completion (the landscape) from those which do not admit it (the swampland). This logic can in principle lead to statements like, ”the physical property A is in the swampland of effective field theories derived from string theory”. In this case, if property A is experimentally measured in our universe, then it cannot be described by an effective field theory arising from string theory. Logically is not yet clear whether string theory is really falsified or our logic about effective field theories does not apply. In any case, by better understanding properties that lie in the swampland of effective field theories derived from string theory, we learn a lot on the underlying conceptual principles of quantum gravity and string theory, respectively.

One potential candidate for such an enterprise arises in cosmology, namely large field inflation. Interestingly triggered by the, at first misinterpreted, BICEP2 measurement of the cosmic microwave background (CMB), during the last four years the community made a big effort in realizing large field inflation in string theory. Such models feature a tensor-to-scalar ratio in a regime that can be measured with current or planed experiments. As we will briefly review, there exist various proposals for getting a controllable model of string inflation. They all exploit the shift symmetry of axions and can be divided into two classes. In the first class, the shift symmetry of the axion is broken to a discrete one by non-perturbative effects. In the second class, a branch structure arises, where each branch features a polynomial potential for the axion and the shift symmetry is still present but shifts also the branch for the potential. In other words, by a choice of branch the shift symmetry is spontaneously broken and the axion is not periodic any more. These models go under the name of axion monodromy inflation [9, 10] (see [11, 12, 13] for field theory analogues). The question arises is whether the large (trans-Planckian) field regime of such models is under control in the string derived effective field theory.

In this talk, for a broader audience I will review recent attempts to clarify this issue. Methodologically, we will employ a combination of concrete string model building attempts and abstracted swampland conjectures on UV complete quantum gravity derived effective field theories. In section 2, a brief reminder of some basic notions of large single field inflation and also of quintessence models is presented. In section 3, I review string theory proposals for the realization of axion inflation via non-perturbative effects and how they are bound to fail, if the Weak Gravity Conjecture (WGC) holds. A second swampland conjecture on the validity of quantum gravity derived effective field theories is introduces in section 4. This is called the Refined Swampland Distance Conjecture (RSDC) and quantitative evidence for it is summarized, that arose from a detailed study of distances in concrete Calabi-Yau moduli spaces. In section 5, models of axion monodromy inflation are introduced and after providing a concrete example it is argued that such models are challenged by an axionic extension of the RSDC. Section 6 contains a few comments on how also models of axionic quintessence face the same challenges. In section 7 we summarize a few more swampland conjectures that have recently been proposed in the literature, the most dramatic one stating that meta-stable de Sitter minima might also be in the swampland.

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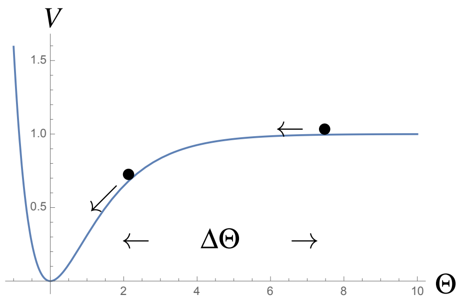
2. Preliminaries

Not only provides it a natural theoretical explanation of the homogeneity and flatness properties of the observable universe, but there is now also mounting detailed experimental evidence for the existence of an inflationary epoch in the early universe. In particular, the temperature fluctuations in the CMB are predicted by such a scenario in an impressive manner. The combined PLANCK 2015 and BICEP2/Keck Array results [14] confirm that these fluctuations are almost scale invariant and Gaussian. Moreover, these measurements provide an upper bound to the tensor-to-scalar ratio. For the corresponding parameters they find

- tensor-to-scalar ratio: $r < 0.07$
- spectral index: $n_s = 0.9667 \pm 0.004$ and its running $\alpha_s = -0.002 \pm 0.013$
- amplitude of the scalar power spectrum $\mathcal{P} = (2.142 \pm 0.049) \cdot 10^{-9}$.

These findings are consistent with a simple model of inflation, namely that of a single slowly rolling scalar field as shown in figure 2.



Here $\Delta\Theta$ denotes the distance in field space along which the inflaton rolls, i.e. where the slow-roll conditions

$$\varepsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1\,, \qquad \eta = M_{\rm pl}^2\left(\frac{V''}{V}\right) \ll 1 \tag{2.1}$$

are satisfied. The measured parameter can be expressed in terms of the slow-roll parameters as

$$n_s = 1 + 2\eta - 6\varepsilon\,, \qquad r = 16\varepsilon\,, \qquad \mathcal{P} = \frac{H_{\rm inf}^2}{8\pi^2\varepsilon M_{\rm pl}^2} \tag{2.2}$$

where $H_{\rm inf}$ denotes the Hubble constant during inflation. Due to the Lyth bound [15]

$$\frac{\Delta\Theta}{M_{\rm pl}} > O(1)\sqrt{\frac{r}{0.01}} \tag{2.3}$$

the distance in field space traversed during inflation is directly related to the tensor-to-scalar ratio. Thus, if r is detected by the current experiments then theoretically a model of large field inflation will be favored, i.e. $\Delta\Theta > M_{\rm pl}$. Moreover, in this case the Hubble constant during inflation comes out as $H_{\rm inf} \sim 10^{14}\,{\rm GeV}$. Via $V_{\rm inf} = 3M_{\rm pl}^2 H_{\rm inf}^2$ one can infer that the inflationary mass scale is of the order of the GUT scale

$$M_{\rm inf} = (V_{\rm inf})^{\frac{1}{4}} \sim \left(\frac{r}{0.1}\right)^{\frac{1}{4}} \times 1.8 \cdot 10^{16}\,{\rm GeV}\,. \tag{2.4}$$

The mass of the inflaton is given by $M_\Theta^2 = 3\eta M_{\rm pl}^2 H_{\rm inf}^2$ and comes out as $M_\Theta \sim 10^{13}\,{\rm GeV}$. Thus, all mass scales are close to the GUT scale and only one order of magnitude apart.

Recalling that inflation describes a period of a slowly rolling scalar field in the early universe giving rise to an effective positive cosmological constant, it is tempting to speculate that also the very tiny positive cosmological constant in our present universe can be originating via a similar mechanism. This proposal goes under the name of quintessence [16] and conceptually it is very similar to inflation, the main difference being that the mass scales of quintessence are much tinier. The Hubble parameter today is of the order $H \sim 10^{-33}$ eV and the measured cosmological constant is

$$V_{\rm quint} = 10^{-120} M_{\rm pl}^4 \sim \left(10^{-3}\,{\rm eV}\right)^4 . \tag{2.5}$$

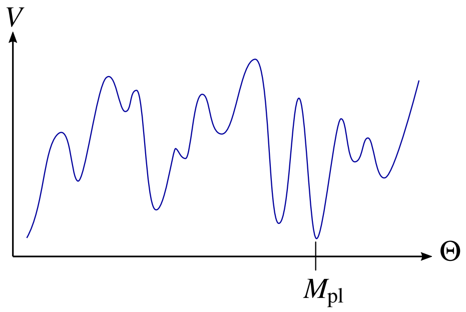
Note that quite non-trivial, these two scales are consistent with the slow-roll expectation

$$V_{\rm quint} \sim M_{\rm pl}^2 H^2\,. \tag{2.6}$$

The mass of the quintessence field is then of the order of the Hubble scale, thus very tiny with respect to e.g. the heavy moduli masses that due to the cosmological moduli problem must be larger than 30 TeV. The equation of state is $p = w\rho$ where the parameter w has been measured by the Planck satellite $w = -1.006 \pm 0.045$. In the slow-roll regime, this is related to the slow-roll parameter ε as

$$w \sim \frac{\varepsilon/3 - 1}{\varepsilon/3 + 1}\,. \tag{2.7}$$

The main issue with such slow-rolling models of inflation (or quintessence) is that they are very sensitive to corrections to the scalar potential of the inflaton. In particular, one expects that the effective field theory model has to be embedded into a UV complete theory of quantum gravity. In such a theory, like string theory, one expects corrections by Planck-suppressed operators of the form $(\Theta/M_{\rm pl})^n$. However, for large field inflation, such corrections are substantial and will change the potential in an uncontrolled way, as shown in figure 2.



For models of quintessence, one has to control the Planck suppressed corrections to the mass of the scalar up to order $n = 6$ in

$$\Delta M^2 \sim M_{\rm susy}^2 \left(\frac{M_{\rm susy}}{M_{\rm pl}}\right)^n \tag{2.8}$$

for a supersymmetry breaking scale $M_{\rm susy} = 1\,{\rm TeV}$. Higher scales of supersymmetry breaking imply the control of even higher orders.

In order to make progress, one really has to discuss these issues in an understood UV complete theory of quantum gravity. In particular, one has to study whether one can identify mechanisms to control Planck suppressed operators. As usual such mechanism will be related to symmetries that will be broken in a controlled way. Natural candidates are the shift symmetry of axions that can be broken by non-perturbative corrections. Let us discuss how this can in principle be realized in the context of string theory compactifications.

3. Natural inflation and the weak gravity conjecture

As mentioned in the introduction, to protect the potential from receiving Planck suppressed corrections, one can employ the shift symmetry of axions. Such four-dimensional axions arise naturally in string compactifications by either directly resulting from ten-dimensional axion-like fields, like the NS-NS and R-R p-forms. In addition, at special points in the moduli space of e.g. the complex structure moduli space, accidental shift symmetries of some of the scalar fields can arise. For instance, around the large complex structure point of a Calabi-Yau (CY) moduli space, the Kähler potential takes the form

$$K = -\log\Big(i\kappa^{ijk}(U_i - \overline{U}_i)(U_j - \overline{U}_j)(U_k - \overline{U}_k) + \ldots\Big) \tag{3.1}$$

where κ^{ijk} denote the triple intersection numbers of the mirror dual CY threefold. This Kähler potential is shift symmetric in the real part of the complex structure moduli U_i. This shift symmetry is broken by higher order terms, that correspond to world-sheet instanton corrections on the mirror dual side.

The simplest class of models assumes that, after stabilizing all heavy moduli except the lightest axion θ, the continuous shift symmetry of the latter is broken to a discrete one by a non-perturbative correction to the potential. Thus, the effective Lagrangian takes the form

$$\mathscr{L} = f^2(\partial\theta)^2 + \Lambda(1-\cos\theta) \tag{3.2}$$

where f is the axion decay constant and $\Lambda \sim \exp(-S_{\rm inst})$ the size of the (leading order) non-perturbative instanton correction. Here we have chosen natural units where $M_{\rm pl} = 1$. Note that one has assumed that $S_{\rm inst} > 1$ so that higher order instanton corrections can be neglected. Moreover, from the string theory point of view, we have implicitly assumed that moduli stabilization can be achieved such that at the end one gets a Minkowski minimum at $\theta = 0$. Whether this is possible in string theory is a not yet completely settled issue, as in most concrete cases AdS-minima seem to be preferred and one needs to rely on the existence of some sort of uplift mechanism. More on this will be mentioned in section 5.

However, this is not the point here, as already the Lagrangian (3.2) has an inherent problem in the large field regime. To see this let us first transform it to a canonically normalized kinetic term. With $\Theta = f\theta$ we get

$$\mathscr{L} = (\partial\Theta)^2 + \Lambda\Big(1 - \cos\Big(\frac{\Theta}{f}\Big)\Big)\,. \tag{3.3}$$

In the regime $\Theta/f \ll 1$ this potential reduces to the one of chaotic inflation $V \sim \Lambda\Theta^2/f^2$, which gives rise to large field inflation with $\Delta\Theta \sim 10$ and $r \sim 0.2$. Therefore also f has to be larger than one. This field theory model is called natural inflation [17]. Now in string theory, the (pseudo-scalar) axion is combined with a scalar ϕ (also called saxion) to a complex field $T = \theta + i\phi$ and the parameters in (3.2) are given by

$$f = \frac{1}{\phi}\,, \qquad S_{\rm inst} = \phi \tag{3.4}$$

up to numerical factors of order one[1]. Thus, for this string derived effective action, one has the relation

$$f\, S_{\rm inst} \sim 1\,. \tag{3.5}$$

As a consequence, the large field regime $\Theta > 1$ (presuming $f > 1$) requires that the instanton action is smaller than one, spoiling the validity of the instanton expansion. Therefore, the large field regime lies outside the regime of validity of the pre-assumed effective action. The same logic also applies if one wants to interprete the axion Θ as a quintessence field.

Of course this is a just a simple example, but it has been conjectured that the reason behind this failure is a property of any consistent theory of quantum gravity. Namely, as argued in [18, 19, 20] the relation (3.5) is just the generalization of the so-called Weak Gravity Conjecture (WGC) [21], to more general p-forms (here 0-form). The WGC is formulated for a Maxwell theory with gauge coupling g coupled to gravity and it says that gravity must be the weakest force, i.e. for a U(1) gauge theory there must exist a particle of mass m and charge q such that $m \le gqM_{\rm pl}$. Arguments in favor of this conjecture have been given in [21], some of them being:

- The WGC guarantees that any non-BPS subextremal charged black hole with $M \ge Q$ can decay.
- It is satisfied for perturbative states in toroidal heterotic string theory compactifications.
- It has a magnetic version $\Lambda < gM_{\rm pl}$ relating the cut-off of the theory. This can be derived by requiring that a magnetic monopole is not a black-hole. It implies that the $g \to 0$ limit is dramatic so that one cannot simply generate a continuous global symmetry from a gauge symmetry[2].

[1] The first relation can be seen as a consequence of a Kähler potential of the type (3.1). Indeed the Kähler potential $K = -3\log(-i(T-\overline{T}))$ implies for the metric $G_{T\overline{T}} = \partial_T\partial_{\overline{T}}K = \frac{3}{4\phi^2}$.

[2] It is expected that quantum gravity theories do not admit continuous global symmetries.

Via T-duality it has been argued that there should exist such a relation for any p-form gauge field [18, 19, 20]. For a 0-form with

$$m \to S_{\rm inst} \qquad gq \to 1/f \tag{3.6}$$

one precisely arrives at (3.5)

$$f S_{\rm inst} \le 1 \tag{3.7}$$

where we have set $M_{\rm pl} = 1$. It is beyond the scope of this article to discuss this in detail, but let us mention that there exist proposals to generalize the WGC to multiple $U(1)$ gauge factors[3] [22] leading to strong constraints also for inflationary models invoking multiple axion fields [23, 24].

The lesson is that guided by string theory experience and arguments from black hole physics, one has arrived at a general conjecture discriminating in simple terms effective field theories that admit a consistent UV completion (the landscape) from those that do not (the swampland). In the here derived context one can say that the WGC conjecture forbids consistent effective field theory models of natural inflation.

More generally, whenever one claims to have found a working 4D effective field model of large (single) field inflation derived from string theory, one really has to ensure that in particular the pre-assumed hierarchy of mass scales

$$M_{\rm pl} > M_{\rm s} > M_{\rm KK} > M_{\rm mod} > H_{\rm inf} > M_{\Theta} \tag{3.8}$$

is satisfied. Clearly, for working in an effective 4D field theory for discussing moduli stabilization, all final masses of the former moduli have to be lighter than the Kaluza-Klein and the string scale. Moreover, for having a model of single field inflation the masses of all heavy moduli (excluding potentially very light sub-meV axions) have to be larger than the Hubble scale during inflation. It is important to emphasize that the realization of this hierarchy in a final minimum is not yet sufficient, it really has to hold throughout the entire slow rolling phase.

Before we discuss the second approach of realizing axion inflation, namely axion monodromy, we spend some time on discussing a second conjecture for discriminating the landscape from the swampland.

4. The Swampland Distance Conjecture

In 2006 Ooguri and Vafa had already proposed more conjectures of this type [26]. The most quantitative one has meanwhile been termed the Swampland Distance Conjecture (SDC). The inspiration for it comes from a simple dimensional reduction of Einstein gravity on a circle. Choosing the ansatz for a 5D metric

$$G_{MN} dX^M dX^N = g_{\mu\nu}(x) dx^\mu dx^\nu + r(x)^2 dy^2 \tag{4.1}$$

[3] A generalization to include also scalar fields was proposed in [25].

leads to an effective action for the modulus field $r(x)$

$$S = \int d^4x\sqrt{g}\, \frac{1}{(\lambda r)^2}\, \partial_\mu r\, \partial^\mu r \tag{4.2}$$

where λ denotes a numerical constant. Note that this is the same form of the metric on moduli space as already encountered in (3.4), where however here it is the kinetic term for a saxionic field $r(x)$ and not an axion. It followed that the canonically normalized field $\rho(x)$ is

$$\rho = \lambda^{-1} \log r \tag{4.3}$$

which scales logarithmically with r. The mass of Kaluza-Klein modes along the circle are therefore given as

$$M_{\rm KK} \sim \frac{n}{r} \sim n\, e^{-\lambda\rho} \tag{4.4}$$

Thus, for $\rho > \rho_c = \lambda^{-1}$ infinitely many states become exponentially light indicating a breakdown of the effective action that has only kept the massless four-dimensional metric and the radial modulus[4].

Motivated by this and similar observations, Ooguri/Vafa proposed that this behavior is a general property of any effective theory derived from string theory (quantum gravity).

Swampland Distance Conjecture:

For any point p_0 in the continuous scalar moduli space of a consistent quantum gravity theory, there exist other points p at arbitrarily large distance. As the distance $d(p_0, p)$ diverges, an infinite tower of states exponentially light in the distance appears, i.e. the mass scale of the tower varies as

$$m \sim m_0\, e^{-\lambda d(p_0,p)}\,. \tag{4.5}$$

Therefore, beyond the critical distance $d_c = \lambda^{-1}$ the effective field theory breaks down.

Let us make a few comments:

- The distance is measured by the metric on the moduli space and is given by the length of the shortest geodesic.
- The conjecture is about the flat moduli space of a compactification, in which axionic fields are compact. Thus, the directions in which arbitrarily large distances appear involve saxionic directions.
- The original SDC conjecture makes no reference to the value of the critical distance $d_c = \lambda^{-1}$ beyond which the effective field theory breaks down.
- Note that the SDC describes a property of models in the landscape!

If one uses the Kähler potential (3.1), one would find $\lambda = \sqrt{2/3}M_{\rm pl}$ where we have reintroduced the Planck scale. This and similar results for simple models of axion monodromy inflation [29, 30] led Kläwer/Palti [31] to go one step further and conjecture that the critical distance is always of the order of the natural built in mass scale, namely $\lambda = \alpha M_{\rm pl}$, where α is a number of order one. Adding this extra piece to the SDC was termed the Refined Swampland Distance Conjecture (RSDC). Its relation to models of axion monodromy inflation will be discussed in section 5.

[4] In [27, 28] it was shown that integrating out an infinite tower of exponentially light states is closely related to the appearance of infinite distances in field space.

Testing the RSDC on Calabi-Yau moduli spaces

In [32] this RSDC conjecture was challenged by a detailed computation of distances in the Kähler moduli space of Calabi-Yau compactifications (see also [33, 34]). These Kähler moduli spaces do not only contain (geometric) regions/phases, where points at infinite distance exist, but also non-geometric phases, like the Landau-Ginzburg (LG) phase. These often do only have a finite radius and therefore do not admit points at infinite distance. To reach the latter one first has to cross the non-geometric phase and move to another geometric phase where such a point do exist.

If the distances one can travel along geodesics in such non-geometric phases were already larger than $M_{\rm pl}$, it would directly falsify the RSDC [5]. Let us explain this potential challenge for the concrete example of the Kähler moduli space of the quintic, which looks like as shown in figure 1. There is only a single complexified Kähler modulus $t = B + iJ$. As indicated, there are three

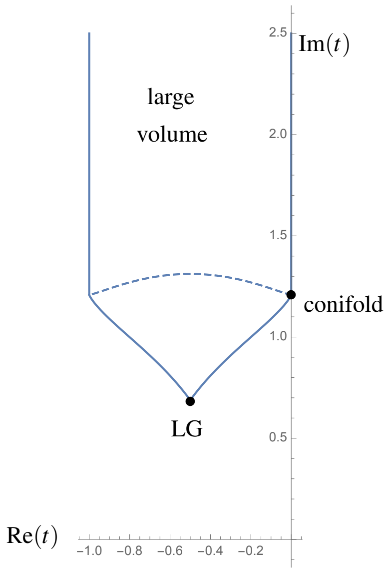


Figure 1: Sketch of the Kähler moduli space of the quintic.

distinguished points: the large volume point, the conifold and the Landau-Ginzburg (LG) point. The LG or Gepner point is the one of minimal radius. To cover the whole moduli space, one needs at least two charts, whose radii of convergence are shown by the dashed arc. Now one can ask the question whether the RSDC still holds for points p_0 in the small volume regime. Following a geodesic from the LG point to the large volume regime, one expects that the proper field distance depends on $\mathrm{Im}\, t$ like shown in figure 2.

As long as one stays in the small volume regime the proper field distance scales polynomially with $\mathrm{Im}\, t$ and at some point $(\mathrm{Im}\, t_0, \Theta_0)$ the logarithmic scaling becomes dominant. As a consequence, we define the critical field distance as the sum

$$\Theta_c = \Theta_0 + \lambda^{-1}, \tag{4.6}$$

[5]Similar in spirit, in [35] axion decay constants have been evaluated in the geometric phases for $h^{11} \in \{1,2\}$ CY threefolds. Is was found that they are bounded from above by an order one parameter times the Planck-scale.

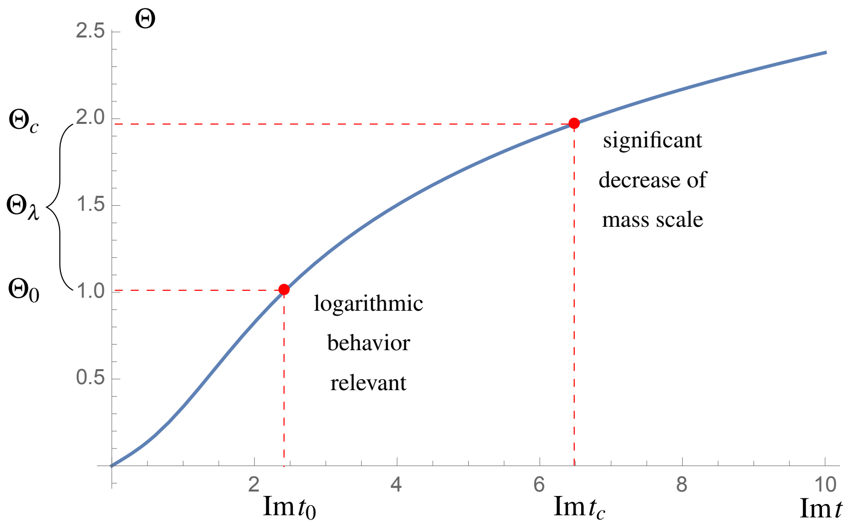


Figure 2: Expected relation between proper field distance Θ and $\mathrm{Im}\,t$.

which includes the distance Θ_0. Clearly, if Θ_0 determined in this way was already much larger than the Planck-scale, the RSDC would be falsified. Said the other way around, if the RSDC is correct, the proper field distance that can be traveled in the small volume regime must be smaller than $M_{\rm pl}$.

Following this idea, using two different methods to determine the Kähler potential on the Kähler moduli space, in [32] the following two tasks were treated:

- Compute the periods and Kähler potential in non-geometric phases (LG and hybrid) of CY manifolds with $h_{11} \in \{1, 2, 101\}$
- Compute the critical distances λ and Θ_0 for various geodesics in these highly curved moduli spaces .

For the quintic, the Kähler potential in the large volume regime takes the familiar form

$$K = -\log\Big(i(t-\bar{t})^3 + C + O(e^{-2\pi t})\ldots\Big) \tag{4.7}$$

whereas in the Landau-Ginzburg phase we obtain ($\psi = |\psi|\exp(i\theta)$ and $|\psi| < 1$)

$$K = -\log\Big(\alpha\,|\psi|^2 + \beta\,|\psi|^4 + \gamma|\psi|^6 + \delta\,|\psi|^7\cos(5\theta) + \ldots\Big)\,. \tag{4.8}$$

Note that both expressions feature a shift symmetry for the leading order terms, namely in $\mathrm{Re}(t)$ and $\mathrm{Arg}\,\psi$, that are broken by higher order corrections. The resulting Kähler metric as a function of ψ is shown in figure 3. Note that the variable t and ψ are related via the mirror map.
The bisectrix is indeed a geodesic for which we obtain

$$\Theta_0^{\rm LG} = 0.43\,, \qquad \Theta_\lambda = \lambda^{-1} = \sqrt{\tfrac{3}{4}} \tag{4.9}$$

in accordance with the RSDC. This ist just the simplest example that has been computed in [32]. There, many more tests have been made for other Calabi-Yau threefolds and more general geodesics.

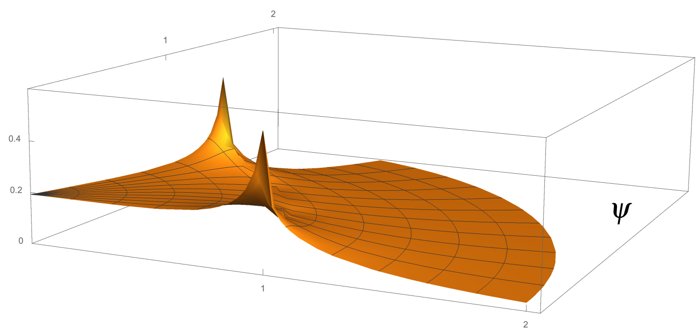


Figure 3: Kähler metric for the quintic.

The findings can be summarized by saying that in all examples one obtains $\Theta_0 < 1$ and $\lambda^{-1} < 1$. Moreover, studying the recently determined 204 periods of the quintic [36], it was observed that the distances in the LG phase become an order 10^{-2} smaller than in the one-dimensional Kähler moduli space of the quintic. Thus, it is compelling to propose the scaling relation

$$\left\langle \frac{\Theta_0}{\text{phase}} \right\rangle \cdot \#(\text{phases}) < M_{\text{pl}} \tag{4.10}$$

for the average distance that can be traversed in a non-geometric phase. Then, even crossing more non-geometric phases before reaching the large volume phase does not help in collecting an overall distance that is much larger than one. If true, this would be a compelling piece of evidence for the RSDC.

5. Axion monodromy inflation

Let us come back to the question of generating a potential for the initially shift symmetric axion. As opposed to taking non-perturbative effects into account, that are there once the classical string background is specified, one can also add new classical ingredients. In the pioneering work [9, 10] these were D-branes, whose backreaction on the geometry were inducing a non-trivial potential for the axion (this arose from the Kalb-Ramond field contribution to the Dirac-Born-Infeld action $S_{\text{DBI}} = \int d^p x \sqrt{G+B}$).

F-term axion monodromy

An alternative mechanism was proposed in [37, 38, 39], namely that the same type of fluxes that one introduces for moduli stabilization already induces a potential for the axion. This can be seen from the dimensional reduction of the kinetic terms of the R-R forms in the ten-dimensional effective supergravity action

$$S_{10} = \int d^{10}x\, G_p \wedge \star G_p \tag{5.1}$$

with

$$G_p = F_p - H_3 \wedge C_{p-3} + \mathscr{F} \wedge e^B \tag{5.2}$$

where $F_p = dC_{p-1}$, $H = dB_2$ and $\mathscr{F}$ denotes a formal sum of all R-R field strengths F_p. Note that the fluxes F_p and H are closed and therefore take values in $H^p(M,\mathbb{Z})$. Moreover, the dimensionally reduced higher gauge fields C_p and B give rise to axions θ in four-dimensions. For instance, in type IIA, the axions $\int_{\Sigma_2^{(i)}} B$ appear in the complexified Kähler moduli $T^{(i)} = \int_{\Sigma_2^{(i)}} B + i\int_{\Sigma_2^{(i)}} J$. Since the the gauge fields appear explicitly in G_p, the dimensional reduction of the kinetic terms (5.1) directly lead to a scalar potential for axions.

Motivated by the earlier field theory proposals by [11, 12, 13], it was explicitly shown in [40] (see also the review [41]) that the resulting scalar potential takes a very peculiar form, namely it can be expressed in terms of 4D space-time filling four-forms coupled to the axions

$$S_4 = \int d^4x \Big(-Z_{ab}(\phi)\, F_4^a \wedge \star F_4^b + 2F_4^a\, \rho(\theta)\Big) + \ldots . \tag{5.3}$$

Here $Z_{ab}(\phi)$ depends only on the saxionic fields ϕ and $\rho(\theta)$ depends on the axionic fields and the fluxes turned on. Integrating out the 4D three-forms in $F_4^a = dC^a$ and choosing the integration constant to be vanishing, one arrives at the scalar potential

$$V = \big(Z^{-1}(\phi)\big)^{ab}\, \rho_a(\theta)\, \rho_b(\theta)\,, \tag{5.4}$$

that nicely separates the dependence on the saxions and the axions. To illustrate what is going here, let us consider a simplified field theory model, where the Lagrangian of a single axion θ coupled to a three-form gauge field reads

$$\mathscr{L} = -f^2 d\theta \wedge \star d\theta - F_4 \wedge \star F_4 + 2F_4(m\theta + f_0)\,. \tag{5.5}$$

As we have seen, in string theory f_0 and m can be thought of as quantized background fluxes. There, the axion decay constant f will also be (saxionic) moduli dependent. The resulting equation of motion for C_3 takes the simple form

$$d\star F_4 = d(m\theta + f_0) \quad \Rightarrow \quad \star F_4 = f_0 + m\theta \tag{5.6}$$

where the integration constant was set to zero. Introducing this back into the action leads to an effective potential

$$V = (f_0 + m\theta)^2\,. \tag{5.7}$$

We observe that the scalar potential and F_4 are invariant under the extended shift symmetry

$$\theta \to \theta - c/m \qquad f_0 \to f_0 + c\,. \tag{5.8}$$

Therefore, the system still preserves the shift symmetry, that is broken spontaneously by a choice of branch f_0, m. Moreover, the shift symmetry and the gauge symmetry of C_3 strongly constrain higher order corrections. They must be functions of F_4, i.e.

$$\delta V \sim \sum (F_4)^{2n} \sim \sum (V_0)^n \tag{5.9}$$

so that even in the trans-Planckian regime $\delta\theta \gg 1$, as long as $\delta V \ll 1$ one controls the expansion. In other words, this way of breaking the shift symmetry (on a branch) is not generic but of a special kind that still allows to control the Planck suppressed operators.

Inflation

Using these flux induced potentials for an axion to realize inflation goes under the name of axion monodromy inflation. On a fixed branch the scalar potential increases by a certain amount when one traverses over one period of the former shift symmetric axion. Compared to the models of natural inflation the range of the axion θ is now disentangled from the axion decay constant (depending on the saxions) so that the WGC conjecture does not provide any direct constraint on the field range of the axion.

The question is whether there exist other problems of controlling the effective field theory along a trajectory that involves trans-Planckian field ranges for the axion. First, one realizes that in full examples the issue of axion inflation cannot be disentangled from the issue of moduli stabilization. It is the same flux induced potential that is responsible for both. Therefore, one has to make sure that the fluxes can be turned on in such a way, as to allow that the lightest (heavy) modulus is an axion (see e.g. [42]). This would then be the candidate for the inflaton. Since the axion/inflaton receives its small mass from a tree-level effect, it is unnatural to expect that moduli getting their mass from a non-perturbative effect come out more massive than the inflaton. In other words, *all* other heavy moduli should better also be stabilized by tree-level fluxes.

Let us assume that using a 4D low-energy effective action, one has stabilized all moduli, so that a canonically normalized axion Θ is the lightest one with mass M_Θ and all the remaining moduli have a higher mass $M_\Theta \ll M_{\rm heavy} \ll M_{\rm KK}$. For inflation, the axion gets displaced from its minimum and, as mentioned already before, along the entire slow rolling phase the hierarchy (3.8) must be intact. However, when θ is displaced from its minimum, generically the minima of the saxions will also change,

$$r(\theta) = r_0 + \delta r(\theta) \tag{5.10}$$

where r_0 denotes the vacuum expectation value of the scalar r at the minimum of the potential, i.e. when θ is also at its minimum. By plugging this back into the effective theory, the scalar potential and the kinetic term for the inflaton can be substantially modified. In other words, the inflationary trajectory is no longer only along θ but corresponds to a combination of θ and r. It has been pointed out in [43] that this backreaction leads to a flattening of the inflaton potential.

Studying concrete examples, in [29, 30] it was realized that the displacement of the saxions will generically backreact on the kinetic metric of the inflaton leading at best to a logarithmic behavior of the proper field distance at large field. More concretely,

$$\Theta = \int \sqrt{K_{\theta\theta}(s)}\, d\theta \sim \int \frac{1}{r(\theta)} \sim \frac{1}{\lambda} \log(\theta) \tag{5.11}$$

where we have used that $K \sim -\log(r)$ with r being the saxionic partner of the inflaton, and that for large field excursions $\delta r(\theta) \simeq \lambda\theta$ (which was found for concrete examples). In (5.11), Θ is the canonically normalized inflaton field. This behavior is reminiscent of the logarithmic scaling in the SDC (4.3), with the difference that now we are dealing with axions that are not flat directions but are moving in a potential. As in the SDC, the mass of Kaluza-Klein modes scales exponentially with the field excursion of the axion

$$M_{\rm KK} \sim \frac{n}{r} \sim \frac{n}{\lambda} e^{-\lambda\rho}\,. \tag{5.12}$$

Whether trans-Planckian field ranges in Θ are under control now depends on the value of λ. In concrete examples this is related to the ratio of light to heavy moduli masses.

$$\lambda = \left(\frac{M_\Theta}{M_{\text{heavy}}}\right)^p \tag{5.13}$$

where values $p = 1/2, 1$ have been found in concrete models. Therefore a mass hierarchy between the inflaton and the saxionic moduli can help to delay the backreaction effects which are not anymore tied to the Planck mass in an obvious way. To have parametrical control over this mass hierarchy, requires the minimum of the potential to satisfy the following condition [42, 44]: $\Theta_c = \lambda^{-1}$ will be tuneable if one can parametrically set the inflaton mass to zero without destabilizing the other scalars. In order to see whether this is possible in (quasi-) realistic set-ups, we consider a simple example from [45], that realizes many of the ingredients that we need.

An example with a D-brane modulus

We consider the so-called type IIB STU-model extended by a complex open string modulus Φ that parametrizes the transversal deformation of a D7-brane. Such a model has also been studied in [46]. We work in the framework of effective four-dimensional $\mathrm{N} = 1$ supergravity, where all string and Kaluza-Klein modes have been integrated out. The four complexified moduli are

$$S = c + is\,, \qquad T = \rho + i\tau\,, \qquad U = v + iu\,, \qquad \Phi = \theta + i\varphi \tag{5.14}$$

where the real parts are axion-like scalars. At large values of the saxions (s, τ, u), the Kähler potential is given at leading order as

$$K = -3\log(-i(T-\overline{T})) - 2\log(-i(U-\overline{U})) - \log\left[-(S-\overline{S})(U-\overline{U}) + \tfrac{1}{2}(\Phi-\overline{\Phi})^2\right]. \tag{5.15}$$

Now we turn on fluxes to generate the superpotential

$$W = \mathfrak{f}_0 - 3\mathfrak{f}_2 U^2 + hSU + qTU + \mu\Phi^2 \tag{5.16}$$

where the fluxes are considered to be integers. This model is of the flux-scaling type discussed in [47, 48]. In type IIB the term involving the Kähler modulus T is generated by a so-called non-geometric Q-flux. This flux breaks the no-scale structure that is present for $q = 0$ and which was the starting point for the KKLT and the LVS scenario of Kähler moduli stabilization via non-perturbative effects. As mentioned, as we want to identify the real part of Φ with the inflation, stabilizing the Kähler modulus in the latter non-perturbative way would lead to the wrong mass hierarchy.

The Kähler- and the superpotential data specify the supergravity model and one can compute the scalar potential. It admits an analytically solvable non-supersymmetric tachyon-free AdS minimum at

$$\begin{aligned} &s_0 = \frac{2^{\frac{7}{4}}\cdot 3^{\frac{1}{2}}}{5^{\frac{1}{4}}}\frac{(\mathfrak{f}_0\mathfrak{f}_2)^{\frac{1}{2}}}{h}\,, \qquad \tau_0 = \frac{5^{\frac{3}{4}}\cdot 3^{\frac{1}{2}}}{2^{\frac{1}{4}}}\frac{(\mathfrak{f}_0\mathfrak{f}_2)^{\frac{1}{2}}}{q}\,, \qquad u_0 = \frac{1}{10^{\frac{1}{4}}\cdot 3^{\frac{1}{2}}}\left(\frac{\mathfrak{f}_0}{\mathfrak{f}_2}\right)^{\frac{1}{2}} \\ &\varphi_0 = 0\,, \qquad v_0 = hc_0 + q\rho_0 = \theta_0 = 0\,, \end{aligned} \tag{5.17}$$

leaving one axionic direction unconstrained. The value of the scalar potential in the AdS minimum is

$$V_0 = -\frac{1}{120\cdot 3^{\frac{1}{2}}\cdot 10^{\frac{1}{4}}}\,\frac{h\,q^3}{\mathfrak{f}_0^{\frac{3}{2}}\,\mathfrak{f}_2^{\frac{1}{2}}}\,. \tag{5.18}$$

For realizing inflation, one now needs to make the assumption that this minimum can be uplifted to Minkowski, i.e. $V_0 = 0$, without destabilizing the moduli and only slightly changing their vacuum expectation values. For recent critical discussions on the existence of bona-fide de Sitter vacua in string theory and the consistency of up-lift mechanisms please consult [49, 50]. To this discussion one can add another argument supporting the claim that the correct description of an uplift by an $\overline{D3}$-brane placed in a highly warped throat is not yet fully understood. In most cases, one simply adds to the supergravity generated scalar potential an uplift term

$$V = V_{\rm SUGRA} + V_{\rm up} = V_{\rm SUGRA} + \frac{\varepsilon}{\tau^2} \tag{5.19}$$

with $\varepsilon \ll 1$ in the strongly warped region. However, the complex structure modulus Z governing how close one is to a conifold singularity also needs to be stabilized before. It was shown in [51] that the self-consistency of the used supergravity action requires that the physical volume of the corresponding 3-cycle A must still be larger than one, i.e.

$$\mathrm{Vol}(A) = \mathscr{V}^{\frac{1}{2}}\int_A \Omega_3 = (|Z|^2\,\mathscr{V})^{\frac{1}{2}} > 1 \tag{5.20}$$

so that one stays in the dilute flux limit. Here $\mathscr{V}$ is the total volume of the CY threefold. This means that the strongly warped region is outside the validity of the used supergravity effective action. Therefore, the ansatz (5.19) for the total scalar potential is highly questionable. We note that for the upcoming discussion of the backreaction, the existence of an uplift is not really relevant.

Now coming back to our example, the mass-matrix has the eigenvalues[6]

$$M^2_{\rm closed} = \nu_i\,\frac{h\,q^3}{\mathfrak{f}_0^{\frac{3}{2}}\,\mathfrak{f}_2^{\frac{1}{2}}} \tag{5.21}$$

with $\nu \in \{0, 0.0001, 0.0019, 0.0029, 0.0117, 0.0162\}$ and

$$M^2_\phi \simeq 0.002\,\frac{h\,q^3}{\mathfrak{f}_0^{\frac{3}{2}}\,\mathfrak{f}_2^{\frac{1}{2}}}\,,\qquad M^2_\theta \simeq 0.021\,\frac{\mu\,q^3}{\mathfrak{f}_0^{\frac{3}{2}}\,\mathfrak{f}_2^{\frac{1}{2}}}\,. \tag{5.22}$$

Therefore, the open string axion Θ can be *parametrically* lighter than all the other massive moduli, indeed

$$\frac{M_{\rm heavy}}{M_\Theta} \sim \sqrt{\frac{h}{\mu}} = \lambda^{-1}\,. \tag{5.23}$$

From the former discussion, one expects that $\lambda = \sqrt{\mu/h}$ is the flux dependent parameter that controls the backreaction of the inflaton onto the other moduli. Let us analyze this in more detail

[6]Introducing back factors of $M_{\rm pl}$ we note that $|V_0| \sim M^2_{\rm heavy}\,M^2_{\rm pl}$.

under the assumption $\lambda \ll 1$. Up to subleading corrections of order $\mathscr{O}(\lambda^2)$, the conditions for the backreacted minima can be solved

$$\begin{aligned} s_0(\theta) &\sim \frac{2^{\frac{7}{4}}\, 3^{\frac{1}{2}}}{5^{\frac{1}{4}}} \frac{(\mathfrak{f}_0+\mu\theta^2)^{\frac{1}{2}}\, \mathfrak{f}_2^{\frac{1}{2}}}{h}\,, \qquad \tau_0(\theta) \sim \frac{5^{\frac{3}{4}}\, 3^{\frac{1}{2}}}{2^{\frac{1}{4}}} \frac{(\mathfrak{f}_0+\mu\theta^2)^{\frac{1}{2}}\, \mathfrak{f}_2^{\frac{1}{2}}}{q} \\ u_0(\theta) &\sim \frac{1}{10^{\frac{1}{4}}\, 3^{\frac{1}{2}}} \left(\frac{\mathfrak{f}_0+\mu\theta^2}{\mathfrak{f}_2}\right)^{\frac{1}{2}} \end{aligned} \tag{5.24}$$

with all other fields sitting in their minimum at zero. Thus, the critical value of θ where the backreaction becomes significant is $\theta_c = \sqrt{\frac{f_0}{\mu}}$. The kinetic term for the inflaton becomes

$$\mathscr{L}^{\rm ax}_{\rm kin} = K_{\Phi\overline{\Phi}}\, \partial_\mu\theta\partial^\mu\theta = \frac{1}{8}\sqrt{\frac{5}{2}}\, \frac{h}{f_0+\mu\theta^2}\, (\partial\theta)^2 \tag{5.25}$$

so that the critical value for the canonically normalized inflaton field Θ is

$$\Theta_c = \gamma\sqrt{\frac{h}{f_0}}\,\theta_c = \gamma\sqrt{\frac{h}{\mu}} = \gamma\lambda^{-1} \tag{5.26}$$

with $\gamma = \frac{1}{2}\left(\frac{5}{2}\right)^{\frac{1}{4}} = 0.63$. Therefore, from this perspective, for $\lambda \ll 1$ and $\Theta \ll \Theta_c$ the backreaction can be neglected and one gets a polynomial effective potential for the inflaton (after adding a constant uplift). Thus, it seems that by parametrically choosing $\Theta_c \sim \lambda^{-1} > 10$ one can achieve a stringy model featuring large field inflation[7].

Beyond the critical value, the kinetic term for the inflaton takes the form

$$\mathscr{L}^{\rm ax}_{\rm kin} = \frac{1}{8}\sqrt{\frac{5}{2}}\, \frac{h}{\mu} \left(\frac{\partial\theta}{\theta}\right)^2 \tag{5.27}$$

so that the canonically normalized inflaton shows the logarithmic behavior

$$\Theta = \Theta_c \log\left(\frac{\theta}{\theta_c}\right) \simeq \frac{1}{\lambda}\log\theta \simeq \frac{M_{\rm heavy}}{M_\Theta}\log\theta\,. \tag{5.28}$$

This is familiar from the RSDC. The difference is that here it is an axionic field that can go to large distances and that the parameter $\lambda^{-1} \sim \sqrt{\frac{h}{\mu}}$ can be parametrically larger than one so that trans-Planckian distances seem to be controlled. The reason behind it is that for $\lambda^{-1} \gg 1$ the inflationary trajectory goes mostly in the axionic direction with only a slight movement in the (dangerous) saxionic directions

However, we have only analyzed one aspect of the quantum gravity embedding of trans-Planckian axionic field directions. Since we have more mass scales in the problem like the hierarchy between the heavy moduli masses and the Kaluza-Klein mass scale, other issues might occur. To see what happens let us compute the various mass scales, like string scale, Kaluza-Klein scales, heavy moduli masses and the inflaton mass. Since we are heading for systematic effects,

[7] This is consistent with the observation already made in [46] for a more complicated, only numerically treatable open string model (without non-geometric fluxes).

we will not be concerned with model dependent numerical prefactors, but will focus on desired mass hierarchies that are guaranteed or spoiled parametrically. The relevant masses scale in the following way with the fluxes (recall that we set $M_{\rm pl}=1$): The string scale is

$$M_{\rm s}^2 \sim \frac{1}{\tau^{\frac{3}{2}}\, s^{\frac{1}{2}}} \sim \frac{h^{\frac{1}{2}}\, q^{\frac{3}{2}}}{\mathfrak{f}_0\, \mathfrak{f}_2}\,. \tag{5.29}$$

Moreover, considering our model as being realized on the isotropic T^6, we now have *two* Kaluza-Klein scales for $u>1$, yielding a *heavy* and a *light* Kaluza-Klein mass

$$M_{\rm KK}^2 \sim \frac{1}{\tau^2}\, u^{\pm 1} \qquad \Longrightarrow \qquad M_{\rm KK,h}^2 \sim \frac{q^2}{\mathfrak{f}_0^{\frac{1}{2}}\, \mathfrak{f}_2^{\frac{3}{2}}}\,, \qquad M_{\rm KK,l}^2 \sim \frac{q^2}{\mathfrak{f}_0^{\frac{3}{2}}\, \mathfrak{f}_2^{\frac{1}{2}}}\,. \tag{5.30}$$

Recall that the mass of the heavy moduli and the inflaton scaled as

$$M_{\rm heavy}^2 \sim \frac{h\, q^3}{\mathfrak{f}_0^{\frac{3}{2}}\, \mathfrak{f}_2^{\frac{1}{2}}}\,, \qquad M_{\Theta}^2 \sim \frac{\mu\, q^3}{\mathfrak{f}_0^{\frac{3}{2}}\, \mathfrak{f}_2^{\frac{1}{2}}}\,. \tag{5.31}$$

We can also evaluate the various mass-scales in the large field regime. Due to (5.24), this means that we just have to change

$$\mathfrak{f}_0 \to \mathfrak{f}_0 \left(\frac{\theta}{\theta_{\rm c}}\right)^2 \to \mathfrak{f}_0 \exp\left(2\frac{\Theta}{\Theta_{\rm c}}\right) \tag{5.32}$$

so that the various mass scales become

$$M_i^2 = M_i^2\big|_0 \exp\left(-\mu_i \frac{\Theta}{\Theta_{\rm c}}\right)\,. \tag{5.33}$$

with the integer coefficient $\mu_s = 2, \mu_{\rm KK,h}=1, \mu_{\rm KK,l}=\mu_{\rm heavy}=\mu_\Theta = 3$. Note that all these mass scales become exponentially light for large Θ. We observe that the quotient of the *light* KK-mass and the heavy moduli mass is constant along the trajectory[8]

$$\frac{M_{\rm KK,l}^2}{M_{\rm mod}^2} \sim \frac{1}{h\, q}\,. \tag{5.34}$$

Now, for quantized fluxes, one can only tune $\lambda^{-1} \sim \sqrt{\frac{h}{\mu}}$ large by choosing a large flux parameter $h \gg \mu$. However, in this regime parametrically one gets $M_{\rm KK,l}^2 \ll M_{\rm mod}^2$ so that the light KK modes become lighter than the moduli masses, thus spoiling the use of the low energy effective supergravity action. Note that we are not talking here about model dependent numerical coefficient, but about the parametrical dependence and this goes in the wrong direction and makes trans-Planckian field ranges of Θ at least unnatural.

Thus, in this concrete (admittedly not completely perfect) string motivated model, we found evidence that the distance in proper field space Θ, where the logarithmic behavior sets in, is around

[8]For the flux supported AdS vacuum, the curvature radius of the AdS space is given by $L_{\rm AdS} \sim M_{\rm pl}/V_0^{\frac{1}{2}} \sim M_{\rm heavy}^{-1}$. Therefore for the ratio of the length scales $L_{\rm AdS}/L_{\rm KK,l} \sim M_{\rm KK,l}/M_{\rm heavy}$ one finds the same ratio of mass scales as in (5.34). This means that parametrically the initial AdS minimum does not allow a clear separation of the length scales of the compact and the non-compact space.

the Planck-scale and cannot be much increased without invalidating the effective theory. Of course this is just a single example but a similar behavior has been found in all of the models studied in [45, 47]. There, e.g. for the KKLT and LVS type models from [52], another issue was observed namely that for the anti D3-brane uplifted models the backreacted valley along which inflation is supposed to happen destabilizes at around the scale Θ_c. This behavior is shown in figure 4.

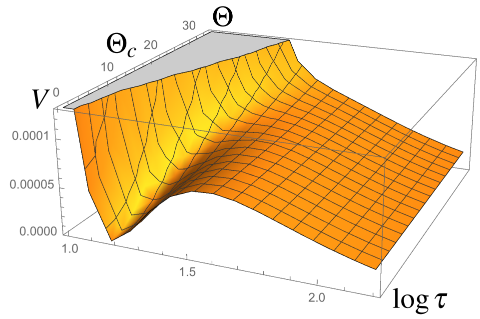


Figure 4: A backreacted trajectory that destabilizes at Θ_c. τ denotes a saxionic field.

All these findings point towards the axionic extension of the RSDC that says: *Even for axion monodromy models with potentially infinite distances in axionic directions, the validity of the effective field theory breaks down for field excursions of the order of the Planck-scale.*

As we have seen, the validity of this conjecture is closely related to the possibility to stabilize the moduli such that a single axion is parametrically lighter than all the other moduli. Note that here one is not dealing with geodesic distances in a moduli space, but with actual trajectories in a field space with a scalar potential. Thus, one has two competing effects and the true trajectory will be somewhere between a geodesic and a valley in the potential. Since the axionic RSDC conjecture is a very general and very strong statement, more evidence is needed to support it or to disprove it by a convincing counter-example. In the moment we can say that so far no fully compelling string model of axion monodromy inflation has been worked out that admits controllable trans-Planckian field ranges. If the axionic RSDC is correct one gets a picture like shown in the figure 5.

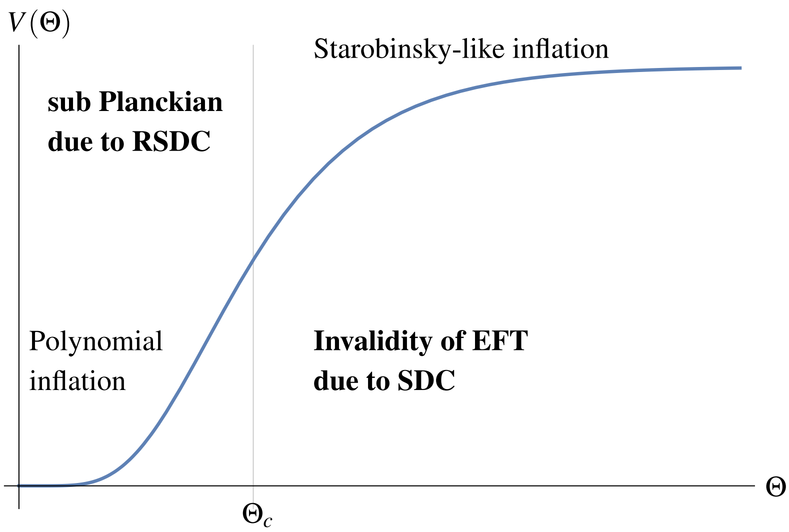


Figure 5: Large field inflation EFTs derived from UV completion.

Therefore, all these developments could culminate in a no-go theorem of the type: In string theory (quantum gravity) it is impossible to achieve a *parametrically* controllable effective field theory model of large (single) field inflation. The tensor-to-scalar ratio is thus bounded from above by $r \lessapprox 10^{-3}$. If true, it offers an experimentally testable way to falsify string theory. However, it could also be that the highly non-trivial axionic extension of the RSDC is biased by the examples one has studied so far. More work is needed to settle this important question.

6. Comments on quintessence

As we mentioned in section 2, the mass scales appearing in quintessence models are extremely tiny

$$H \sim M_\Theta \sim 10^{-33}\,\mathrm{eV}\,, \qquad V_{\rm quint} \sim (10^{-3}\,\mathrm{eV})^4 \tag{6.1}$$

and therefore Planck-suppressed operators are even more dangerous to destroy the control over the slow-roll regime. If the quintessence field is a saxion then there is no mechanisms to control the dangerous Planck suppressed operators and during the rolling some of the coupling constants could be time dependent. Moreover, there is the general concern of a long range fifth force.

Thus, again one is led to axionic fields with an initial shift symmetry to realize quintessence (see [53, 54]). For such models with a potential $V(\Theta) \sim \Theta^p$ the slow-roll conditions are met for $\Theta \gg pM_{\rm pl}/\sqrt{2}$. From the measured value of the parameter in the equation of state $w = -1.006 \pm 0.045$, one can infer that $2.8pM_{\rm pl} \lessapprox \Theta < \infty$. Moreover, the number of e-foldings up to the present time is of order one, as $N_e \sim Ht_0 \sim 1$. Therefore, depending on the values of w and p, one could stay closer to Planck-size field excursion than for large field inflation[9].

For models with a non-perturbatively generated periodic potential, one faces the same problem as for inflation. In the trans-Planckian regime one still needs $f > 1$ and a very small $\exp(-S_{\rm inst})$ factor, which is not consistent with the WGC. For models of axion monodromy, even though the distance the field has to roll is smaller than for inflation, one still has to work in the (close to) trans-Planckian regime. The main challenge though is that in a full string theory setup one needs to stabilize the moduli such that the mass of the axionic quintessence field is by many orders of magnitude smaller than the heavy saxionic moduli masses. Due to the cosmological moduli problem these must satisfy $M_{\rm heavy} > 30$ TeV so that

$$\lambda \sim \frac{M_\Theta}{M_{\rm heavy}} \sim 10^{-45}\,. \tag{6.2}$$

For the previously discussed concrete example this involves huge fluxes that are certainly not consistent with tadpole cancellation and the hierarchy between the moduli and the KK-masses. In more general words, the axionic RSDC will also impose very strong constraints for axionic models of quintessence to be realizable in effective theories derived from string theory. Therefore, effective axionic quintessence models might be in the swampland, as well.

[9] I thank Nemanja Kaloper for pointing this out.

7. Outlook

In the moment, we are witnessing a paradigm shift in the development of string theory or better string phenomenology where people take the message from string model building obstacles seriously and make an effort to extract general conceptual rules that govern any theory of quantum gravity. Here, besides general arguments based on black holes, string theory provides an important source of insight as it is a theory of quantum gravity that methodologically and technically is quite well understood. However, behind all its technical ingredients it seems to hide some basic principles of quantum gravity. As we have reviewed, once these more general principles are revealed, they could potentially lead to no-go theorems that in principle allow one to experimentally falsify string theory.

In this article we were mostly concerned with a UV completion of models of large field inflation with a measurable tensor-to-scalar ratio. The arguments follow the logic that the interesting parametric region for realizing it lies out of control of the effective action one is using. This is not a full no-go theorem, as it might only indicate that one is using an unsuitable framework and should better use the full string theory without referring to an effective action. Of course, how this can be done in practice is unexplored.

Finally, let us also mention two more conjectures discriminating properties of the string landscape from those of the swampland. For more details we refer to the original literature. In [55] it has been proposed that the equal sign in the WGC is only for BPS states and that as a consequence every (flux supported) non-supersymmetric AdS vacuum is (non-perturbatively) unstable. This has immediate consequences for non-supersymmetric AdS-CFT duality, as on the field theory side a finite life-time sets a scale and therefore violates the conformal symmetry.

Maybe even more drastic, in [56] a potential conjecture was mentioned that claims that meta-stable de Sitter vacua are also in the swampland of string theory. This was also argued for in [50] where a critical look on existing attempts to construct dS vacua was given. Such a no-go theorem would imply that our universe cannot be in a meta-stable state but must have some still rolling scalar fields. This is reminiscent of models of quintessence. As we have just mentioned, if the conjectured axionic RSDC indeed holds, string derived effective models of this type will be plagued with the same kind of control issues as large field inflationary models[10]. Admittedly, the axionic RSDC is not fully established yet and is motivated by considering a few examples. More work is needed to see whether our intuition is maybe misguided by the lamppost we were looking under.

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[10]As an analogy let us mention that by analyzing the question of the (non-)existence of an S-matrix [57], it was found that rolling eternal quintessence models behave analogously to de Sitter vacua

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