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Anomalous gauge-U(1), 't Hooft mechanism, and "invisible" QCD axion from string

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We are players in a theory theater. A play we show here is the 't Hooft mechanism: if a gauge symmetry and a global symmetry are broken by one complex scalar field by the Higgs mechanism then there survives a global symmetry below the breaking scale. In the string compactification, it is realized when an anomalous gauge U(1) symmetry is created. This is a good example of obtaining a global PQ symmetry at an intermediate scale, realizing an "invisible" axion with mass around 10^{-4} eV for a cold dark matter candidate.

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1. ’t Hooft mechanism

The ’t Hooft mechanism is: If a gauge symmetry and a global symmetry are broken by one complex scalar field by the Higgs mechanism [1], then the gauge symmetry is broken and a global symmetry remains unbroken.

We can consider the following cases for two phase directions: Two phases accompany two generators Q_1 and Q_2 for which we can consider three cases,

(i) both Q_1 and Q_2 are generators of two global symmetries,

(ii) both Q_1 and Q_2 are generators of two gauge symmetries,

(iii) one is a gauge generator and the other is a global generator.

The ’t Hooft mechanism belongs to (iii), which is an elementary and simple concept, but it seems that it is not known widely in the community. It is obvious that the gauge symmetry is broken because the corresponding gauge boson obtains mass, and only one phase or pseudoscalar is absorbed to the gauge boson. Then, there remains one continuous direction. To see this clearly, let us introduce a field ϕ on which charges $Q_{\rm gauge}$ and $Q_{\rm global}$ act, which define the local and global transformations,

$$\phi \to e^{i\alpha(x)Q_{\rm gauge}} e^{i\beta Q_{\rm global}}\phi. \tag{1.1}$$

Redefining the local direction as $\alpha'(x) = \alpha(x) + \beta$, the transformations are

$$\phi \to e^{i\alpha'(x)Q_{\rm gauge}} e^{i\beta (Q_{\rm global}-Q_{\rm gauge})}\phi. \tag{1.2}$$

So, the $\alpha'(x)$ direction becomes the longitudinal mode of heavy gauge boson. Now, the charge $Q' = Q_{\rm global} - Q_{\rm gauge}$ is reinterpreted as the new global charge and is not broken by the VEV, $\langle\phi\rangle$. The kinetic energy term of ϕ is

$$\left|D_\mu\phi\right|^2 = \left|(\partial_\mu - igQ'A^\mu)\phi\right|^2 = \frac{g^2}{2}Q'^2v^2\left(A_\mu - \frac{1}{gQ'v}\partial_\mu a_\phi\right)^2 = \frac{g^2}{2}Q'^2v^2\left(A'_\mu\right)^2, \tag{1.3}$$

where $\phi = (v+\rho)e^{ia_\phi/v}$ and $v = \langle\phi\rangle$. A'_μ is the heavy gauge boson and there is no kinetic energy term of a_ϕ in the right hand side of Eq. (1.3). This is the essence of the ’t Hooft mechanism. This theorem has a profound effect in obtaining the intermediate scale “invisible” axion from string compactification [2, 3, 4].

The ’t Hooft mechanism can be applied at any step. When one global symmetry survives below a high energy scale, we consider another gauged U(1) and one more complex scalar to break two U(1)’s. Then, one global symmetry survives. There are many anomaly free U(1) gauge symmetries from string compactification. The global symmetry from the ’t Hooft mechanism at the string scale goes through breaking of these anomaly free U(1) gauge symmetries at the GUT scale.

2. Model-independent axion in string theory

The gravity anomaly in 10D requires 496 spin-$\frac{1}{2}$ fields. Possible non-Abelian gauge groups are rank 16 groups SO(32) and $\mathrm{E}_8\times\mathrm{E}'_8$. The anti-symmetric tensor field B_{MN} has the field strength

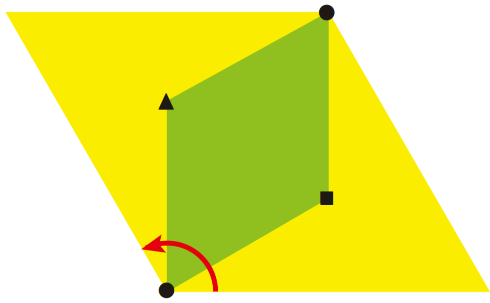

Figure 1: Three fixed points at the corners in the fundamental region (limegreen parallelogram) of $\mathbf{Z}_3$ orbifold.

H_{MNP}. The Green-Schwarz (GS) form [5] is $H = dB + \omega_{3Y} - \omega_{3L}$ in differentialform where ω_{3L} is the connection and $d\omega_{3Y} = \mathrm{tr}F^2$ with

$$\omega_{3Y} = \mathrm{tr}\left(AF - \frac{1}{3}A^3\right). \tag{2.1}$$

One needs the following counter-term to cancel the gauge and gravitational anomalies [5], in the example of $\mathrm{E}_8\times\mathrm{E}_8'$ heterotic string,

$$S_1' = \frac{c}{108\,000}\int\left\{30B[(\mathrm{tr}_1F^2)^2 + (\mathrm{tr}_2F^2)^2 - \mathrm{tr}_1F^2\mathrm{tr}_2F^2] + \cdots\right\} \tag{2.2}$$

where the subscript 1 (2) corresponds to E_8 (E_8'). The kinetic energy term in 10D is

$$-\frac{3\kappa^2}{2g^4\varphi^2}H_{MNP}H^{MNP} \tag{2.3}$$

where $M,N,P = 1,2,\cdots,10$. In 10D gauge theory, there is the hexagon anomaly. It is cancelled by the GS term. One may look this in the following way. The 10D supergravity quantum field theory with SO(32) and $\mathrm{E}_8\times\mathrm{E}_8'$ gauge groups has gauge and gravitational anomalies. Let us believe that string theory is consistent, effectively removing all divergences, *i.e.* removing all anomalies. Therefore, the point particle limit of 10D string theory should not allow any anomalies. There must be some term in the string theory removing all these anomalies. It is the GS term. Suppose the fermion theory with the global group G which is spontaneously broken to $G' \in G$. Below the spontaneous symmetry scale, we integrate out the fermions, resulting with the Goldstone bosons corresponding to the coset space G/G'. In this regard, one may remember the Wess-Zumino term [6, 7], removing $\mathrm{SU(3)}_L\times\mathrm{SU(3)}_R$ global anomalies in the strongly interacting meson theory, by some term involving pseudoscalar fields. For the GS term, already there is the field B_{MN} needed for the anomaly cancellation. The dual of $H_{\mu\nu\rho}$ in the tangential space $(\mu,\nu,\rho = 1,2,3,4)$ is the so-called model-independent (MI) axion [8],

$$H_{\mu\nu\rho} = M_{\rm MI}\,\varepsilon_{\mu\nu\rho\sigma}\partial^\sigma a_{\rm MI}. \tag{2.4}$$

In the orbifold compactification, e.g. at a $\mathbf{Z}_3$ torus, there are three fixed points as shown in the limegreen fundamental region in Fig. 1. The torus is the yellow parallelogram and the area of the

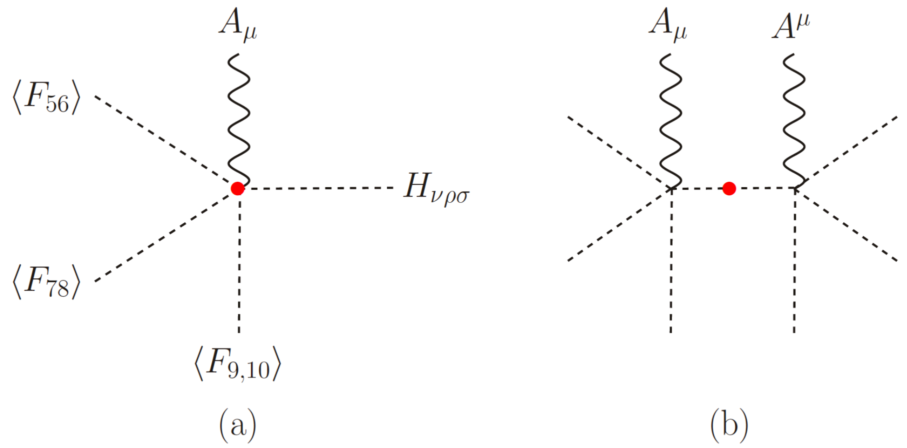

Figure 2: Anomalous U(1) in string compactification: (a) the mixing term of the anomalous U(1) gauge boson A_μ with the anti-symmetric tensor field $H_{\nu\rho\sigma}$, and (b) the gauge boson mass term.

fundamental region is $\frac{1}{3}$ (due to $\mathbf{Z}_3$) of the torus. Here, we interpret that the flux is located at the fixed points such as the bullet, black triangle and black square in Fig. 1. We take the limit of string loop almost sitting at the fixed points. It involves the 2nd rank antisymmetric field B_{MN}. A two dimensional torus of Fig. 1 gives $\langle F_{ij}\rangle$. A product of three two-tori fluxes leads to

$$\mathscr{L}_1' \propto \frac{c'}{108\,000}\{H_{\mu\nu\rho}A_\sigma\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{ijklmn}\langle F_{ij}\rangle\langle F_{kl}\rangle\langle F_{mn}\rangle+\cdots\} \to \frac{1}{3!}H_{\mu\nu\rho}A_\sigma\varepsilon^{\mu\nu\rho\sigma}. \tag{2.5}$$

There is the kinetic energy term

$$\frac{1}{2\cdot 3! M_{MI}^2}H_{\mu\nu\rho}H^{\mu\nu\rho}. \tag{2.6}$$

With the duality relation (2.4), it gives the coupling

$$M_{MI}A^\mu\partial_\mu a_{MI}+\frac{1}{2}(\partial_\mu a_{MI})^2+\frac{1}{2\cdot 3!}A_\mu A^\mu, \tag{2.7}$$

as depicted in Fig. 2 (a) from which we have the mass term of A_μ shown in Fig. 2 (b). Summing up these terms, we have

$$\to \frac{1}{2}M_{MI}^2\left(A_\mu+\frac{1}{M_{MI}}\partial_\mu a_{MI}\right)^2, \tag{2.8}$$

which is the 't Hooft mechanism working in string compactification. Namely, the continuous direction $a_{MI}\to a_{MI}+$costant survives as a global symmetry at low energy, providing a necessary symmetry for the "invisible" axion at the intermediate scale. Two continuous directions in string compactification are U(1)$_{\rm anom}$ gauge symmetry and the shift $a_{MI}\to a_{MI}+$costant. The breaking terms can be considered as the value M_{MI} and a complex scalar ϕ which appears in the Fayet-Iliopoulos term in string compactification [9]. M_{MI} and $\langle\phi\rangle$ break both symmetries, which may be the reason that many believe "there does not survive a global symmetry below the GUT scale." However, one can easily figure out this problem in a hierarchcal set up [4]. Suppose that $\langle\phi\rangle \ll M_{MI}$. Below the scale M_{MI}, there survives a global symmetry out of U(1)$_{\rm anom}$ gauge symmetry and the shift $a_{MI} \to a_{MI} +$ costant. Now consider $\langle\phi\rangle$ at the GUT scale. Certainly, it breaks the global symmetry, but $\langle\phi\rangle$ also carries an anomaly free gauge charge and present in the compactification. Below $\langle\phi\rangle$, again a global symmetry survives. Even if $\langle\phi\rangle \approx M_{MI}$ [10], counting the number of degrees does not change and the appearance of a global symmetry is realized. This process continues until all anomaly free gauge groups, except U(1)$_Y$ of the Standard Model (SM), are broken at the GUT scale. Then, there results two U(1) symmetries, the gauge U(1)$_Y$ and a global U(1)$_{\rm PQ}$. U(1)$_{\rm PQ}$ is broken by a SM singlet σ at the intermediate scale. U(1)$_Y$ is broken by a Higgs doublet at the electroweak scale.

Figure 3 shows two relevant points from string compactification. If there does not appear an anomalous U(1) gauge symmetry from string compactification, then the axion decay constant is determined to be around $10^{15-16}\,$GeV [11] which is shown as the white square. On the other hand, if there appears an anomalous U(1) gauge symmetry from string compactification [9], then the axion decay constant can be lowered to the intermediate scale. It was argued that the ratio between intermedtiate and compactification scales $f_{\rm int\ scale}/M_{\rm comp} \approx 10^{-7}$ is not considered to be a fine tuning [4]. This is because $f_{\rm int\ scale}$ is determined by some scalar field breaking the global symmetry at the intermediate scale rather than by some VEV ratios at the compactification scale.

3. Approximate global symmetry

Symmetry is beautiful not only in the world which we see every day but also at very small scales where the fundamental laws respect symmetries. The most studied symmetry is the simplest one: parity or $\mathbf{Z}_2$ symmetry. In many beautiful parity symmetric objects, we observe that parity is slightly broken. In weak interactions, parity is slightly broken. When we consider symmetries, therefore, it is a good strategy to consider feeble breaking of such symmetries except for the well-known gauge symmetries, U(1)$_{\rm em}$ and QCD.

Figure 4 illustrates interaction terms in the potential V respecting and breaking some symmetries. Terms symbolized by the vertical (red and lavender) column correspond to some discrete symmetry. Terms symbolized by the horizontal (green and lavender) column correspond to some global symmetry. Global symmetries are differentiated from the discrete symmetry in that all global symmetries are broken at least by gravitational interaction [12, 13]. For discrete symmetries, there is no such a gravitational enigma. Therefore, we distinguished them in Fig. 4. The common laventer part respects both discrete and global symmetries. So, out of all terms respected by the discrete symmetry, keep only some lowest order terms symbolized by the lavender square. Then, the discrete symmetry is promoted to a global symmetry shown as the lavender plus green parts. The green part, respecting the global symmetry, denotes terms breaking the discrete symmetry. The red part, respecting the discrete symmetry, denotes terms breaking the global symmetry.

Consider the following potential composed of two complex scalar fields σ_1 and σ_2, respecting the discrete symmetry $D: \sigma_1 \to -\sigma_1$, and $\sigma_2 \to -\sigma_2$,

$$V = V_0 + \Delta V \tag{3.1}$$

where

$$V_0 = m_1^2\sigma_1^*\sigma_1 + m_2^2\sigma_2^*\sigma_2 + \lambda_1(\sigma_1^*\sigma_1)^2 + \lambda_2(\sigma_2^*\sigma_2)^2 + \lambda_3[(\sigma_1^*)^2\sigma_2^4 + \text{h.c.}] \tag{3.2}$$

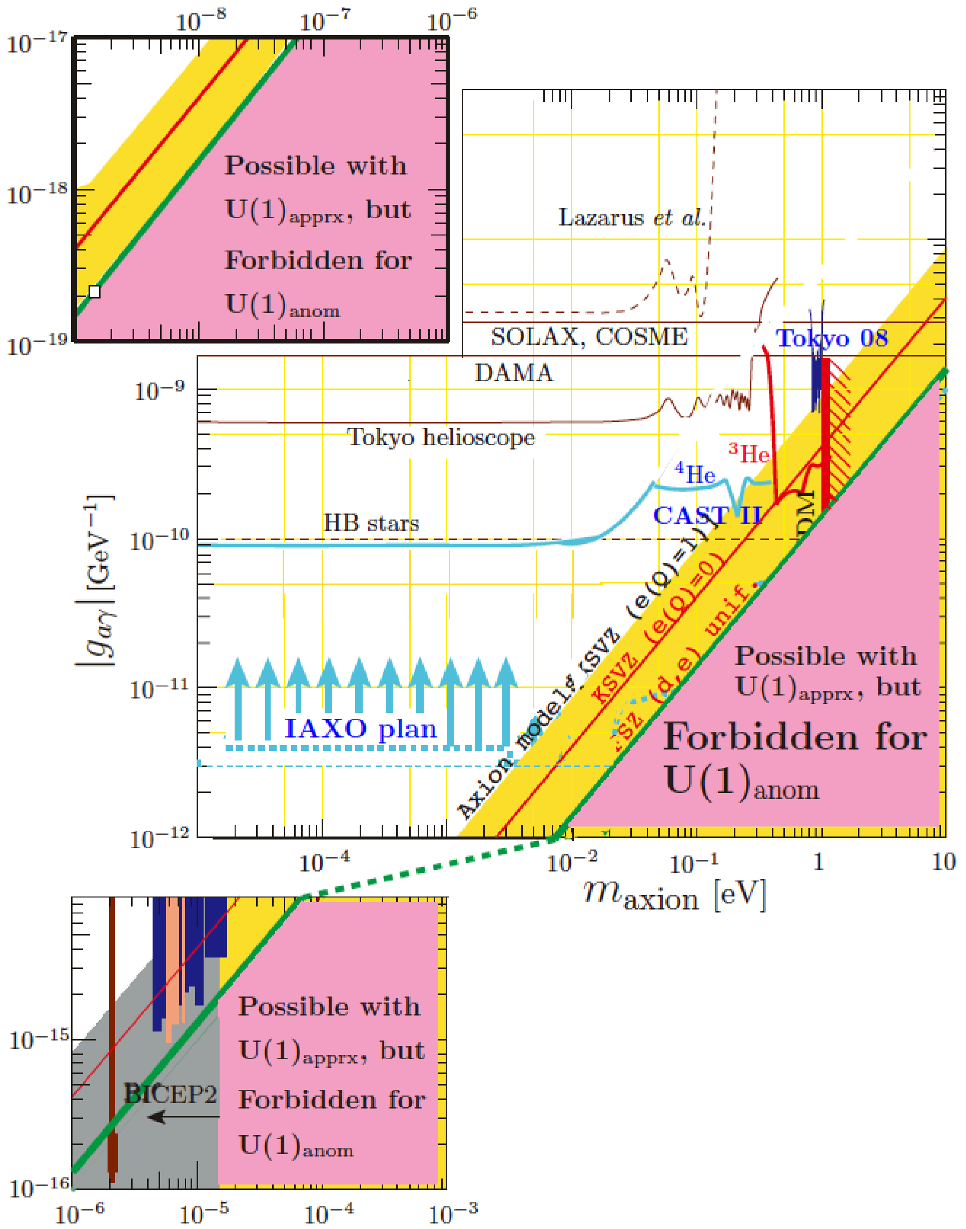

Figure 3: A summary of axion search expermiments and points of "invisible" axions from string. The white square around $m_a \approx 10^{-9}$ eV is the case without an anomalous U(1) [11], and the yellow triangle in $10^{[-5,-4]}$ eV region is with an anomalous U(1).

and

$$\Delta V = \lambda_4(\sigma_1^* \sigma_1^* \sigma_2 \sigma_2 + \text{h.c.}) \tag{3.3}$$

V_0 respects a global symmetry $\sigma_1 \to e^{2i\alpha}\sigma_1$ and $\sigma_2 \to e^{i\alpha}\sigma_2$. This global symmetry is broken by ΔV. In this example, V_0 belongs to the lavender square and ΔV belongs to the red. Terms in the green may contain $\mu(\sigma_1^* \sigma_2 \sigma_2 + \text{h.c.})$.

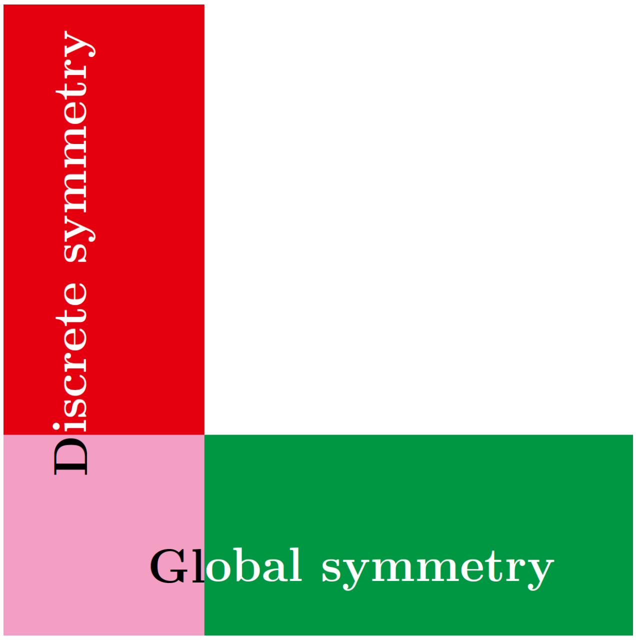

Figure 4: A cartoon for representing effective terms, $V_0 + \Delta V$.

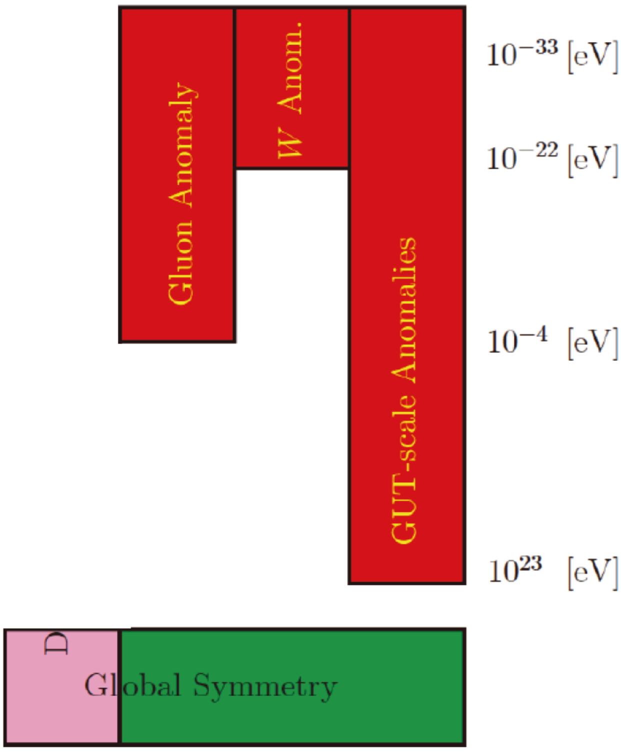

Figure 5: A cartoon for anomaly breaking.

Breaking global symmetries are also appearing as global anomalies from the fermion loops [15]. In this case, it is not present in the form of ΔV of Fig. 4. The breaking term appears in the form of $aG_{\mu\nu}\tilde{G}^{\mu\nu}$ where $G_{\mu\nu}$ is the field strength of gauge boson G_μ and a is the global symmetry direction. In 4D it is called the triangle anomaly, and in 10D it is called the hexagon anomaly. With the instanton solutions in non-Abelian gauge groups, the breaking terms are sketched in Fig. 5, showing that it is not related to ΔV. The most well-known example is the Peccei-Quinn (PQ) global symmetry which are broken by the QCD anomaly [16].

The important question is, "Even if one allows a discrete symmetry from string compactification, at which level is the effective global symmetry bfroken?" Several scales of the masses of the corresponding pseudoscalar particles are marked on the right hand side of Fig. 5.

Pseudoscalars arising from the anomaly breaking are widely used in cosmology. If the non-Abelian gauge group confines around the GUT scale, the pseudoscalar mass is of order $10^{23}\,$eV which is a good choice for the naural inflation [17, 18]. For QCD, the "invisible" axion mass of order $10^{-4}\,$eV is used for CDM [4]. For a ULA with the SU(2) gauge group in the SM, the estimated mass is of order $10^{-26}\,$eV, a bit too small compared to $10^{-22}\,$eV.

Some very very light pseudoscalars applicable in cosmology are excluding the possibility of non-Abelian anomaly, especially the QCD anomaly, as depicted in Fig. 4. For example, the mass scale $10^{-22}\,$eV is used for solving the problem of too many satellites in the N-body simulation [4]. These pseudoscalars are called ULAs [19]. The pseudoscalar mass of order $10^{-33}\,$eV is used for dark energy in the Universe [20]. Even though the required mass for the N-flation is very large, the N-flation [21] may belong here because it is not likely that there are numerous confining non-Abelian gauge groups beyond QCD [22].

The extremely light pseudoscalars such as ULAs and quintessential axions require that the mother global symmetries do not allow non-Abelian anomalies. This requirement interrelates the ULAs/quintessences with the QCD axion. Technically, it is achieved by defining the mother global symmetry at a sufficiently higher order toward a quintessential axion [23]. For a ULA, an example is presented from discrete symmetries [24]. This kind of definition of a global symmetry is well-known in the Kim-Nilles μ term in supersymmetric field theory [25]. It was required to introduce a SM singlet complex scalar σ toward an "invisible" axion. But, at dimension 3 superpotential, there is no term allowing a σ coupling to SM quarks to provide the global charge of σ. The required coupling is possible at dimension 4 superpotential level [25]. Breaking of the global symmetry is via the QCD anomaly, which is considered to be a kind of strong breaking, realizing an "invisible" axion. Similarly, global symmetries toward quintessential axions and ULAs can be defined at sufficiently high orders, but in these cases there are no anomaly terms, breaking them strongly. Breaking these global symmetries very feebly must occur at much more higher orders in ΔV, which enables one to obtain the extremely small masses of quintessential axions and ULAs.

4. Conclusion

In the compactification in string theory, if an anomalous gauge U(1) is created, then the 't Hooft mechanism works and a global PQ symmetry comes down to the low energy scale, realizing a CDM candidate, through an "invisible" axion with mass around $10^{-4}\,$eV, in the Universe.

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