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Theory of Mixing and CP violation

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We review the current status of B-mixing observables and point out the crucial importance of a control of the hadronic uncertainties for ruling out or confirming hints of BSM physics. In addition we introduce a rating system for theory predictions for lifetimes and mixing observables, that classifies the quality of the corresponding SM values ranging from no star to ****.

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1. Introduction

In the Standard Model (SM) mixing of neutral B_q-mesons is governed by the famous box-diagrams, with internal W-bosons and internal *up*-, *charm*- and *top*-quarks, see Fig. 1 for the case of B_s-mesons - for a more detailed introduction into B-mixing, see e.g. [1]. The contribution of

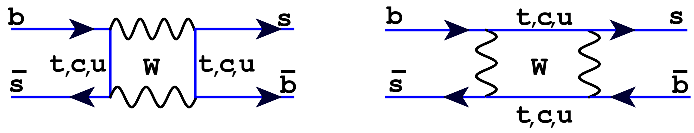

Figure 1: Standard Model diagrams for the transition between B_s and $\bar{B}_s$ mesons.

internal on-shell particles (only the *charm*- and the *up*-quark can contribute) is denoted by Γ_{12}^q; the contribution of internal off-shell particles (all depicted particles can contribute) is denoted by M_{12}^q. In the B-system there are simple relations[1] between Γ_{12}^q, M_{12}^q and the physical observables mass difference ΔM_q, the decay rate difference $\Delta\Gamma_q$ and the semi-leptonic asymmetries a_{sl}^q:

$$\Delta M_q \approx 2\left|M_{12}^q\right|, \qquad \Delta\Gamma_q \approx 2\left|\Gamma_{12}^q\right|\cos\phi_{12}^q, \qquad a_{sl}^q \approx \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right|\sin\phi_{12}^q. \tag{1.1}$$

The calculation of M_{12}^q gives

$$M_{12}^q = \frac{G_F^2}{12\pi^2}\lambda_t^2 M_W^2 S_0(x_t) B f_{B_q}^2 M_{B_q}\hat{\eta}_B, \tag{1.2}$$

where λ_t denotes the CKM elements $V_{tq}^* V_{tb}$ and the Inami-Lim function S_0 [5] contains the result of the 1-loop box diagram in the SM. The bag parameter B and the decay constant f_{B_q} quantify the hadronic contribution to B-mixing, the uncertainties of their numerical values make up the by far biggest uncertainty in the SM prediction of the mass difference. Perturbative 2-loop QCD corrections have been calculated by [6] and they are compressed in the factor $\hat{\eta}_B$. The calculation of Γ_{12}^q is more involved and is based on the Heavy Quark Expansion (HQE) (see [7] for a review and the original references). According to the HQE the total decay rate of a heavy hadron can be expanded in the inverse of the heavy quark mass as

$$\frac{1}{\tau} = \Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \frac{\Lambda^4}{m_b^4}\Gamma_4 + \ldots. \tag{1.3}$$

The hadronic scale Λ is of order Λ^{QCD}, its numerical value has to be determined by direct computation. For hadron lifetimes it turns out that the dominant correction to Γ_0 is the third term Γ_3. Each of the Γ_i's can be split up in a perturbative part and non-perturbative matrix elements - it can be formally written as

$$\Gamma_i = \left[\Gamma_i^{(0)} + \frac{\alpha_S}{4\pi}\Gamma_i^{(1)} + \frac{\alpha_S^2}{(4\pi)^2}\Gamma_i^{(2)} + \ldots,\right]\langle O^{d=i+3}\rangle \tag{1.4}$$

[1]This holds not for D-mixing, see e.g. [2, 3, 4].

where $\Gamma_i^{(0)}$ denotes the perturbative LO-contribution, $\Gamma_i^{(1)}$ the NLO one and so on; $\langle O^{d=i+3} \rangle$ is the non-perturbative matrix element of $\Delta B = 0$ operators of dimension $i+3$. The mixing quantity Γ_{12}^q obeys a very similar HQE, but now the operators change the b-quantum number by two units, $\Delta B = 2$:

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3} \Gamma_3 + \frac{\Lambda^4}{m_b^4} \Gamma_4 + \ldots . \tag{1.5}$$

2. Current Status

We introduce in this section a rating system for the robustness of lifetime and mixing predictions. Any calculation of a perturbative term ($\Gamma_i^{(j)}$) or a non-perturbative matrix element ($\langle O^{d=k} \rangle$) gets a $''+''$; if the calculation is confirmed by an independent collaboration it gets a $''++''$. In the case of non-perturbative matrix elements one can even gain a $''+++''$ for two independent lattice evaluations and one sum rule evaluation. A missing non-perturbative matrix element of dimension 6 is punished by a $''--''$ contribution. Non-perturbative estimates different from lattice or sum rules (like quark models) will be valued by a $''0''$. Partial perturbative calculations will be rated with a $''+/2''$. The possible number of 15 "+" will be classified in 5 categories: **** (at least 12 "+"), *** (at least 8 "+"), ** (at least 4 "+"), * (at least 2 "+") and no star for 1 or less "+".
For the lifetimes of heavy hadrons we get the following overview:

Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	Σ
$\tau(B^+)/\tau(B_d)$	$++$	$++$	0	$+$	$++$	0	0	$**$ (7+)
$\tau(B_s)/\tau(B_d)$	$++$	$++$	0	$\frac{+}{2}$	$++$	0	0	$**$ (6.5+)
$\tau(\Lambda_b)/\tau(B_d)$	$++$	$\frac{+}{2}$	0	$\frac{+}{2}$	$+$	0	0	$**$ (4+)
$\tau(b-baryon)/\tau(B_d)$	$++$	0	0	0	$+$	0	0	$*$ (3+)
$\tau(B_c)$	$+$	0	0	$+$	0	0	0	$*$ (2+)
$\tau(D^+)/\tau(D^0)$	$++$	$++$	0	$+$	$++$	0	0	$**$ (7+)
$\tau(D_s^+)/\tau(D^0)$	$++$	$++$	0	$\frac{+}{2}$	$++$	0	0	$**$ (6.5+)
$\tau(c-baryon)/\tau(D^0)$	$++$	0	0	0	$+$	0	0	$*$ (3+)

The LO-QCD part $\Gamma_3^{(0)}$ was first done with the full charm quark mass dependence in 1996 by Uraltsev [8] and Neubert and Sachrajda [9]. For the B_c-meson one has to estimate also the leading HQE term Γ_0 - the full estimate of the lifetime was done by Beneke and Buchalla [10] - to some extent this quantity does not perfectly fit in our list. The NLO-QCD corrections $\Gamma_3^{(1)}$ to B^+, B_d and B_s were done by [11] and the Rome group [12] - the Rome group also presented part of the NLO-QCD corrections for the Λ_b. In the charm system the NLO-QCD corrections were done by [13] for D-mesons. The dimension 6 matrix elements for mesons (except for small corrections arising in B_s and D_s) were recently calculated via HQET sum rules [14] - here a complementary lattice evaluation would be very important, either for looking for BSM effects in the very precisely predicted ratio $\tau(B_s)/\tau(B_d)$ - this could point towards new effects in hadronic tree-level decays [15] - , or for testing the convergence of the HQE in the b- and in particular in the *charm*-system. For baryons we do not have a complete first principle determination of the non-perturbative matrix elements - there are sum rule determinations of the condensate contribution for the Λ_b [16] - we

Source	$f_{B_s}\sqrt{\hat{B}}$	$\Delta M_s^{\rm SM}$
HPQCD14 [21]	(247 ± 12) MeV	$(16.2 \pm 1.7)\,\mathrm{ps}^{-1}$
HQET-SR [14]	(261 ± 8) MeV	$(18.1 \pm 1.1)\,\mathrm{ps}^{-1}$
ETMC13 [22]	(262 ± 10) MeV	$(18.3 \pm 1.5)\,\mathrm{ps}^{-1}$
HPQCD09 [23] = FLAG13 [24]	(266 ± 18) MeV	$(18.9 \pm 2.6)\,\mathrm{ps}^{-1}$
FLAG17 [25]	(274 ± 8) MeV	$(20.01 \pm 1.25)\,\mathrm{ps}^{-1}$
Fermilab16 [26]	(274.6 ± 4) MeV	$(20.1 \pm 0.7)\,\mathrm{ps}^{-1}$
HPQCD06 [27]	(281 ± 20) MeV	$(21.0 \pm 3.0)\,\mathrm{ps}^{-1}$
RBC/UKQCD14 [28]	(290 ± 20) MeV	$(22.4 \pm 3.4)\,\mathrm{ps}^{-1}$
Fermilab11 [29]	(291 ± 18) MeV	$(22.6 \pm 2.8)\,\mathrm{ps}^{-1}$

Table 1: List of predictions for the non-perturbative parameter $f_{B_s}\sqrt{\hat{B}}$ and the corresponding SM prediction for ΔM_s. The current FLAG average is dominated by the FERMILAB/MILC value from 2016.

have, however, some estimates [7, 18] of the size of the matrix elements using spectroscopy as an input (based on [17]). LO dimension 7 contributions to B^+, B_s, B_d and Λ_b were done in [19]. These authors also considered dimension 8 contribution, but since there are operators arising where we even cannot use vacuum insertion approximation, we did not include these corrections in our list. There are unpublished calculations of the dimension 7 terms to B^+, B_s and B_d by Uli Nierste and myself, that agree with [19], therefore the "++" in the table. Perturbative dimension 7 contributions to D mesons were determined in [13] and to charmed baryons in [18]. So far there exists no non-perturbative determination of the matrix elements of dimension 7 operators. In Fig. 2, taken from [14], we compare the most solid SM predictions for heavy lifetimes with experiment and find an excellent agreement.

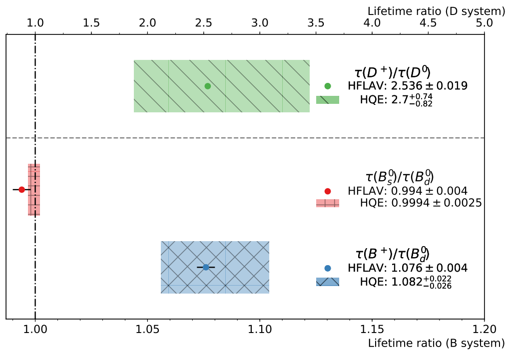

Figure 2: Comparison of the most solid SM predictions for heavy lifetimes with experiment.

The SM prediction for the mass difference is completely dominated by the non-perturbative input for the matrix element of the dimension 6 operator with a V-A Dirac structure. Depending on this input we get the range of predictions for the mass difference in the B_s-system as indicated in Table 1, taken from [20].

For the SM predictions of the decay rate differences in the B_d and B_s-system we get the following list:

Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	Σ
Γ_{12}^s	$++$	$++$	$\frac{+}{2}$	$++$	$++$	0	0	$8.5 + (***)$
Γ_{12}^d	$++$	$++$	0	$+++$	$++$	0	0	$9 + (***)$

The NLO-QCD corrections $\Gamma_3^{(1)}$ have been calculated in [30, 31, 32], recently also a part of the NNLO-QCD has been determined [33]. At dimension 6 two additional operators to the one appearing in the mass difference are arising. We have currently a HQET sum rule determination for B_d mesons [34, 14] and lattice determinations from 2016 [26] ($N_f = 2+1$) and 2013 [22] ($N_f = 2$). The dimension 7 perturbative part has been determined already in 1996 by Buchalla and Beneke [35] for B_s and in [36] for B_d. For numerical values of the mixing observables see e.g. the *aggressive scenario* of [2]

$$\Delta\Gamma_s = (0.098 \pm 0.014)\mathrm{ps}^{-1}\,, \quad a_{sl}^s = (2.27 \pm 0.25) \cdot 10^{-5}\,, \tag{2.1}$$
$$\Delta\Gamma_d = (2.99 \pm 0.52) \cdot 10^{-3}\mathrm{ps}^{-1}\,, \quad a_{sl}^d = -(4.90 \pm 0.54) \cdot 10^{-4}\,. \tag{2.2}$$

3. One constraint to kill them all

The importance of the precise value of SM predictions and a strict control of the corresponding uncertainties was highlighted recently in [20]. Lepto-quarks and Z' models are popular explanations of the B anomalies[2]; these new models would also affect B-mixing - in the case of Z' models already at tree-level. In Fig. 3 (from [20]) we show the allowed parameter range for a Z' model: in order to explain e.g. $R_{K^{(*)}}$ the mass of the Z' and the coupling to the b- and s-quark should lie within the black parabola-like shape (the 1 sigma bound is a solid line, the 2 sigma one a dotted line). Taking the FLAG inputs from 2013 for the mass difference one can exclude the blue region. Taking the new FLAG average, that is dominated by the 2016 FNAL/MILC we are left with the red exclusion region and almost all of the possible parameter space of the Z' model is excluded.

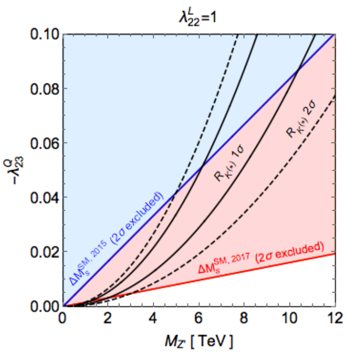

Figure 3: Allowed parameter space of Z' models that try to explain the B anomalies.

[2] Due to time and space restrictions I will not attempt to cite the numerous relevant papers in that field.

4. Conclusion

We presented an overview of the current theoretical status of lifetime and mixing predictions. $\Delta\Gamma_q$ and a_{sl}^q get the highest ranking (***). Γ_{12}^s is slightly less precise known, because the HQET sum rule calculation does not include yet m_s-effects. To improve further the reliability of these predictions one needs a non-perturbative determination of the dimension 7 matrix elements (first steps have been done in [37]) and perturbative evaluations of the α_s^2- and α_s/m_b-corrections. The next solid class of theoretical rigidness is (**) for $\tau(B^+)/\tau(B_d)$ and $\tau(D^+)/\tau(D^0)$. Here an independent lattice determination of the dimension 6 matrix elements is urgently needed. $\tau(B_s)/\tau(B_d)$ and $\tau(D_s^+)/\tau(D^0)$ is slightly less well known, because the m_s corrections to the HQET sum rule are not yet available. Finally Λ_b is considerably less well-known but still a (**) - here we need urgently a first non-perturbative determination of the dimension 6 matrix element. Finally we have the (*) class, which one should consider more an estimate than a precise SM prediction with well-defined uncertainties. We pointed out the crucial significance of a precise non-perturbative input for ΔM_q and related BSM studies - here an independent $N_f = 2+1$ or $N_f = 2+1+1$ confirmation of the FNAL/MILC result of 2016 would be desirable.

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