PROCEEDINGS OF SCIENCE

Realization of Spin-dependent p-C Scattering in GEANT4 and Its Application To Storage Ring Experiment For pEDM

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The Center for Axion and Precision Physics Research (CAPP) of the Institute for Basic Science (IBS) is attempting to develop prototype polarimeter detector using gas electron multiplier (GEM) technology for the storage ring proton EDM (SR pEDM) experiment. Systematic errors in the polarimeter mainly come from geometric characteristics of the polarimeter and beam dynamics. Such as beam tilt, shift or detector misalignment etc. can be sources of errors. This investigation of systematic errors has been done with GEANT4 simulation package modified by ourselves. The modified GEANT4 can calculate spin-dependent hadron elastic cross-section of proton-carbon scattering, unlike the original one. This research will explain how to realize spin-dependent hadronic elastic scattering in the azimuth angle and demonstrate its application to the geometric error related systematic error study of polarimeter.

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1. Introduction

Understanding our universe has been developed remarkably since decades. However, the world keeps bringing up considerable issues like CP problem, matter-antimatter asymmetry. [1] To search clue for these problems, a storage ring experiment has been proposed. The experiment aims to measure a permanent intrinsic electric dipole moment of proton (pEDM). [2]

2. The Storage Ring Experiment

The electric dipole moment d and magnetic moment ν are proportional to the rest frame spin $\boldsymbol{s}$ with several factors.

$$\boldsymbol{d} = \frac{\eta e}{2mc}\boldsymbol{s}, \tag{2.1}$$

$$\boldsymbol{\mu} = \frac{ge}{2m}\boldsymbol{s}, \tag{2.2}$$

where e is a electric charge, m is a mass, c is a speed of light, and these relations define η and g. These two moments interact with electromagnetic field and makes spin precess. At rest frame, the spin precession of a particle in electric field $\boldsymbol{E}$ and magnetic field $\boldsymbol{B}$ is described by

$$\frac{d\boldsymbol{s}}{dt} = \boldsymbol{\mu}\times\boldsymbol{B} + \boldsymbol{d}\times\boldsymbol{E}. \tag{2.3}$$

From Eq. (2.3), spin dynamics in electromagnetic field is derived known as T-BMT euqation. [3]

$$\frac{d\boldsymbol{s}}{dt} = -\frac{e}{m}\left[G\boldsymbol{B} + \left(\frac{1}{\gamma^2-1} - G\right)\boldsymbol{\beta}\times\frac{\boldsymbol{E}}{c} + \frac{\eta}{2}\left(\frac{\boldsymbol{E}}{2} + \boldsymbol{\beta}\times\boldsymbol{B}\right)\right] \tag{2.4}$$

$G = (g-2)/2$, $\boldsymbol{\beta} = \boldsymbol{v}/c$ is the velocity of a particle, and $\gamma = \sqrt{1-v^2/c^2}$ is relativistic factor. The spin dynamics can be considered as a sum of two parts; by electric dipole and by magnetic dipole. Let us call these two angular velocity $\boldsymbol{\omega}_a$ and $\boldsymbol{\omega}_e$ so that $d\boldsymbol{s}/dt = \boldsymbol{\omega}_a + \boldsymbol{\omega}_e$.

$$\boldsymbol{\omega}_a = \frac{e}{m}\left[G\boldsymbol{B} - \left(\frac{1}{\gamma^2-1} - G\right)\boldsymbol{\beta}\times\frac{\boldsymbol{E}}{c}\right] \tag{2.5}$$

$$\boldsymbol{\omega}_e = -\frac{\eta e}{2m}\left(\frac{\boldsymbol{E}}{2} + \boldsymbol{\beta}\times\boldsymbol{B}\right) \tag{2.6}$$

With certain momentum and without magnetic field, the $(g-2)$ precession $\boldsymbol{\omega}_a$ vanishes. And EDM precession $\boldsymbol{\omega}_e$ is just directly proportional to the magnitude of the particle's EDM.

$$\boldsymbol{\omega}_a = 0 \tag{2.7}$$

$$\boldsymbol{\omega}_e = -\frac{\eta e}{2mc}\boldsymbol{E} \tag{2.8}$$

The storage ring experiment uses this basic principle of spin precession inside field. If a particle rotates with constant velocity in an pure radial electric storage ring, the field will cause spin precession out of the storage ring plane. And the angular speed of spin rotation will be constant. If the initial state of spin direction is parallel to momentum, which means on the storage plane, spin precession angle will change linearly from 0 measured from the momentum direction. The strategy is to measure spin polarization over time expecting linear increase over time. In case of proton, "magic" parameters to make pure EDM precession in an pure electric ring are $\gamma = 1.248107$, $\beta = 0.598379$, $p = 0.7007$ GeV/c when radius of the ring is 52.3 m and $E = 1.171$ GeV. The goal of the experiment is to reach sensitivity of the pEDM measurement to the 10^{-29} e·cm within a year.

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3. Measurement of Spin Precession and Polarimetry

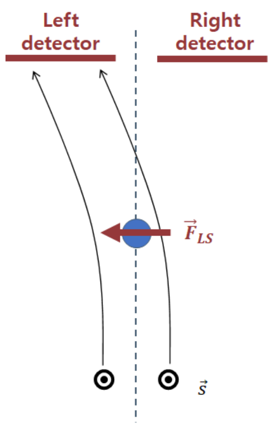

Figure 1: Left-biased scattering between proton and carbon

Scattering by solid carbon target is an option to measure proton spin precession. Non-zero polarization makes asymmetric scattering due to spin-orbit coupling. So measuring asymmetry is an indirect method to measure polarization. With vertically polarized proton with polarization p, asymmetric cross sections of left and right are written such as

$$\sigma_L = \sigma_{UNP}\,(1 + pA)\,, \tag{3.1}$$

$$\sigma_R = \sigma_{UNP}\,(1 - pA)\,, \tag{3.2}$$

where σ_{UNP} is the cross section of unpolarized proton and A is a physical quantity called analyzing power that gives the strength of the symmetry. Left-right asymmetry of the scattering is defined as

$$\varepsilon = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = pA. \tag{3.3}$$

Known A values enable to get p as ε divided by A. Experimentally, σ is replaced by number of hit on the detectors. Because the number of hit is proportional to the cross section if any other conditions were all the same such as luminosity or detector efficiency and so forth.

$$\varepsilon = \frac{N_L - N_R}{N_L + N_R} \tag{3.4}$$

Fortunately analyzing power of hadronic elastic scattering at around magic momentum is fairly large. [4]

Based on the discussion so far, polarimeter to measure spin polarization is essential for the storage ring experiment. The polarimeter is going to be put along the beam axis of the storage ring, and it has to be able to measure left-right or up-down asymmetry of scattering feature. Because stored particles go around both counter clockwise and clockwise directions, two identical polarimeters put symmetrically about the target. The polarimetry system also has to be able to measure not only hit count and also tracking. The target is to be carbon block.

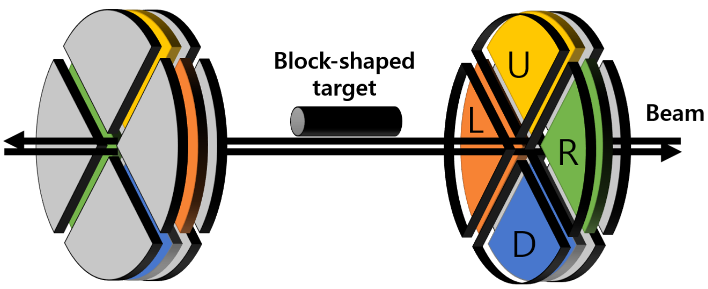

Figure 2: A diagram of polarimeter concept

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4. Systematic Errors

All detections have systematic errors. The polarimeter for the proposed storage ring proton EDM (SR pEDM) experiment also has too. The main errors come from unexpected direction and magnitude of electromagnetic field, geometric miss-alignment, and so on. On this topic of polarimetry, geometric and rate-induced errors are dominant. Among them, we focus on the management of systematic errors due to geometry. There are two main sources of geometry related errors in the polarimeter. One is mis-located detectors with beam axis. The other one is tilted or shifted beam. For example, the error due to beam tilt or shift can be described by the series expansion of $\Delta\theta$ as shown in Eq. (4.1).

$$\varepsilon(\Delta\theta) = \varepsilon_0 + [\text{first order term}]\,\Delta\theta + [\text{second order term}]\,(\Delta\theta)^2 + \cdots \tag{4.1}$$

Figure 3 is top view of polarimetry showing asymmetrical particle scattering with two opposite

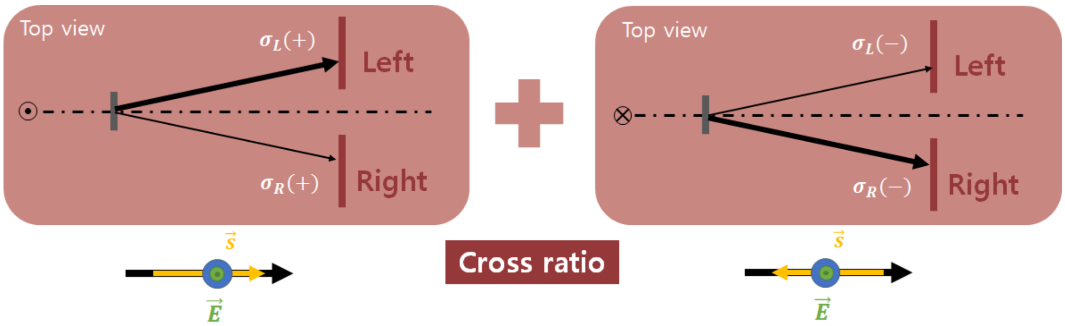

Figure 3: A diagram of cross ratio concept. Positive helicity causes up polarization in the storage ring and negative one does inversely.

polarization directions. If the spin direction is up, scattering will be left-biased. And if spin is down, scattering will be right-biased. Each situation in the storage ring corresponds to parallel spin and anti-parallel spin. With ideal detector and beam alignment, it is exactly same as normal asymmetry, p times A as shown in Eq. (3.3). With non-zero $\Delta\theta$, however, the errors exist as shown in Eq. (4.1). The first order error term in Eq. (4.1) can be eliminated by using the cross ratio method defined as Eq. (4.2). [5]

$$\varepsilon_{CR} \equiv \frac{\sqrt{\sigma_L(+)\sigma_R(-)} - \sqrt{\sigma_R(+)\sigma_L(-)}}{\sqrt{\sigma_L(+)\sigma_R(-)} + \sqrt{\sigma_R(+)\sigma_L(-)}} = pA \tag{4.2}$$

But still there is a second order term. Even though cross ratio is adopted, this term becomes dominant when extremely high precision is required.

$$\varepsilon_{CR}(\Delta\theta) = \varepsilon_0 + [\text{second order term}]\,(\Delta\theta)^2 + \cdots \tag{4.3}$$

5. GEANT4 As a Simulation Tool

To investigate the management of the systematic errors of polarimeter, computer simulation of polarimetry has been performed. We used GEANT4 package to generate physical interactions inside

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the target and detectors. It was confirmed that the cross section of proton and carbon scattering was comparable with experimental data. However, even though the polarization of the beam was set, asymmetry did not occur at all in the simulation. Because the latest version of `GEANT4` does not consider spin effect in hadronic interaction models it has. To realize spin-dependent scattering, we modified `GEANT4`. This upgraded `GEANT4` takes analyzing power data into calculation of cross section. An experimental result by H. O. Meyer, *et. al.* is used. [4] The new `GEANT4` also can set final spin state after scattering. The `GEANT4` uses numerically calculated number set of spin direction as function of CM scattering angle from optical model introduced in Ref. 4. The details are as follows.

5.1 The Original Algorithm of Final State Generation

The basic model to describe elastic hadronic scattering is generated by `HadronElastic.cc` model. This model samples momentum transfer squared t using given parameters like mass of primary particle, mass of the target nucleus, and charge of the target nucleus. [6] This t decides polar scattering angle θ_{CM}. For azimuth angle ϕ, the original model just generates uniformly distributed random number in $[0, 2\pi]$.

5.2 Modified Cross-section Formula

One can maintain original θ generator, or create ones own generator. This θ generator isn't touched in our case. On the other hands, ϕ generation has been changed depending on the polarization. The original one is uniform random as mentioned above, whereas the new p.d.f. is non-uniform, like

$$f(\phi; p, t) = 1 + pA_y(t)\cos(\phi), \tag{5.1}$$

where the polarization is vertical. ϕ is to be sampled from this $f(\phi; p, t)$ by Monte-Carlo method. By making use of transformation method with random number r, ϕ is generated from p.d.f. in Eq. (5.1).

$$\int_0^{\phi'} \frac{1}{2\pi} f(\phi; p, A) d\phi = \frac{1}{2\pi}\left[\phi' + pA(t)\sin\phi'\right] \equiv r \tag{5.2}$$

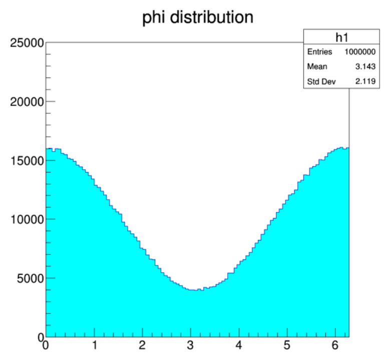

Figure 4: Monte-Carlo generation of ϕ in case of $p = 1$ and $A = 0.6$

Because transformed equation has no analytic solution, we need a numerical solution. Bi-section algorithm was adopted to sample ϕ. The bi-section is quite slow in many root-finding algorithms but always converges with Eq. (5.2).

5.3 Calculating New Final State

Sampled t decides θ and kinetic energy, and the θ and sampled ϕ from Eq. (5.1) go into the final momentum P.

$$\boldsymbol{P_f} = P_f(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \tag{5.3}$$

Likewise, final state of polarization $\boldsymbol{p_f}$ is also generated and applied for the next simulation step.

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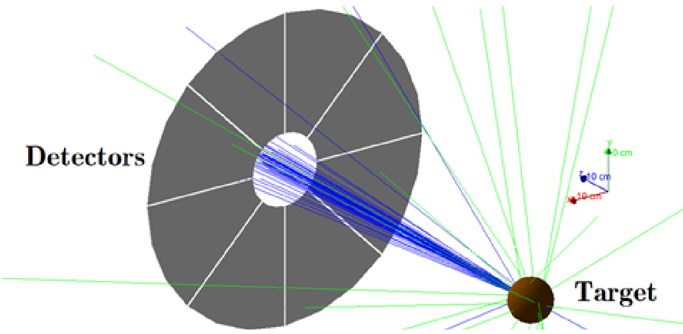
6. Application To Error Study

Upgraded GEANT4 gives us different simulation result. Performed simulation setup is shown in Fig. 5. There is a carbon target and detectors at forward direction. Scattered particles hit detector plates divided by 8 pieces to measure up-down and left-right asymmetry.



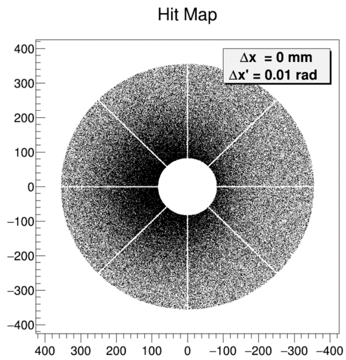


Figure 5: (a) Simulation geometry. Disk-shape detectors are put at forward direction of beam. Diameter of target was 1 mm, but it was drawn larger than its real size for visual help. (b) Hit map on the detector planes

Simulation result by the modified GEANT4 shows asymmetrical proton hits on the detector plane as shown in Fig. 5(b). The corresponding analyzing power can be calculated back from the asymmetry data and can be compared with the original experimental data to check the validity of the simulation. After checking that the GEANT4 can describe spin-dependent scattering, the influence to the polarimetry by beam tilting has been studied. Varying incidence angle to the target block, simulated polarization measurements are plotted on Fig. 4. It is seen that p must be measured as 1, but measured p is going down as tilting larger. It is clear that this effect comes from the second order error of cross ratio. Miss-alignment between beam axis and polarimeter axis also can be simulated by this tool. This study shows that the modified GEANT4 can be used to study geometric misalignment related systematic errors in the polarimeter.

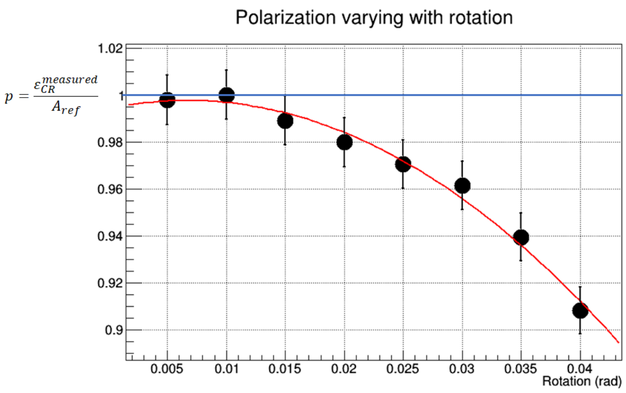


Figure 6: Measured polarization by polarimeter in GEANT4 simulation. The dots are fitted by quadratic function drawn by red curve.

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