Measurement of $\hat{q}$ in RHI collisions using di-hadron correlations

M. J. Tannenbaum

Physics Dept., 510c, Brookhaven National Laboratory, Upton, NY 11973-5000, USA
E-mail: mjt@bnl.gov

In the BDMPSZ model, the energy loss of an outgoing parton in a medium $-dE/dx$ is the transport coefficient $\hat{q}$ times $L$, the length traveled. This results in jet quenching, which is well established. However BDMPSZ also predicts an azimuthal broadening of di-jets also proportional to $\hat{q}L$ which has so far not been observed. The azimuthal width of the di-hadron correlations in p+p collisions, beyond the fragmentation transverse momentum, $j_T$, is dominated by $k_T$, the so-called intrinsic transverse momentum of a parton in a nucleon, which can be measured. The broadening should produce a larger $k_T$ in A+A than in p+p collisions. This presentation introduces the observation that the $k_T$ measured in p+p collisions for di-hadrons with $p_Tt$ and $p_Ta$ must be reduced to compensate for the energy loss of both the trigger and away parent partons when comparing to the $k_T$ measured with the same di-hadron $p_Tt$ and $p_Ta$ in A+A collisions. This idea is applied to a recent STAR di-hadron measurement in Au+Au at $\sqrt{s_{NN}} \approx 200$ GeV, Phys. Lett. B 760, 689 (2016), with result $<\hat{q}L> = 2.1 \pm 0.6$ GeV. This is more precise but in agreement with a theoretical calculation of $<\hat{q}L> = 14_{14}^{+42}$ GeV$^2$ using the same data. Assuming a length $<L> \approx 7$ fm for central Au+Au collisions the present result gives $\hat{q} \approx 0.30 \pm 0.09$ GeV$^2$/fm, in fair agreement with the JET collaboration result from single hadron suppression of $\hat{q} \approx 1.2 \pm 0.3$ GeV$^2$/fm at an initial time $\tau_0 = 0.6$ fm/c in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. There are several interesting details to be discussed: for a given $p_Tt$ the $<\hat{q}L>$ seems to decrease then vanish with increasing $p_Ta$; the di-jet spends a much longer time in the medium ($\approx 7$ fm/c) then $\tau_0 = 0.6$ fm/c which likely affects the value of $\hat{q}$ that would be observed.
1. Jet Quenching: the first QCD based prediction BDMPSZ [1]

The first prediction of how to detect the QGP was via $J/\Psi$ suppression [2] in 1986. However the first QCD based prediction for detecting the QGP was BDMPSZ Jet Quenching [1]. This is produced by the energy loss, via LPM coherent radiation of gluons, of an outgoing parton with color charge fully exposed in a medium with a large density of similarly exposed color charges (i.e. the QGP) (Fig. 1a). Jet quenching was observed quite early at RHIC by suppression of high $p_T$ $\pi^0$ [3], with lots of subsequent evidence (Fig. 1b). It is interesting to note that all identified hadrons generally have different $R_{AA}$ for $p_T \leq 5$ GeV/c but tend to converge to the same value for $p_T \geq 5$ GeV/c. The fact that direct-$\gamma$ are not suppressed indicates that suppression is a medium effect on outgoing color-charged partons as predicted by BDMPSZ [1].

![Figure 1: a) Schematic of $q + q$ scattering with scattered quarks losing energy in the medium. b) Suppression, $R_{AA}(p_T)$, for all identified particles so far measured by PHENIX in Au+Au central collisions at $\sqrt{s_{NN}} = 200$ GeV.](image)

1.0.1 But the BDMPSZ model has two predictions

(I) The energy loss of the outgoing parton, $-dE/dx$, per unit length ($x$) of a medium with total length $L$, is proportional to the total 4-momentum transfer-squared, $q^2(L)$, with the form:

$$-\frac{dE}{dx} \sim \alpha_s \langle q^2(L) \rangle = \alpha_s \mu^2 L/\lambda_{mfp} = \alpha_s \hat{q}L$$

(1.1)

where $\mu$, is the mean momentum transfer per collision, and the transport coefficient $\hat{q} = \mu^2/\lambda_{mfp}$ is the 4-momentum-transfer-squared to the medium per mean free path, $\lambda_{mfp}$.

(II) Additionally, the accumulated momentum-squared, $\langle p_{T,W}^2 \rangle$ transverse to a parton traversing a length $L$ in the medium is well approximated by

$$\langle p_{T,W}^2 \rangle \approx \langle q^2(L) \rangle = \hat{q}L$$

so that

$$\langle \hat{q}L \rangle /2 = \langle k_T^2 \rangle_{AA} - \langle k_T^2 \rangle_{pp}$$

(1.2)

since only the component of $\langle p_{T,W}^2 \rangle$ $\perp$ to the scattering plane affects $k_T$. This is called azimuthal broadening. Here (see Fig. 2) $k_T$ denotes the intrinsic transverse momentum of a parton in a proton plus any medium effect and $\hat{k}_T$ denotes the reduced value correcting for the lost energy of the scattered partons in the QGP, a new idea this year [4].
Measurement of $\hat{q}$ in RHI collisions  

M. J. Tannenbaum

---

Figure 2: Initial configuration: trigger jet $\hat{p}_{Tt}$, associated (away) jet $\hat{p}_{Ta}$ with $k_T$ effect (dashed arrow) and fragments $p_{Tt}$ and $p_{Ta}$, with fragmentation transverse momentum $j_T$, and $p_{out} = p_{Ta}\sin(\pi - \Delta \phi)$.

Even though jet quenching has been established and confirmed for more than 15 years, many experiments have tried to find azimuthal broadening at RHIC e.g. Fig. 3 [5], [6], but have not been able to observe the effect because of systematic uncertainties.

Figure 3: a) STAR measurement of the Gaussian widths $\sigma_{AS}$ of away-side hadron peaks triggered by a jet in collisions of Au+Au (solid symbols) and p+p (open symbols) at $\sqrt{s_{NN}}=200$ GeV [5]. b) Away-peaks in STAR di-jet measurement for two $\hat{p}_{Tt}$ ranges in Au+Au at $\sqrt{s_{NN}}=200$ GeV: (left) peripheral, (right) central collisions, with the same $\sigma$ [6].

1.1 Understanding $k_T$ and $k_T'$.

Following the methods of Feynman, Field and Fox [7], CCOR [8] and PHENIX [9], the $\langle k_T^2 \rangle$ for di-hadrons is computed from Fig. 2 as:

$$\sqrt{\langle k_T^2 \rangle} = \hat{x}_h \sqrt{\langle p_{out}^2 \rangle - \langle j_T^2 \rangle} / \langle j_T^2 \rangle / 2$$

(1.3)

where $p_{Tt}$, $p_{Ta}$ are the transverse momenta of the trigger and away particles, $x_h = p_{Ta}/p_{Tt}$, $\Delta \phi$ is the angle between $p_{Tt}$ and $p_{Ta}$ and $p_{out} \equiv p_{Ta}\sin(\pi - \Delta \phi)$. The di-hadrons are assumed to be fragments of jets with transverse momenta $\hat{p}_{Tt}$ and $\hat{p}_{Ta}$ with ratio $\hat{x}_h = \hat{p}_{Ta}/\hat{p}_{Tt}$. $z_t \approx p_{Tt}/\hat{p}_{Tt}$ is the fragmentation variable, the fraction of momentum of the trigger particle in the trigger jet. $j_T$ is the jet fragmentation transverse momentum and we have taken $\langle \hat{p}_{Tay} \rangle \equiv \langle \hat{p}_{Tay}^2 \rangle = \langle \hat{p}_{Tao}^2 \rangle = \langle \hat{p}_{Tao}^2 \rangle = \langle j_T^2 \rangle / 2$. The variable $x_h$ (which STAR calls $z_T$) is used as an approximation of the variable $x_k = x_h \cos(\pi - \Delta \phi)$ from the original terminology at the CERN ISR where $k_T$ was discovered and measured 40 years ago.
A recent STAR paper [10] on $\pi^0$-hadron correlations in $\sqrt{s_{NN}} = 200$ GeV Au+Au 0-12% central collisions had very nice correlation functions for large enough $12 \leq p_Tt \leq 20$ GeV/c so that the $v_2$, $v_3$ modulation of the background was negligible (Fig. 4). I made fits to these data [4] to determine $\langle p^2_{out} \rangle$ so that I could calculate $k_T$ in $p+p$ and Au+Au using Eq. 1.3. The results for $3 \leq p_{Ta} \leq 5$ GeV/c were $\sqrt{\langle k^2_T \rangle} = 2.5 \pm 0.3$ GeV/c for $p+p$ and $\sqrt{\langle k^2_T \rangle} = 1.4 \pm 0.2$ GeV/c, for Au+Au, exactly the opposite of azimuthal broadening (Eq. 1.2)!

After considerable thought, I finally figured out what the problem was and introduced the new $k'_T$ [4]. For a di-jet produced in a hard scattering, the initial $\hat{p}_T$, and $\hat{p}_{Ta}$ (Fig. 2) will both be reduced by energy loss in the medium to become $\hat{p}'_T$, and $\hat{p}'_{Ta}$ which will be measured by the di-hadron correlations with $p_Tt$ and $p_{Ta}$ in Au+Au collisions. The azimuthal angle between the di-jets, determined by the $\langle k^2_T \rangle$ in the original collision, should not change as both jets lose energy unless the medium induces multiple scattering from $\hat{q}$. Thus, without $\hat{q}$ and assuming the same fragmentation transverse momentum $\langle j^2_T \rangle$ in the original jets and those that have lost energy, the $p_{out}$ between the away hadron with $p_{Ta}$ and the trigger hadron with $p_{Tt}$ will not change; but the $\langle k'^2_T \rangle$ will be reduced because the ratio of the away to the trigger jets $\hat{x}_h = \hat{p}'_{Ta}/\hat{p}'_T$, will be reduced. Thus the calculation of $k'_T$ from the di-hadron $p+p$ measurement to compare with Au+Au measurements with the same di-hadron $p_Tt$ and $p_{Ta}$ must use the values of $\hat{x}_h$, and $\langle z_t \rangle$ from the Au+Au measurement to compensate for the energy lost by the original dijet in $p+p$ collisions.

The same values of $\hat{x}_h$, and $\langle z_t \rangle$ in Au+Au and $p+p$ simplify Eqs. 1.2 and 1.3 to:

$$\langle \hat{q}L \rangle / 2 = \left[ \frac{\hat{x}_h}{\langle z_t \rangle} \right]_{AA}^2 \left[ \frac{\langle p^2_{out} \rangle_{AA} - \langle p^2_{out} \rangle_{PP}}{x^2_h} \right]$$  (1.4)
Measurement of \( \hat{q} \) in RHI collisions

M. J. Tannenbaum

from which one could immediately get a reasonable answer for \( \langle \hat{q}L \rangle / 2 \) from the \( \langle p_{out}^2 \rangle \) results indicated on Fig. 4 if the values of \( \hat{x}_h \) and \( \langle z_t \rangle \) in the Au+Au measurement are known.

2. How to calculate \( \langle \hat{q}L \rangle \) from the Au+Au (and p+p) measurements for di-hadrons with a trigger \( p_T \) and away-side \( p_{Ta} \) distribution.

From Eq. 1.4, we need \( \langle p_{out}^2 \rangle_{pp} \), \( \langle p_{out}^2 \rangle_{AA} \) plus \( \hat{x}_h \) and \( \langle z_t \rangle \) in Au+Au. This will be illustrated with the STAR data [10].

a) \( \hat{x}_h \) for a given \( p_T \) can be calculated from the \( p_{Ta} \) distribution: The ratio of the away jet to the trigger jet transverse momenta, \( \hat{x}_h = \hat{p}_{Ta}/\hat{p}_{Tt} \), can be calculated (Fig. 5) from the away particle \( x_h = p_{Ta}/p_{Tt} \) distributions, which were also given in the STAR paper. The formula is [9], where \( n \) is the power of the \( p_T \) spectra:

\[
\frac{dP}{dp_{Ta}} \bigg|_{p_{Tt}} = N(n-1) \frac{1}{\hat{x}_h} \left(1 + \frac{\hat{x}_h}{\hat{x}_h} \right)^n.
\]  

(2.1)

\[ n = 8.1 \]

**Figure 5:** a) Plots of Eq. 2.1 for the values of \( \hat{x}_h \) indicated. b) Fits of Eq. 2.1 [4] to the STAR away-side \( z_T \) distributions [10] in Au+Au 0-12% centrality, and p+p, for \( 12 < p_{Tt} < 20 \) GeV/c. The Au+Au curve is a fit with \( \hat{x}_{AA} = 0.36 \pm 0.05 \) with error corrected by \( \sqrt{\chi^2/d.o.f.} \). The points with the open circles are the \( \gamma_i \) and systematic errors \( \sigma_{b_i} \) of the data points while the filled points are \( \gamma_i + \epsilon_{b_i} \sigma_{b_i} \) with errors \( \tilde{\sigma}_i \) and \( \epsilon_{b_i} = -1.3 \pm 0.5 \). [4]

b) Fit the away-side peaks in the Au+Au and p+p correlation functions to gaussians in \( p_{out} \): The gaussian fit directly gives \( \langle p_{out}^2 \rangle \) as was nicely shown for the STAR data in Fig. 4.

c) The power of hard scattering: the Bjorken parent-child relation and "trigger bias":

The hard-scattering \( p_T \) spectra, \( d\sigma/p_T dp_T \), at RHIC in the range \( 3 \leq p_T \leq 20 \) GeV/c for p+p
and Au+Au for all centralities follow the same power law \(1/p_T^2\) with \(n = 8.10 \pm 0.05\) [11]. This is why \(R_{AA}(p_T)\) for \(\pi^0\) and \(\eta\) in Fig. 1b are relatively constant over the same \(p_T\) range. The Bjorken parent-child relation [12] proved that the power \(n\) in \(p_T^{-n}\) is the same in the jet and fragment (\(\pi^0\)) \(p_T\) spectra. This is why \(\pi^0\) can be used in place of the parent jet. However because the trigger \(\pi^0\) spectrum for a given \(p_T\), in Au+Au for 0–10% centrality is shifted down by \(\delta p_T / p_T^{\text{pp}} = 20\%\) in \(p_T\) compared to \(\text{p}+\text{p}\) [13], the \(\langle z_i \rangle\) for A+A and compensated \(\text{p}+\text{p}\) should be calculated [9] from the measured \(\text{p}+\text{p}\ \pi^0\) \(p_T\) spectrum at \(p_T^{\text{pp}} / (1 - \delta p_T / p_T^{\text{pp}})\).

(For the present discussion, STAR measured \(\langle z_i \rangle = 0.80 \pm 0.05\) from their \(\text{p}+\text{p}\) data [10].)

This method enabled me to calculate \(\langle \hat{q}L \rangle\) from the \(\langle p_{\text{out}}^2 \rangle\) values indicated on Fig. 4, now with sensible results (Table 1). The results in the two \(p_{Ta}\) bins are at the edge of agreement, different by 2.4\(\sigma\); but both are \(> 2.6\sigma\) from zero. These results leave several open issues as mentioned in the abstract.

<table>
<thead>
<tr>
<th>Table 1: Tabulations for (\hat{q}) [4] from STAR (\pi^0)-h data [10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{s_{NN}} = 200\text{GeV})</td>
</tr>
<tr>
<td>Reaction</td>
</tr>
<tr>
<td>Au+Au 0-12%</td>
</tr>
<tr>
<td>Au+Au 0-12%</td>
</tr>
</tbody>
</table>

3. Homework

However there is a nice prediction of \(\Delta\phi\) for for 35 GeV Jets at RHIC [14] for several values of \(\langle \hat{q}L \rangle\) (Fig. 6). An amusing test would be to see if the present method gives the same answers for \(\langle \hat{q}L \rangle\) by calculating \(\langle p_{\text{out}}^2 \rangle\) of the predictions.

![Dijet Angular Correlation at RHIC](image1)

![Dijet Angular Correlation at the LHC](image2)

**Figure 6:** Prediction by Al Mueller and collaborators [14] of the di-jet azimuthal decorrelation as a function of \(\hat{q}L\) for a) 35 GeV jets at RHIC \(\sqrt{s_{NN}} = 200\text{GeV}\); and b) 50 GeV jets at the LHC \(\sqrt{s_{NN}} = 2.76\text{TeV}\) where “\(p_T\) broadening effects are negligible” [14].
References


