PROCEEDINGS OF SCIENCE

Probing violations of CPT with B_d mesons

Miguel Nebot*
Instituto Superior Técnico (IST), U. de Lisboa (UL),
Av. Rovisco Pais, P-1049-001 Lisboa, Portugal.
E-mail: miguel.r.nebot.gomez@tecnico.ulisboa.pt

We establish the precise connection between the theoretical T, CP, CPT asymmetries, in terms of transition probabilities between the neutral meson B_d states, and the experimental asymmetries, in terms of the double decay rate intensities for Flavour-CP eigenstate decay products in a B-Factory of entangled states. Genuine asymmetry parameters in the time distribution of the asymmetries are identified and their measurability analysed, disentangling genuine and possible fake terms. The nine asymmetry parameters – three different observables for each one of the three symmetries – are expressed in terms of the ingredients of the Weisskopf-Wigner dynamical description of the entangled B_d-meson states and we obtain a global fit to their values from the BaBar Collaboration experimental results. The possible fake terms are all compatible with zero. The information content of the nine asymmetry parameters is indeed different. The non-vanishing $\Delta\mathscr{S}_c^{\mathrm{T}} = -0.687 \pm 0.020$ and $\Delta\mathscr{S}_c^{\mathrm{CP}} = -0.680 \pm 0.021$ are impressive separate direct evidence of Time-Reversal-Violation and CP-Violation in these transitions (and compatible with Standard Model expectations). A 2σ effect for the Re(θ) parameter responsible of CPT-Violation appears; interpreted as an upper limit, it leads to $|M_{\bar{B}^0\bar{B}^0} - M_{B^0B^0}| < 4.0\times 10^{-5}$ eV at 95% C.L. for the diagonal flavour terms of the mass matrix. Finally, we consider scenarios where, in the presence of quantum gravity fluctuations (space-time foam), the CPT operator may be ill-defined. Its perturbative treatment leads to a modification of the Einstein-Podolsky-Rosen correlation of the neutral meson system by adding an Entanglement-weakening term of the wrong exchange symmetry, the ω-effect. The analysis is extended to identify how to probe the complex ω when the connection between the Intensities for the two time-ordered decays (f,g) and (g,f) is lost (f flavour and g CP eigenstate decay channels), and how the ω-effect is disentangled from CPT violation in the evolution Hamiltonian.

Corfu Summer Institute 2017 'School and Workshops on Elementary Particle Physics and Gravity'
2-28 September 2017
Corfu, Greece

*Speaker.

PoS(CORFU2017)199

1. Introduction

The BaBar Collaboration demonstrated direct evidence of Time Reversal Violation in the time evolution of the B_d^0–$\bar{B}_d^0$ meson system [1], independent of CP Violation or CPT Invariance. The result does not dependent on a particular dynamical description of the B_d^0–$\bar{B}_d^0$ system: it is established in terms of asymmetries of observable transition rates. The essential quantum mechanical ingredients involved are (i) the entanglement of the B_d^0–$\bar{B}_d^0$ pair in a B-Factory before the first decay, (ii) the use of decays as filtering measurements to (a) prepare the initial, and (b) detection the final, B meson states participating in the transition, (iii) the time dependence of the double decay rates. The concept behind this approach originates in [2,3] while the application to an actual experimental analysis was addressed in [4], avoiding, in particular, the need of the T-reversal of the decay (since, as emphasized by L.Wolfenstein [5], the T-reverse of a decaying state is not a physical state). The considered transitions are between Flavour and CP eigenstate decay products. Orthogonality between the meson states filtered by both types of decay products (the Flavour and CP "Tags" [6]), is well defined under certain conditions [4, 7, 8]; this opens the possibility of constructing Genuine Asymmetries without contaminating fake terms. The eight different Flavour $\leftrightarrows$ CP transitions provide separate and independent asymmetries for the T, CP and CPT transformations. Our main objectives are [9]: (1) provide the precise connection between theoretical asymmetries (in terms of transition probabilities for the meson states), and experimental asymmetries (in terms of double decay intensities), giving *genuine asymmetry parameters* for model-independent T, CP, CPT time dependent asymmetries; (2) analysing these asymmetry parameters within the Weisskopf-Wigner Approach (WWA) [10,11] for the description of the dynamics/time evolution of the B_d^0–$\bar{B}_d^0$ system. In the following, we recall some generalities of the B_d^0–$\bar{B}_d^0$ effective hamiltonian, the evolution of the initial entangled state and the double decay rate intensities. In section 3, the conditions for the channels used in the Babar analysis of [1] to be truly appropriate for time reversal genuine asymmetries are addressed. Section 4 analyses the experimental asymmetries of [1] in detail, focusing on the connection with section 2. The reconstruction of complete genuine asymmetries beyond the BaBar ratios is shown in section 5. The contamination of genuine T and CPT asymmetries from deviations in the conditions discussed in section 3 is discussed and quantified in section 6. A summary of the most relevant results from a global fit to the Babar data is included in 7. Section 8 is devoted to a condensed discussion of the sensitivity to the ω-effect and to present selected results derived when the previous analysis is extended through the inclusion of ω.

2. Entanglement, Time evolution and double decay rates

The effective Hamiltonian for the two meson system B_d^0–$\bar{B}_d^0$ is $\hat{H} = \hat{M} - i\hat{\Gamma}/2$ with $\hat{M}$ and $\hat{\Gamma}$ 2×2 hermitian matrices, respectively the hermitian and the antihermitian parts of $\hat{H}$. In the notation of [12] the eigenvalues and eigenvectors are

$$\mu_{H,L} = M_{H,L} - \frac{i}{2}\Gamma_{H,L}; \qquad \hat{H}|B_H\rangle = \mu_H|B_H\rangle; \qquad |B_H\rangle = p_H|B_d^0\rangle + q_H|\bar{B}_d^0\rangle; \tag{2.1}$$

$$\hat{H}|B_L\rangle = \mu_L|B_L\rangle; \qquad |B_L\rangle = p_L|B_d^0\rangle - q_L|\bar{B}_d^0\rangle. \tag{2.2}$$

In general, with $\hat{H}$ not a normal operator, $[\hat{M},\hat{\Gamma}]\neq 0$: then, the states (2.1) and (2.2) are not orthogonal. The averages and differences of masses and widths[1] are more convenient to use:

$$\mu \;=\frac{M_H+M_L}{2}-\frac{i}{2}\frac{\Gamma_H+\Gamma_L}{2}\equiv M-\frac{i}{2}\Gamma, \tag{2.3}$$

$$\Delta\mu = M_H-M_L-\frac{i}{2}(\Gamma_H-\Gamma_L)\equiv \Delta M-\frac{i}{2}\Delta\Gamma, \tag{2.4}$$

together with the complex parameters θ and q/p:

$$\frac{q_H}{p_H}=\frac{q}{p}\sqrt{\frac{1+\theta}{1-\theta}}, \qquad \frac{q_L}{p_L}=\frac{q}{p}\sqrt{\frac{1-\theta}{1+\theta}}, \tag{2.5}$$

and with

$$\delta=\frac{1-|q/p|^2}{1+|q/p|^2}. \tag{2.6}$$

From $\hat{H}$, $\theta=(\hat{H}_{22}-\hat{H}_{11})/\Delta\mu$, $(q/p)^2=\hat{H}_{21}/\hat{H}_{12}$; θ is a CP and CPT violating complex parameter; δ violates CP and T. Interestingly, $\hat{H}$ can be written in terms of physical parameters [13] (except for the phase of q/p, which is convention dependent),

$$\hat{H}=\begin{pmatrix}\mu-\frac{\Delta\mu}{2}\theta & \frac{p}{q}\frac{\Delta\mu}{2}\sqrt{1-\theta^2}\\ \frac{q}{p}\frac{\Delta\mu}{2}\sqrt{1-\theta^2} & \mu+\frac{\Delta\mu}{2}\theta\end{pmatrix}. \tag{2.7}$$

In a B factory, operating at the $\Upsilon(4S)$ peak, the initial two-meson state is EPR [14] entangled,

$$|\Psi_0\rangle=\frac{1}{\sqrt{2}}\Big(|B_d^0\rangle|\bar{B}_d^0\rangle-|\bar{B}_d^0\rangle|B_d^0\rangle\Big)=\frac{1}{\sqrt{2}(p_Lq_H+p_Hq_L)}\Big(|B_L\rangle|B_H\rangle-|B_H\rangle|B_L\rangle\Big), \tag{2.8}$$

which retains its entangled antisymmetric character in terms of $\hat{H}$ eigenstates. For the decay of the first state into $|f\rangle$ at time t_0, and then the second state into $|g\rangle$ at time $t+t_0$, the transition amplitude is

$$\langle f,t_0;g,t+t_0|T|\Psi_0\rangle=\frac{e^{-i(\mu_H+\mu_L)t_0}}{\sqrt{2}(p_Lq_H+p_Hq_L)}\Big(e^{-i\mu_Ht}\mathscr{A}_f^L\mathscr{A}_g^H-e^{-i\mu_Lt}\mathscr{A}_f^H\mathscr{A}_g^L\Big), \tag{2.9}$$

with $\mathscr{A}_f^{H,L}\equiv\langle f|T|B_{H,L}\rangle$ the decay amplitudes of the eigenstates into the final state f. The double decay rate $I(f,g;t)$ is obtained squaring, and integrating over t_0:

$$I(f,g;t)=\frac{e^{-\Gamma t}}{4\Gamma|p_Lq_H+p_Hq_L|^2}\left|e^{i\Delta Mt/2}e^{\Delta\Gamma t/4}\mathscr{A}_f^H\mathscr{A}_g^L-e^{-i\Delta Mt/2}e^{-\Delta\Gamma t/4}\mathscr{A}_f^L\mathscr{A}_g^H\right|^2. \tag{2.10}$$

Equation (2.10) makes transparent the following expected symmetry property: up to the global exponential decay factor $e^{-\Gamma t}$, the combined transformations $t\to -t$ and $f\rightleftarrows g$ give the identity. Expansion of the t dependence with the approximation $\Delta\Gamma=0$, (valid for the B_d^0 states) gives

$$I(f,g;t)=e^{-\Gamma t}\frac{\langle\Gamma_f\rangle\langle\Gamma_g\rangle}{\Gamma}\Big\{\mathscr{C}_h[f,g]+\mathscr{C}_c[f,g]\cos(\Delta Mt)+\mathscr{S}_c[f,g]\sin(\Delta Mt)\Big\}, \tag{2.11}$$

[1] Subindices "H" and "L" label the "heavy" and "light" states, then $\Delta M>0$ and the sign of $\Delta\Gamma$ is not a matter of convention, it is not fixed.

with $\langle\Gamma_f\rangle$ defined after eq. (2.13). Therefore, with the initial entangled state in eq. (2.8) and the evolution in eq. (2.9), the following symmetry properties follow:

$$\mathscr{C}_h[f,g]=\mathscr{C}_h[g,f],\quad \mathscr{C}_c[f,g]=\mathscr{C}_c[g,f],\quad \mathscr{S}_c[f,g]=-\mathscr{S}_c[g,f]. \tag{2.12}$$

They are central in assessing the independent observables present in the Babar measurements. For the expressions to follow in the next sections, we define as usual the parameters of mixing times decay amplitudes

$$\lambda_f\equiv\frac{q}{p}\frac{\bar{A}_f}{A_f},\quad C_f\equiv\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2},\quad S_f\equiv\frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2},\quad R_f\equiv\frac{2\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}, \tag{2.13}$$

where $\langle f|T|B_d^0\rangle\equiv A_f$, $\langle f|T|\bar{B}_d^0\rangle\equiv\bar{A}_f$ and $\langle\Gamma_f\rangle=\frac{1}{2}(|A_f|^2+|\bar{A}_f|^2)$. Furthermore, for flavour specific channels $f=\ell^\pm+X$ ($f=\ell^\pm$ for short in the following), and assuming no wrong lepton charge sign decays, $C_{\ell^\pm}=\pm1$, $R_{\ell^\pm}=S_{\ell^\pm}=0$. It is also convenient to introduce the reduced intensity $\hat{\mathrm{I}}(f,g;t)$,

$$\hat{\mathrm{I}}(f,g;t)\equiv\frac{\Gamma}{\langle\Gamma_f\rangle\langle\Gamma_g\rangle}I(f,g;t)=e^{-\Gamma t}\left\{\mathscr{C}_h[f,g]+\mathscr{C}_c[f,g]\cos(\Delta M t)+\mathscr{S}_c[f,g]\sin(\Delta M t)\right\}. \tag{2.14}$$

3. Conditions for motion reversal asymmetries

The original proposal of [2, 3] to observe T violation independently of CP violation, implemented by BaBar [1] after [4], contained three ingredients:

I - Time reversal in the B_d^0–$\bar{B}_d^0$ Hilbert space: define a reference transition $P_1\to P_2(t)$ among meson states, and compare with the reversed transition $P_2\to P_1(t)$. The probability that an initially prepared state P_1, evolved to $P_1(t)$, behaves like a P_2, is:

$$P_{12}(t)=|\langle P_2|U(t,0)|P_1\rangle|^2\,. \tag{3.1}$$

The proposed T violating asymmetry is

$$P_{12}(t)-P_{21}(t)\,. \tag{3.2}$$

II - Beyond the use of $P_1,P_2=B_d^0,\bar{B}_d^0$ states. If the transitions $B_d^0\rightleftarrows\bar{B}_d^0$ are used, the asymmetry is not independent of CP: by construction, it is both CP and T violating (and very small since it comes from the parameter δ). A new reference transition $B_d^0\to B_+$ was introduced, to be compared with $B_+\to B_d^0$. In a decay channel with well-defined $CP=+$ and negligible CP violation, the reference transition can be accessed by looking for decay events f_1 where a B meson decays to a self-tagging channel of $\bar{B}_d^0$ and the other B meson decays later to a CP eigenstate $f_{CP=+}$ decay where one can neglect CP violation. Then, the main problem is how to measure the reverse transition.

III - The entangled character of the initial state is crucial to (i) connect double decay rates with meson transition rates and (ii) identify the reverse transition. If we assume that observing a $f_{CP=-}$ one filters in that side a B_-, then, due to the entanglement, we tag the orthogonal state to B_- in the opposite side. This state should be a B_+. In general, from eq. (2.8) we can say that, if at time t_1 we observe in one side the decay product f, the (still living) meson at time t_1 is tagged as *the state that does not decay into* f, $|B_{\nrightarrow f}\rangle$,

$$|B_{\nrightarrow f}\rangle = \frac{1}{\sqrt{|A_f|^2+|\bar{A}_f|^2}}\left(\bar{A}_f|B_d^0\rangle - A_f|\bar{B}_d^0\rangle\right). \tag{3.3}$$

The orthogonal state $\langle B^{\perp}_{\nrightarrow f}|B_{\nrightarrow f}\rangle = 0$, the one filtered by a decay f, is

$$|B^{\perp}_{\nrightarrow f}\rangle = \frac{1}{\sqrt{|A_f|^2+|\bar{A}_f|^2}}\left(A_f^*|B_d^0\rangle + \bar{A}_f^*|\bar{B}_d^0\rangle\right). \tag{3.4}$$

The *filtering identity* [8, 15] defines the precise meaning of the last statement:

$$\left|\langle B^{\perp}_{\nrightarrow f}|B_1\rangle\right|^2 = \frac{|\langle f|T|B_1\rangle|^2}{|A_f|^2+|\bar{A}_f|^2}. \tag{3.5}$$

Notice that, if $B_1 = B_{\nrightarrow g}(t)$, this quantity is the reduced intensity $\hat{\mathrm{I}}(g,f;t)$ in eq. (2.14):

$$\hat{\mathrm{I}}(g,f;t) = \frac{|\langle f|T|B_{\nrightarrow g}(t)\rangle|^2}{|A_f|^2+|\bar{A}_f|^2} = \left|\langle B^{\perp}_{\nrightarrow f}|B_{\nrightarrow g}(t)\rangle\right|^2. \tag{3.6}$$

Eq. (3.6) is, therefore, the precise connection between meson transition probabilities and double decay rates. By measuring $\hat{\mathrm{I}}(f_1,f_2;t)$, – we use the shorthand notation (f_1,f_2) in the following to refer to the considered first and second decays – we study probabilities $P_{12}(t)$ for transitions between meson states (B_1,B_2) given by

$$|B_1\rangle = |B_{\nrightarrow f_1}\rangle, \quad |B_2\rangle = |B^{\perp}_{\nrightarrow f_2}\rangle, \tag{3.7}$$

i.e. transition probabilities for $(B_1,B_2) = (B_{\nrightarrow f_1}, B^{\perp}_{\nrightarrow f_2})$. To compare with $P_{21}(t)$ one needs the reverse transition $(B^{\perp}_{\nrightarrow f_2}, B_{\nrightarrow f_1})$, but the applied filtering and tagging methods do not give this transition. Two new decay channels f_1' and f_2' in (f_2', f_1') give the transition $(B_{\nrightarrow f_2'}, B^{\perp}_{\nrightarrow f_1'})$. Then, provided they fulfill

$$|B_{\nrightarrow f_i'}\rangle = |B^{\perp}_{\nrightarrow f_i}\rangle, \tag{3.8}$$

this new transition (f_2', f_1') gives the reversed meson transition. For flavour specific decay channels, with no wrong lepton charge sign decays, $|B_d^0\rangle = |B_{\nrightarrow \ell^-}\rangle$ and $|\bar{B}_d^0\rangle = |B_{\nrightarrow \ell^+}\rangle$, the identity is obviously $|\bar{B}_d^0\rangle = |(B_d^0)^{\perp}\rangle$: if $f_1 = X\ell^+\nu_\ell$, then $f_1' = X'\ell^-\bar{\nu}_\ell$ ($f_1 = \ell^+$ and $f_1' = \ell^-$ for short). For the CP channel, equations (3.3), (3.4) and (3.8), give the condition these channels should satisfy:

$$\lambda_{f_2}\lambda^*_{f_2'} = -\left|\frac{q}{p}\right|^2. \tag{3.9}$$

Following eq. (3.9), $f_2 = J/\psi K_+$ and $f_2' = J/\psi K_-$ were proposed in [2–4] and used by BaBar in [1], where $K_\pm$ are the neutral kaon states filtered by the CP eigenstate decay channels. Consequently, the states $B_\mp$ are well defined and given by equation (3.4) for each of the two decay channels. From now on, we use K_S for K_+ and K_L for K_- as it is an accurate approximation up to CP violation in the kaon system. Considering that $\lambda_{J/\psi K_S} \equiv \lambda_{K_S} \sim \left|\frac{q}{p}\right| e^{-i2\beta}$ and $\lambda_{J/\psi K_L} \equiv \lambda_{K_L} \sim -\left|\frac{q}{p}\right| e^{-i2\beta}$, we use the general parameterisation

$$\lambda_{K_S} = \left|\frac{q}{p}\right| \rho\,(1+\varepsilon_\rho)\, e^{-i(2\beta+\varepsilon_\beta)}, \quad \lambda_{K_L} = -\left|\frac{q}{p}\right| \frac{1}{\rho}\,(1+\varepsilon_\rho)\, e^{-i(2\beta-\varepsilon_\beta)}, \tag{3.10}$$

in terms of the real parameters $\{\rho,\beta,\varepsilon_\rho,\varepsilon_\beta\}$, which allows us to control deviations from condition (3.9). By properly comparing double decay rates corresponding to two channels, one from $\{\ell^+,\ell^-\}$ and the other from $\{J/\psi K_S, J/\psi K_L\}$ (K_S and K_L for short in the following), we will therefore measure genuine time-reverse processes provided

$$\varepsilon_\rho = 0, \quad \varepsilon_\beta = 0. \tag{3.11}$$

Deviations from (3.11) contaminate time reversal asymmetries and should be conveniently subtracted out. Notice that eq. (3.9) is fulfilled even if $\rho \neq 1$. Equation (3.9) guarantees that the considered channels allow to truly compare the transition $P_2 \to P_1(t)$ with the reversed transition $P_1 \to P_2(t)$. Nevertheless, in order to ensure that this motion reversal asymmetry is truly a time reversal asymmetry, one needs decay channels f such that in the limit of T invariance, $S_f = 0$ [8,15]. For CP eigenstates, T invariance implies $S_f = 0$ provided there is no CPT violation in the corresponding decay amplitude, in accordance with [7]. This is equivalent to no CP violation in the decay, in the T invariant limit, giving, in addition to eq. (3.11), the condition $\rho = 1$. We then conclude that we should perform the data analysis with arbitrary ρ, ε_ρ and ε_β and that any deviation from

$$\rho = 1\,,\ \varepsilon_\rho = 0\,,\ \varepsilon_\beta = 0\,, \tag{3.12}$$

is a source of fake T violation. Notice that in the absence of CP violation in the decays filtering $B_\pm$, these states would be orthogonal, implying eq. (3.12), the orthogonality condition (3.9) would be automatically satisfied. Finally, it is convenient to clarify that without wrong flavour decays in $B_d^0 \to J/\psi K^0$ and $\bar{B}_d^0 \to J/\psi \bar{K}^0$, $\lambda_{K_S} + \lambda_{K_L} = 0$ [16], implying

$$\rho = 1\,,\ \varepsilon_\beta = 0\,, \tag{3.13}$$

clearly showing full compatibility among eq. (3.9) and the absence of wrong flavour decays. In terms of C_{K_S}, C_{K_L}, S_{K_S}, S_{K_L}, R_{K_S} and R_{K_L} (eq. (2.13)), no wrong flavour decays imply $C_{K_S} - C_{K_L} = 0$, $S_{K_S} + S_{K_L} = 0$ and $R_{K_S} + R_{K_L} = 0$; if, in addition, we impose eq. (3.9), $C_{K_S} = C_{K_L} = \delta$.

4. Independent asymmetries at BaBar

To reduce dependences on detection efficiencies in the different channels, in [1], instead of measuring $\mathscr{C}_h[f,g]$, $\mathscr{C}_c[f,g]$ and $\mathscr{S}_c[f,g]$ in eq. (2.11) or eq. (2.14), the BaBar collaboration fixed the normalization of the constant term and used the decay intensity

$$\mathbf{g}_{f,g}(t) \propto e^{-\Gamma t}\{1 + \mathrm{C}[f,g]\cos(\Delta M t) + \mathrm{S}[f,g]\sin(\Delta M t)\}\,, \tag{4.1}$$

and thus two quantities,

$$\mathrm{C}[f,g] = \frac{\mathscr{C}_c[f,g]}{\mathscr{C}_h[f,g]}, \quad \mathrm{S}[f,g] = \frac{\mathscr{S}_c[f,g]}{\mathscr{C}_h[f,g]}, \tag{4.2}$$

are measured for each pair (f,g). According to eq. (2.12), they verify

$$\mathrm{C}[f,g]=\mathrm{C}[g,f],\quad \mathrm{S}[f,g]=-\mathrm{S}[g,f]\,. \tag{4.3}$$

Genuine discrete asymmetries can be constructed combining one flavour specific channel and one CP channel: starting from one reference transition, one can generate another three by means of T, CP and CPT transformations. Because of eq. (4.3), the four transitions $\bar{B}_d^0\to B_-$, $B_-\to\bar{B}_d^0$, $B_d^0\to B_-$ and $B_-\to B_d^0$ saturate all the independent parameters measurable with one flavour specific and one CP decays. Table 1 shows the different meson state transitions, the corresponding decay channels, and the effect of the discrete symmetry transformations. Only eight parameters are independent: they are the $\mathrm{C}[f,g]$ and $\mathrm{S}[f,g]$ corresponding to the decays (ℓ^+,K_S), (K_L,ℓ^-), (ℓ^-,K_S) and (K_L,ℓ^+). BaBar has, of course, at least two independent ways of measuring the same parameter by means of the time-ordering of the decays. This operation is not a symmetry transformation from the left to the right-hand side of Table 1. Only *six* independent asymmetries can

Table 1: Double decay channels, the associated filtered meson states and their transformed transitions under the three discrete symmetries.

	Transition	$\mathbf{g}_{f,g}(t)$	$\mathbf{g}_{g,f}(t)$	Transition	
Reference	$\bar{B}_d^0\to B_-$	(ℓ^+,K_S)	(K_S,ℓ^+)	$B_+\to B_d^0$	Reference
T-transformed	$B_-\to\bar{B}_d^0$	(K_L,ℓ^-)	(ℓ^-,K_L)	$B_d^0\to B_-$	T-transformed
CP-transformed	$B_d^0\to B_-$	(ℓ^-,K_S)	(K_S,ℓ^-)	$B_+\to\bar{B}_d^0$	CP-transformed
CPT-transformed	$B_-\to B_d^0$	(K_L,ℓ^+)	(ℓ^+,K_L)	$\bar{B}_d^0\to B_+$	CPT-transformed

be constructed out of the eight independent parameters, corresponding to three time dependent asymmetries

$$A_{\mathrm{T}}(t)=\mathbf{g}_{K_L,\ell^-}(t)-\mathbf{g}_{\ell^+,K_S}(t)\,, \tag{4.4}$$

$$A_{\mathrm{CP}}(t)=\mathbf{g}_{\ell^-,K_S}(t)-\mathbf{g}_{\ell^+,K_S}(t)\,, \tag{4.5}$$

$$A_{\mathrm{CPT}}(t)=\mathbf{g}_{K_L,\ell^+}(t)-\mathbf{g}_{\ell^+,K_S}(t)\,, \tag{4.6}$$

which can be explicitely expanded as

$$A_{\mathrm{S}}(t)=e^{-\Gamma t}\left\{\Delta C_{\mathrm{S}}[\ell^+,K_S]\cos(\Delta M t)+\Delta S_{\mathrm{S}}[\ell^+,K_S]\sin(\Delta M t)\right\},\ \mathrm{S}=\mathrm{T},\mathrm{CP},\mathrm{CPT}\,, \tag{4.7}$$

where

$$\Delta S_{\mathrm{T}}^+\equiv\Delta S_{\mathrm{T}}[\ell^+,K_S]=\mathrm{S}[K_L,\ell^-]-\mathrm{S}[\ell^+,K_S]\,, \tag{4.8}$$

$$\Delta S_{\mathrm{CP}}^+\equiv\Delta S_{\mathrm{CP}}[\ell^+,K_S]=\mathrm{S}[\ell^-,K_S]-\mathrm{S}[\ell^+,K_S]\,, \tag{4.9}$$

$$\Delta S_{\mathrm{CPT}}^+\equiv\Delta S_{\mathrm{CPT}}[\ell^+,K_S]=\mathrm{S}[K_L,\ell^+]-\mathrm{S}[\ell^+,K_S]\,, \tag{4.10}$$

and the corresponding ones for the $\cos(\Delta M t)$ terms, are the six independent asymmetries that can be constructed (with the same notation of reference [1] for easy comparison). To illustrate the difference among asymmetries which would be equivalent in a CPT invariant world, one can write (expanding to linear order in $\mathrm{Re}(\theta)$, $\mathrm{Im}(\theta)$):

$$\Delta S^{+}_{\mathrm{T}} \simeq S_{K_S} - S_{K_L} - \mathrm{Re}(\theta)(S_{K_S}R_{K_S} + S_{K_L}R_{K_L}) + \mathrm{Im}(\theta)(S^2_{K_S} - S^2_{K_L} + C_{K_S} + C_{K_L}), \quad (4.11)$$

$$\Delta S^{+}_{\mathrm{CP}} \simeq 2S_{K_S} + 2\mathrm{Im}(\theta)(S^2_{K_S} - 1), \quad (4.12)$$

No matter whether CPT Violation is expected to be small, conceptually it is very important to emphasize that $\Delta S^{+}_{\mathrm{T}} \neq \Delta S^{+}_{\mathrm{CP}}$ for several reasons. For $\Delta S^{+}_{\mathrm{T}}$ to be truly T violating, eq. (3.12) should be fulfilled. Then, the dominant term in eqs. (4.11)–(4.12) should be equal: $S_{K_S} - S_{K_L} = 2S_{K_S}$. But, in general, $\Delta S^{+}_{\mathrm{T}}$ and $\Delta S^{+}_{\mathrm{CP}}$ differ by terms that are CPT violating and CP invariant in $\Delta S^{+}_{\mathrm{T}}$, and by terms that are CPT violating and T invariant in $\Delta S^{+}_{\mathrm{CP}}$. Only the pieces that do not depend on θ are identical. See [9] for an extended discussion including $\Delta C^{+}_{\mathrm{T}} \neq \Delta C^{+}_{\mathrm{CP}}$ and the CPT asymmetries.

5. Genuine asymmetry parameters

Although the reduced intensity $\hat{\mathrm{I}}(f,g;t)$ involves three coefficients $\mathscr{C}_h$, $\mathscr{C}_c$ and $\mathscr{S}_c$, the BaBar analysis [1] focused on the ratios $\mathrm{C} = \mathscr{C}_c/\mathscr{C}_h$ and $\mathrm{S} = \mathscr{S}_c/\mathscr{C}_h$ in eq. (4.2). From the experimental point of view those ratios might be more appropriate, but the theoretical side, access to the three independent coefficients would be more desirable: for instance, while an asymmetry in the ratios does imply a symmetry violation, no asymmetry in the ratios may nevertheless come from asymmetries in both the numerator and the denominator. Obtaining the three independent coefficients $\mathscr{C}_h$, $\mathscr{C}_c$ and $\mathscr{S}_c$ for each pair of decay channels might be particularly interesting for asymmetries in the ratios with values that are, within uncertainties, compatible with zero, like e.g. CPT asymmetries. Fortunately, using input information for $|q/p|$ or, equivalently δ, it can be achieved. We have

$$\mathscr{C}_h[\ell^{\pm},K_{S,L}] + \mathscr{C}_c[\ell^{\pm},K_{S,L}] = \frac{(1\pm\delta)(1\mp C_{K_{S,L}})}{2(1-\delta C_{K_{S,L}})} = \mathscr{C}_h[\ell^{\pm},K_{S,L}]\left(1+\mathrm{C}[\ell^{\pm},K_{S,L}]\right). \quad (5.1)$$

Equation (5.1) is interpreted in the following way: while $\mathrm{C}[\ell^{\pm},K_{S,L}]$ and $C_{K_{S,L}}$ will be constrained or extracted from data, including information on δ, we can compute $\mathscr{C}_h[\ell^{\pm},K_{S,L}]$, and thus $\mathscr{C}_c[\ell^{\pm},K_{S,L}]$ and $\mathscr{S}_c[\ell^{\pm},K_{S,L}]$ separately. One can then build T, CP and CPT complete time-dependent asymmetries analog to eqs. (4.4), (4.5) and (4.6),

$$\mathscr{A}_{\mathrm{S}}(t) = e^{-\Gamma t}\left\{\Delta\mathscr{C}_h^{\mathrm{S}} + \Delta\mathscr{C}_c^{\mathrm{S}}\cos(\Delta M t) + \Delta\mathscr{S}_c^{\mathrm{S}}\sin(\Delta M t)\right\}, \qquad \mathrm{S} = \mathrm{T},\mathrm{CP},\mathrm{CPT}. \quad (5.2)$$

We refer to $\Delta\mathscr{C}_h^{\mathrm{S}}$, $\Delta\mathscr{C}_c^{\mathrm{S}}$ and $\Delta\mathscr{S}_c^{\mathrm{S}}$ in these asymmetries as "genuine asymmetry parameters" since they collect the full time-dependent difference of probabilities in transitions among meson states given in eq. (3.2). For detailed expressions expanded up to linear order in θ and δ and a detailed discussion of the physical interpretation of (5.1) we refer to [9].

6. Genuine T-reverse and fake asymmetries

In section 3 we have discussed how asymmetries like eqs. (4.4) and (4.6) are "contaminated" through contributions which are not truly T-violating (this also applies to the genuine asymmetry parameters from section 5). It occurs when the conditions in eq. (3.12) are not fulfilled. The challenge is to disentangle *fake* effects in T and CPT asymmetries due to deviations from the requirements of eq. (3.12). The reasoning is illustrated using, for example, the asymmetry $\Delta\mathscr{S}_c^{\mathrm{T}}$. First, we remind that in terms of all parameters – δ, ρ, β, ε_ρ and ε_β in eq. (3.10), plus the complex θ parameter –, $\Delta\mathscr{S}_c^{\mathrm{T}}$ is simply a function $\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\varepsilon_\rho,\varepsilon_\beta,\delta,\theta)$. $\Delta\mathscr{S}_c^{\mathrm{T}}$ would be a *true* T-violation asymmetry if $\varepsilon_\rho=\varepsilon_\beta=0$ and $\rho=1$ (eq. (3.12)). It is then possible to do the following separation, at each point in parameter space, when performing a fit to the BaBar observables:

$$\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\varepsilon_\rho,\varepsilon_\beta,\delta,\theta)=\left[\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\varepsilon_\rho,\varepsilon_\beta,\delta,\theta)-\Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)\right]+\Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)\,. \tag{6.1}$$

The term within square brackets, $\Delta\mathscr{S}_c^{\mathrm{T}}(\rho,\beta,\varepsilon_\rho,\varepsilon_\beta,\delta,\theta)-\Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)$ has exactly the desired properties for the *fake* contribution: independently of β, δ and θ, it vanishes when the conditions eqs. (3.9) and (3.12) are fulfilled. Then, the last term, $\Delta\mathscr{S}_c^{\mathrm{T}}(1,\beta,0,0,\delta,\theta)$ is the truly T-violating contribution, the *genuine T-reverse* one. It is then possible to quantify the amounts of fake and genuine T-reverse contributions to T and CPT asymmetries like the BaBar ones $\Delta S_{\mathrm{T}}^{+}$, $\Delta C_{\mathrm{T}}^{+}$, $\Delta S_{\mathrm{CPT}}^{+}$, $\Delta C_{\mathrm{CPT}}^{+}$, and also, of course, to the T and CPT genuine asymmetry parameters involving the individual $\mathscr{C}_h$, $\mathscr{C}_c$ and $\mathscr{S}_c$ coefficients. In terms of δ, ρ, β, ε_ρ and ε_β, the genuine T-reverse asymmetries are obtained for

$$\begin{Bmatrix} C_{K_S} \\ C_{K_L} \end{Bmatrix}\to\delta,\quad \begin{Bmatrix} S_{K_S} \\ -S_{K_L} \end{Bmatrix}\to-\sqrt{1-\delta^2}\sin 2\beta,\quad \begin{Bmatrix} R_{K_S} \\ -R_{K_L} \end{Bmatrix}\to\sqrt{1-\delta^2}\cos 2\beta\,. \tag{6.2}$$

7. Results

With the information on the $\mathrm{C}[\ell^\pm,K_{S,L}]$ and $\mathrm{S}[\ell^\pm,K_{S,L}]$ coefficients provided in [1], including full covariance information and separate statistical and systematic uncertainties, supplemented with information on $|q/p|$ from [17], we perform a fit in terms of $\{\mathrm{Re}(\theta),\mathrm{Im}(\theta),\delta,\rho,\beta,\varepsilon_\rho,\varepsilon_\beta\}$. We can also address a more restricted situation where no wrong flavour decays (i.e. $\Delta F=\Delta Q$) are allowed in $B_d^0,\bar{B}_d^0\to J/\Psi K_{S,L}$, that is imposing $\lambda_{K_S}+\lambda_{K_L}=0$: in terms of the previous set of parameters, that means setting $\rho=1$ and $\varepsilon_\beta=0$. For a details of the fit procedure, more detailed results, tables and plots, we refer to [9], and focus here on a few significant aspects. Starting with the CPT violating θ parameter, from the fits,

$$\begin{Bmatrix} \mathrm{Re}(\theta)=\pm(5.92\pm3.03)\times10^{-2} \\ \mathrm{Im}(\theta)=(0.22\pm1.90)\times10^{-2} \end{Bmatrix}$$

$$\text{and}\quad\begin{Bmatrix} \mathrm{Re}(\theta)=\pm(3.92\pm1.43)\times10^{-2} \\ \mathrm{Im}(\theta)=(-0.22\pm1.64)\times10^{-2} \end{Bmatrix}\ \text{with}\ \lambda_{K_S}+\lambda_{K_L}=0, \tag{7.1}$$

which improve significantly on the uncertainty of the real part quoted by the Particle Data Group (PDG) in [17], based on BaBar [18, 19] and Belle [20] results. For similar results based on the samed Babar data, see [21]. Figure 1 shows the result of the fit for the imaginary vs. real part of θ. To illustrate the difference between the genuine T-reverse and CP asymmetry parameters figures 2(a), 2(b), show true T-reverse asymmetries versus CP asymmetries for ΔS and for the genuine

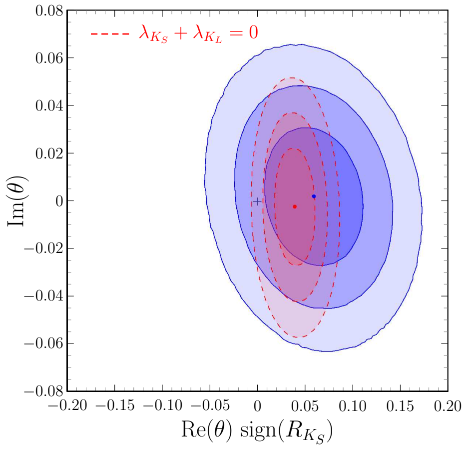
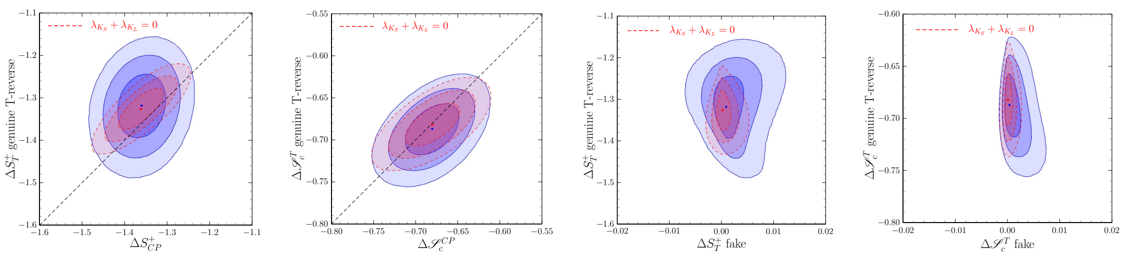


Figure 1: $\mathrm{Im}(\theta)$ vs. $\mathrm{Re}(\theta)\mathrm{sign}(R_{K_S})$ in the full fit (blue regions, solid contours), and in the fit with $\lambda_{K_S} + \lambda_{K_L} = 0$ (red regions, dashed contours); dark to light regions correspond, respectively, to two-dimensional 68%, 95% and 99% C.L..

asymmetry coefficients $\Delta\mathscr{S}_c$ and $\Delta\mathscr{C}_c$. The dashed diagonal line would correspond to strict equality among both observables. In figures 2(c), 2(d), we show genuine T-reverse vs. fake contributions for $\Delta S_{\mathrm{T}}^{+}$ and for the genuine asymmetry parameter $\Delta\mathscr{S}_c^{\mathrm{T}}$. This is particularly relevant for the $\Delta S_{\mathrm{T}}^{+}$ BaBar asymmetry since sizable fake contributions could have weakened the evidence for the time reversal violation observation independent of CP. It is clear that the T-fake contributions to $\Delta S_{\mathrm{T}}^{+}$ and $\Delta\mathscr{S}_c^{\mathrm{T}}$ are below the percent level.



(a) ΔS. (b) $\Delta\mathscr{S}_c$. (c) $\Delta S_{\mathrm{T}}^{+}$. (d) $\Delta\mathscr{S}_c^{\mathrm{T}}$.

Figure 2: Genuine T-reverse vs. CP asymmetries and genuine T-reverse vs. fake contributions.

8. ω effect

Quantum gravity or in general deviations from any of the three assumptions – Lorentz invariance, locality of interactions and unitary – of the CPT theorem, may lead to (independent) violations of CPT and, accordingly, to an ill-defined CPT quantum mechanical operator [22–24]. The CPT breaking associated to ill-defined particle-antiparticle states modifies the EPR correlation and produces the ω-effect [25–27]. Treating it perturbatively, i.e. still talking the language of B_d^0, $\bar{B}_d^0$, the perturbed two-particle state will contain a component of the “wrong” symmetry at the instant of their production by the decay of $\Upsilon(4S)$: instead of eq. (2.8) it would read

$$|\Psi_0\rangle \propto |B_d^0\rangle|\bar{B}_d^0\rangle - |\bar{B}_d^0\rangle|B_d^0\rangle + \omega\left[|B_d^0\rangle|\bar{B}_d^0\rangle + |\bar{B}_d^0\rangle|B_d^0\rangle\right], \tag{8.1}$$

where $\omega = |\omega|e^{i\Omega}$ is a complex CPT-breaking parameter [25,26], associated with the non-identical particle nature of the neutral meson and antimeson states. The presence of an ω-effect weakens the entanglement of the initial state (8.1), as follows from the fact that when $\omega = \pm 1$ the state simply reduces to a product state, whilst the state is fully entangled when $\omega = 0$. Furthermore, the coefficients in the general time-dependent intensity (2.11) are modified,

$$\mathscr{C}_h[f,g] \to \mathscr{C}_h[f,g], \qquad \mathscr{C}_c[f,g] \to \mathscr{C}_c[f,g], \qquad \mathscr{S}_c[f,g] \to \mathscr{S}_c[f,g], \tag{8.2}$$

where now, contrary to eq. (2.12),

$$\mathscr{C}_h[f,g] \neq \mathscr{C}_h[g,f], \qquad \mathscr{C}_c[f,g] \neq \mathscr{C}_c[g,f], \qquad \mathscr{S}_c[f,g] \neq -\mathscr{S}_c[g,f]. \tag{8.3}$$

The redundancy due to the different time orderings in the Babar data for the determination of $\mathscr{C}_h[f,g]$, $\mathscr{C}_c[f,g]$ and $\mathscr{S}_c[f,g]$ does now provide sensitivity to the presence of ω: $\mathrm{C}[\ell^\pm,g] - \mathrm{C}[g,\ell^\pm]$ and $\mathrm{S}[\ell^\pm,g] + \mathrm{S}[g,\ell^\pm]$ are linear in both the real and imaginary parts of ω. With these basic ingredients, one can develop a full analysis along the lines of section 7, but including ω. For detailed discussions and results, we refer the interested reader to reference [28], from which we only mention here that one can indeed extract or bound both the real and imaginary parts of ω, as Fig. 3 shows: for the first time for $\mathrm{Im}\,(\omega)$, and with less precision than analyses focusing only on flavour specific channels in the case of $\mathrm{Re}\,(\omega)$ [27].

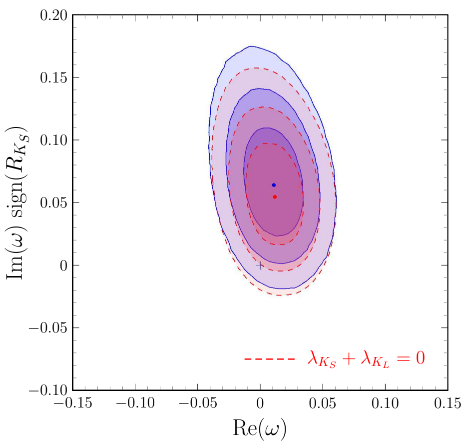


Figure 3: $\mathrm{sign}(R_{K_S})\mathrm{Im}\,(\omega)$ vs. $\mathrm{Re}\,(\omega)$ in the full fit (blue regions, solid contours), and in the fit with $\lambda_{K_S} + \lambda_{K_L} = 0$ (red regions, dashed contours).

Conclusions

We have discussed genuine T, CP and CPT asymmetry parameters at B-factories, the separation of fake contributions to T and CPT violating observables and obtained from a fit to existing data, improved bounds on the CPT violating parameters $\mathrm{Re}\,(\theta)$ in B_d^0–$\bar{B}_d^0$ mixing and $\mathrm{Im}\,(\omega)$.

Acknowledgments

The author thanks the organization of the Corfu 2017 Summer Institute for the excellent development of the workshop. MN also acknowledges support from *Fundação para a Ciência e a Tecnologia* (FCT, Portugal) through postdoctoral grant SFRH/BPD/112999/2015 and through the project CFTP-FCT Unit 777 (UID/FIS/00777/2013) which are partially funded through POCTI (FEDER), COMPETE, QREN and EU.

Bibliography

References

[1] **BaBar** Collaboration, J. P. Lees *et al.*, *Observation of Time Reversal Violation in the B^0 Meson System*, *Phys. Rev. Lett.* **109** (2012) 211801, [1207.5832].

[2] M. Bañuls and J. Bernabéu, *CP, T and CPT versus temporal asymmetries for entangled states of the B(d) system*, *Phys.Lett.* **B464** (1999) 117–122, [hep-ph/9908353].

[3] M. Bañuls and J. Bernabéu, *Studying indirect violation of CP, T and CPT in a B factory*, *Nucl.Phys.* **B590** (2000) 19–36, [hep-ph/0005323].

[4] J. Bernabéu, F. Martínez-Vidal, and P. Villanueva-Pérez, *Time Reversal Violation from the entangled B0-antiB0 system*, *JHEP* **1208** (2012) 064, [1203.0171].

[5] L. Wolfenstein, *Violation of time reversal invariance in K0 decays*, *Phys.Rev.Lett.* **83** (1999) 911–912.

[6] M. C. Bañuls and J. Bernabéu, *The CP conserving direction*, *JHEP* **06** (1999) 032, [hep-ph/9807430].

[7] E. Applebaum, A. Efrati, Y. Grossman, Y. Nir, and Y. Soreq, *Subtleties in the BABAR measurement of time-reversal violation*, *Phys. Rev.* **D89** (2014), no. 7 076011, [1312.4164].

[8] J. Bernabéu, F. J. Botella, and M. Nebot, *Novel T-Violation observable open to any pair of decay channels at meson factories*, *Phys. Lett.* **B728** (2014) 95–98, [1309.0439].

[9] J. Bernabéu, F. J. Botella, and M. Nebot, *Genuine T, CP, CPT asymmetry parameters for the entangled B_d system*, *JHEP* **06** (2016) 100, [1605.03925].

[10] V. Weisskopf and E. Wigner, *Over the natural line width in the radiation of the harmonius oscillator*, *Z.Phys.* **65** (1930) 18–29.

[11] T. Lee, R. Oehme, and C.-N. Yang, *Remarks on Possible Noninvariance Under Time Reversal and Charge Conjugation*, *Phys.Rev.* **106** (1957) 340–345.

[12] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation, International Series of Monographs on Physics, Oxford University Press*.

[13] J. P. Silva, *On the use of the reciprocal basis in neutral meson mixing*, *Phys. Rev.* **D62** (2000) 116008, [hep-ph/0007075].

[14] A. Einstein, B. Podolsky, and N. Rosen, *Can quantum mechanical description of physical reality be considered complete?*, *Phys. Rev.* **47** (1935) 777–780.

[15] J. Bernabéu, F. J. Botella, and M. Nebot, *Novel T-violation observable open to any decay channel at meson factories*, *J. Phys. Conf. Ser.* **631** (2015), no. 1 012017.

[16] Y. Grossman, A. L. Kagan, and Z. Ligeti, *Can the CP asymmetries in $B \to \psi K_S$ and $B \to \psi K_L$ differ?*, *Phys. Lett.* **B538** (2002) 327–334, [hep-ph/0204212].

[17] **Particle Data Group** Collaboration, K. A. Olive *et al.*, *Review of Particle Physics*, *Chin. Phys.* **C38** (2014) 090001.

[18] **BaBar** Collaboration, B. Aubert *et al.*, *Limits on the decay rate difference of neutral−B mesons and on CP, T, and CPT violation in $B^0\overline{B}^0$ oscillations*, *Phys. Rev.* **D70** (2004) 012007, [hep-ex/0403002].

[19] **BaBar Collaboration** Collaboration, B. Aubert *et al.*, *Search for T, CP and CPT violation in B0 anti-B0 mMixing with inclusive dilepton events*, *Phys.Rev.Lett.* **96** (2006) 251802, [hep-ex/0603053].

[20] T. Higuchi *et al.*, *Search for Time-Dependent CPT Violation in Hadronic and Semileptonic B Decays*, *Phys. Rev.* **D85** (2012) 071105, [1203.0930].

[21] **BaBar** Collaboration, J. P. Lees *et al.*, *Tests of CPT symmetry in $B^0 - \overline{B}^0$ mixing and in $B^0 \to c\bar{c}K^0$ decays*, *Phys. Rev.* **D94** (2016), no. 1 011101, [1605.04545].

[22] R. M. Wald, *Quantum Gravity and Time Reversibility*, *Phys. Rev.* **D21** (1980) 2742–2755.

[23] J. R. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, *CPT violation in string modified quantum mechanics and the neutral kaon system*, *Int. J. Mod. Phys.* **A11** (1996) 1489–1508, [hep-th/9212057].

[24] J. R. Ellis, J. L. Lopez, N. E. Mavromatos, and D. V. Nanopoulos, *Precision tests of CPT symmetry and quantum mechanics in the neutral kaon system*, *Phys. Rev.* **D53** (1996) 3846–3870, [hep-ph/9505340].

[25] J. Bernabéu, N. E. Mavromatos, and J. Papavassiliou, *Novel type of CPT violation for correlated EPR states*, *Phys. Rev. Lett.* **92** (2004) 131601, [hep-ph/0310180].

[26] E. Álvarez, J. Bernabéu, N. Mavromatos, M. Nebot, and J. Papavassiliou, *CPT violation in entangled B0 - anti-B0 states and the demise of flavor tagging*, *Phys.Lett.* **B607** (2005) 197–203, [hep-ph/0410409].

[27] E. Álvarez, J. Bernabéu, and M. Nebot, *Delta t-dependent equal-sign dilepton asymmetry and CPTV effects in the symmetry of the B0- anti-B0 entangled state*, *JHEP* **0611** (2006) 087, [hep-ph/0605211].

[28] J. Bernabeu, F. J. Botella, N. E. Mavromatos, and M. Nebot, *The signal of ill-defined CPT weakening entanglement in the B_d system*, *Eur. Phys. J.* **C77** (2017), no. 12 865, [1612.05652].