PoS PROCEEDINGS OF SCIENCE

Which scale for parton densities? Perspective from k_t-factorization

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Using the relationship between collinear and k_t-factorization, we discuss the hard scale uncertainty. While the k_t-factorization is free of this uncertainty, it should be taken into account in the case of collinear factorization. Contrary to the factorization scale uncertainty, the hard scale uncertainty is not reduced by higher order calculations. A more detailed work has been published in [1].

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1. Collinear factorization and uncertainties

For hadron-hadron collisions, the collinear factorization formula is generally written[1]

$$\frac{d\sigma}{dx_1dx_2dp_t^2}(x_1,x_2,p_t^2,Q^2,\mu^2) = f(x_1,\mu^2)f(x_2,\mu^2)\hat{\sigma}\left(x_1,x_2,p_t^2,\frac{Q^2}{\mu^2}\right). \tag{1.1}$$

The functions f and $\hat{\sigma}$ are the parton densities and partonic cross section, respectively. The variable p_t corresponds to the transverse momentum of outgoing partons. We use the generic notation Q^2 for the hard scale which is conventionally identified with p_t^2. The factorization scale μ appears due to the renormalization procedure. It comes inside logarithms of the type $\alpha_s \ln(Q^2/\mu^2)$ and has to be chosen close to Q^2 for an accurate finite order calculation. The dependence on the renormalization scale is not shown, and in this study we take α_s constant. Finally, x_1 and x_2 are the longitudinal momentum fractions carried by the incoming partons. The definition of parton densities is not unique [2], and for our discussion, it is simpler to shift higher-order corrections from $\hat{\sigma}$ to these these functions, leading to the following factorization formula:

$$\frac{d\sigma}{dx_1dx_2dp_t^2}(x_1,x_2,p_t^2,Q^2,\mu^2) = f(x_1,Q^2;\mu^2)f(x_2,Q^2;\mu^2)\hat{\sigma}\left(x_1,x_2,p_t^2\right). \tag{1.2}$$

Taking into account the first higher-order corrections and following [3] we write

$$f(x,Q^2;\mu^2) = f(x,\mu^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi} f(\xi,\mu^2)\left(P\left(\frac{x}{\xi}\right)\ln\frac{Q^2}{\mu^2} + C(x)\right), \tag{1.3}$$

with $C(x)$ a calculable function which is not enhanced by $\ln(Q^2/\mu^2)$. In the following, we will keep the choice and notation of Eqs. (1.2) and (1.3).

In an all-order calculation, the dependence on the unphysical scale μ will disappear in both sides of Eq. (1.2). This is formalized by the DGLAP equation [4] (or [3] for a modern review) which can be written

$$\frac{df(x,Q^2;\mu^2)}{d\mu^2} = 0. \tag{1.4}$$

However, at finite order, the physics does depend on the factorization scale. Consequently, there is a factorization scale uncertainty. The logarithm $\ln(Q^2/\mu^2)$ arises from an integral on the transverse momentum, k_t, of the incoming parton (an example is given in the case of deep inelastic scattering (DIS) in figure 1). The hard scale appears in the upper bound of such an integral.

In this paper, we want to discuss the usual choice[2] $Q^2 = p_t^2$ done in hadron-hadron collisions, giving the factorization formula

$$\frac{d\sigma}{dx_1dx_2dp_t^2}(x_1,x_2,p_t^2,\mu^2) = f(x_1,p_t^2;\mu^2)f(x_2,p_t^2;\mu^2)\hat{\sigma}\left(x_1,x_2,p_t^2\right). \tag{1.5}$$

[1] We will use mainly schematic formulas. The sum over parton flavors is ignored and one can consider that there is only one flavor (it simplifies also the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation). If not necessary, integrals are not written.

[2] Or $Q^2 = m_t^2$, with $m_t^2 = p_t^2 + m^2$ and m the mass of the outgoing parton(s).

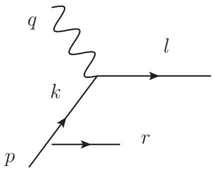

Figure 1: Real emission diagram in DIS.

For definiteness, we consider the case of transverse momentum distribution of heavy quarks in proton-proton collisions at the LHC. This choice for the hard scale means that p_t^2 is assumed to be the upper bound for the k_t^2 integration. Since, for on-shell partons the kinematical constraint is $k_t^2 < \hat{s}/4$ (with $\hat{s} = x_1 x_2 s$), this is a good approximation in the region $p_t^2 \simeq \hat{s}/4$, but it is not correct if $\Lambda_{QCD}^2 \ll p_t^2 \ll \hat{s}/4$. In fact, it is exactly in this region that the k_t-factorization is expected to give important corrections.

In the following, we will argue that the k_t-factorization provides a fundamental explanation on why choosing the hard scale to be p_t^2 is correct. But we will also see that this choice is not unique and gives rise to a theoretical uncertainty (in the collinear factorization case) which is not taken into account in current calculations. This uncertainty is not reduced by higher-order corrections. The other uncertainties come from the choice of the factorization scale, the mass and parton densities.

2. k_t-factorization

For hadron-hadron collisions, the k_t-factorization [5, 6, 7, 8] can be written

$$\frac{d\sigma}{dx_1 dx_1 d^2 p_t}(s, x_1, x_2, p_t^2, \mu^2) = \int^{k_{max}^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2), \tag{2.1}$$

with $\hat{\sigma}$ the off-shell cross section and F the unintegrated parton distribution (uPDF). The variables k_{1t}, k_{2t}, x_1 and x_2 refer to the two spacelike partons entering in the $2 \to 2$ perturbative QCD process. They correspond to the transverse momentum and the hadron longitudinal momentum fraction. The variable p_t is for the transverse momentum of the outgoing parton. For the upper bound k_{max}^2, it is sufficient to know that $k_{max}^2 > x_1 x_2 s/4 = p_{t,max}^2$. Finally, the unintegrated PDF are related to the usual one by

$$f(x, Q^2; \mu^2) = \int^{Q^2} F(x, k_t^2; \mu^2) d^2k_t, \tag{2.2}$$

where we follow the notation used in refs. [9, 10] [3].

3. Relationship between collinear and k_t-factorization: discussion on the hard scale uncertainty

To understand why, in Eq. (1.5), the scale inside the parton density is approximatively p_t^2 and why the collinear factorization still works at $p_t^2 \ll s$, it is interesting to see how the collinear

[3] However our function $F(x, k_t^2; \mu^2)$ is related to their function by a factor x.

factorization can be found as a limit of the k_t-factorization. Eq. (2.1) can be written

$$\frac{d\sigma}{dx_1 dx_2 dp_t^2} = \int^{Q^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2) +$$
$$\int_{Q^2}^{k^2_{max}} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2) \hat{\sigma}(x_1 x_2 s, k_{1t}^2, k_{2t}^2, p_t^2) = I^{cf} + I^{ct}. \quad (3.1)$$

The off-shell cross section is built in order to give the usual on-shell cross section in the limit $k_{it}^2 \ll p_t^2$. Then, if Q^2 is not much bigger than p_t^2, the first term above can be approximately written

$$I^{cf} = \hat{\sigma}_{on-shell}(x_1 x_2 s, p_t^2) \int^{Q^2} d^2k_{1t} d^2k_{2t} F(x_1, k_{1t}^2; \mu^2) F(x_2, k_{2t}^2; \mu^2). \quad (3.2)$$

Finally, using the definition (2.2), we obtain

$$I^{cf} = f(x_1, Q^2; \mu^2) f(x_2, Q^2; \mu^2) \hat{\sigma}_{on-shell}(x_1 x_2 s, p_t^2), \quad (3.3)$$

corresponding to the collinear factorization, Eq. (1.2). It makes sense to cut the integral at Q^2 if the second term in Eq. (3.1) is just a correction. We now define the hard scale Q^2 by

$$I^{ct}(Q^2) \ll I^{cf}(Q^2), \quad (3.4)$$

and we will see that $Q^2 = p_t^2$ respects this condition, explaining in turn why Eq. (1.5) still works at small p_t.

4. Choosing the hard scale

We will now explain qualitatively why $Q^2 = p_t^2$ is an acceptable choice, making the collinear factorization formula Eq. (1.5) accurate. To understand this, we will consider separately the cases of high p_t and low p_t (~ 1 GeV).

The reason why the integral I^{ct} can be small even if the phase space for integration is large is due to the fact that in the region $1 \ll p_t^2 < k_{i,t}^2 < k_{t,max}^2$ the off-shell cross section is slowly decreasing with k_t^2 (figure 2, left), while the unintegrated gluon density is strongly suppressed.

At small p_t, the suppression due to the unintegrated gluon density is not enough to explain why equation (3.4) is true if one chooses $Q^2 = p_t^2$. But, in this region, the off-shell cross section decreases quickly with k_t^2 (figure 2, right), making the integration up to $\sim p_t^2$ sufficient.

Then, we have shown that the dynamical properties of unintegrated PDF and off-shell cross sections can be used to justify the choice $Q^2 = p_t^2$ done in collinear factorization. The same demonstration could be done for $Q^2 = 2p_t^2$, showing that there is an hard scale uncertainty. However, all values are not allowed and the reader can convince himlself that $Q^2 = p_t^2/100$ does not respect the condition (3.4).

5. Conclusion

The k_t-factorization can be decomposed into $I_{kt}() = I^{cf}(Q^2) + I^{ct}(Q^2)$. Eq. (3.4) ensures the accuracy of the collinear factorization contribution, and is regarded as the definition of an

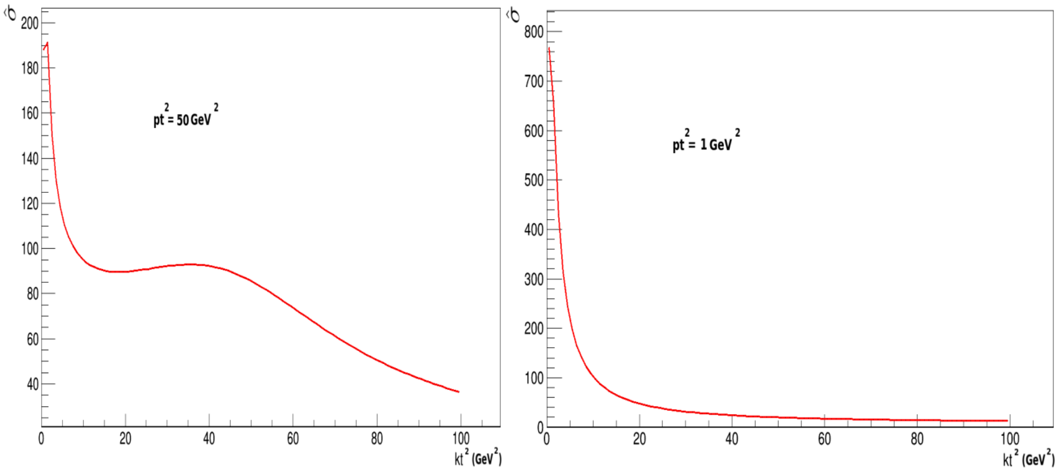

Figure 2: Off-shell cross section for the process $gg \to Q\overline{Q}$ (taken from ref. [6]) as a function of the transverse momentum $k_t^2 = k_{1t}^2 = k_{2t}^2$ of the incoming spacelike partons. Left: For central rapidity, $y = 0$, and $p_t^2 = 50$. Right: $y = 0$, and $p_t^2 = 1$. Other variables have been integrated out.

appropriate hard scale. There is an uncertainty on the choice of the hard scale, and, in the case of heavy quark production, the k_t-factorization can be used to show that $Q^2 = p_t^2$ is a possibility. This hard scale uncertainty is not reduced by higher order calculations (it is a physical scale playing the role of effective cutoff for the k_t^2 integration) and is absent within the k_t-factorization formalism (I_{kt} is independent of Q^2). It is not taken into account in current calculations.

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