PROCEEDINGS OF SCIENCE

Measurements of the CKM angle γ at LHCb

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The latest measurements of the CKM angle γ, using data collected by the LHCb experiment, are presented. These include a time-integrated analysis of $B^{\pm} \to D^0 K^{\pm}$ decays, where $D^0 \to K^0_S hh$, and a decay-time-dependent analysis of $B^0 \to D^{\mp}\pi^{\pm}$ decays, both presented here for the first time. In addition, an updated LHCb combination of the CKM angle γ, produced for this conference, is shown. This combination yields $\gamma = (74.0^{+5.0}_{-5.8})^{\circ}$ which dominates the world average, also presented here. Finally, an overview of the status and future prospects of γ measurements with LHCb future upgrades is discussed.

The International Conference on B-Physics at Frontier Machines - BEAUTY2018
6-11 May, 2018
La Biodola, Elba Island, Italy

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PoS(BEAUTY2018)004

1. Introduction

The angle $\gamma \equiv \arg[-(V_{ub}{}^{*}V_{ud})/(V_{cb}{}^{*}V_{cd})]$ is one of the least well known constraints of the CKM unitarity triangle. It is unique amongst CKM parameters as it is the only one which has no dependence on the top-quark coupling. It is accesible from purely tree-level processes and consequently, within the framework of the SM, has a very small irreducible theoretical uncertainty at the level of $|\delta_\gamma/\gamma| \leq 10^{-7}$ [1]. Experimental access to γ arises via interference between favoured $b \to cW$ and suppressed $b \to uW$ transitions, whose phase difference has a *CP*-violating (γ) and a *CP*-conserving (δ_B) component. The classic example makes use of interference in $B^\pm \to DK^\pm$ decays, where D is a superposition of D^0 and $\bar{D}^0$ which decay to the same final state. The ratio of the suppressed to favoured amplitudes is given by, $A_{\bar{f}}/A_f = r_B e^{i(\delta_B \pm \gamma)}$, where the $\pm$ refers to the intial B-meson charge. The best sensitivity to γ is achevied by reconstructing the D-superposition in several different final states. These proceedings will focus on two specific measurements of γ at LHCb as well as the overall combination of all measurements.

2. GGSZ analysis with $B^\pm \to DK^\pm$, $D \to K^0_S hh$ decays

The GGSZ method makes use of $B^\pm \to DK^\pm$ decays where the intermediate D-meson decays to 3-body self-conjugate final states such as $D \to K^0_S\pi^+\pi^-$ and $D \to K^0_S K^+K^-$ [2]. The partial rate of the B-meson decay depends on the kinematic position of the 3-body D-decay in the Dalitz plane and can be written,

$$\begin{aligned} d\Gamma_{B^\pm}(\mathbf{x}) = \; & A^2_{(\pm,\mp)} + r_B^2 A^2_{(\mp,\pm)} \\ & + 2A_{(\pm,\mp)}A_{(\mp,\pm)}\left[r_B\cos(\delta_B \pm \gamma)\cos(\delta_{D(\pm,\mp)}) + r_B\sin(\delta_B \pm \gamma)\sin(\delta_{D(\pm,\mp)})\right], \end{aligned} \tag{2.1}$$

where $\delta_{D(\pm,\mp)}$ is the *CP*-conserving strong phase difference between the *CP*-conjugate final states of the D decay. The dependence on γ is encoded in four cartesian variables defined as,

$$x_\pm + iy_\pm = r_B e^{i(\delta_B \pm \gamma)}. \tag{2.2}$$

In order to extract the relevant *CP*-information, with the best possible sensitivity, from the distribution of B^+/B^- candidates across the Dalitz-space, the cosine and sine of the strong phase variation for the D decay must be known.

LHCb has recently published a model-independent measurement of γ using these decays with a dataset corresponding to an integrated luminosity of $2\,\text{fb}^{-1}$ collected during 2015 and 2016 [3]. Mass fits to selected $B^\pm \to DK^\pm$ candidates, alongside those from $B^\pm \to D\pi^\pm$ which are used to control the mass shapes and study cross-feed, are shown in Fig. 1. The model-independent method divides the kinematic decay space of the D meson up into $2N$ symmetric bins, chosen to optimise the sensitivity to γ. The bin boundary definitions are shown in Fig. 2. Given the decay amplitude is a superposition of both suppressed and favoured contributions, $A_B(m_-^2, m_+^2) \propto A_D(m_-^2, D_+^2) + r_B e^{i(\delta_B - \gamma)} A_{\bar{D}}(m_-^2, m_+^2)$, the expected number of B^+ and B^- events in bin i is given by,

$$N^+_{\pm i} = h_{B^+}\left[F_{\mp i} + (x_+^2 + y_+^2)F_{\pm i} + 2\sqrt{F_i F_{-i}}(x_+ c_{\pm i} - y_+ s_{\pm i})\right], \tag{2.3}$$

$$N^-_{\pm i} = h_{B^-}\left[F_{\pm i} + (x_-^2 + y_-^2)F_{\mp i} + 2\sqrt{F_i F_{-i}}(x_- c_{\pm i} - y_- s_{\pm i})\right]. \tag{2.4}$$

Here, $h_{B^\pm}$ is the overall normalisation which is insensitive to *CP* information. The parameters $F_{\pm i}$ denote the fraction of D^0 ($\bar{D}^0$) events which end up in bin i. These are obtained from the flavour-specific double-self-tagging control channel $B \to D^{*\pm}\mu^\mp\nu_\mu X$ in data. The parameters c_i and s_i are the cosine and sine of the strong phase δ_D in bin i. These are obtained from measurements made by the CLEO collaboration using quantum-correlated pairs of D^0-$\bar{D}^0$ meson produced by the decay of the $\psi(3770)$ [4]. The parameters $x_\pm$ and $y_\pm$ are the *CP*-violating physical parameters of interest which are extracted by simultaneously fitting for the number of events across each bin. The resulting values are then combined with those extracted by the equivalent Run 1 analysis [5] to obtain a measurement of $\gamma = (80^{+10}_{-9})^\circ$ using these decays alone. The resulting profile likelihood contours for $x_\pm$ and $y_\pm$ are shown in Fig. 3. In Sec. 4 these will be further combined with all other LHCb measurements to better determine γ.

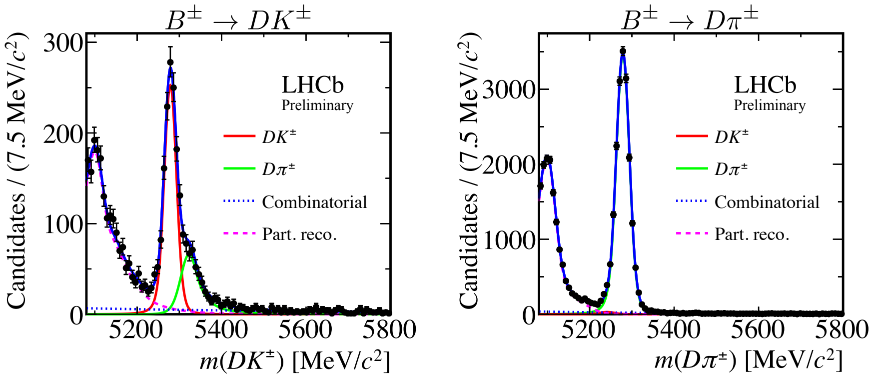

Figure 1: Invariant mass of selected $B^\pm \to D^0K^\pm$ (left) and $B^\pm \to D^0\pi^\pm$ (right) candidates.

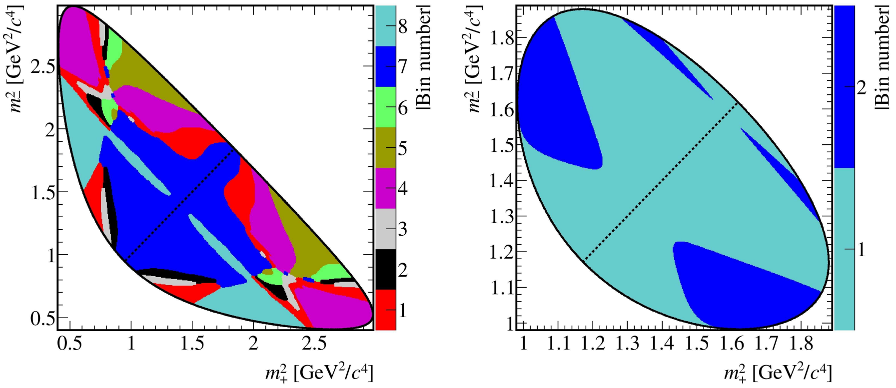

Figure 2: The definition of the bin boundaries for $D^0 \to K^0_S\pi^+\pi^-$ (left) and $D^0 \to K^0_S K^+K^-$ (right) samples

3. Time-dependent analysis of $B^0 \to D^\mp\pi^\pm$

Time-dependent methods can also be used to extract the weak phase γ orginating in the interference between mixing and decay of neutral B-meson decays. The classic example is the $B^0_s \to D_s^\mp K^\pm$ decay which has been analysed with a dataset corresponding to an integrated luminosity of 3 fb^{-1} by LHCb in the past [6]. Recently LHCb has publised a similar analysis with the

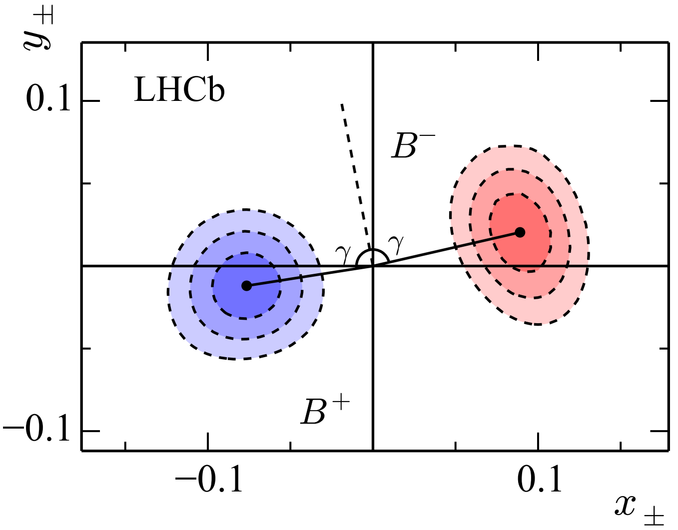

Figure 3: Profile likelihood contours for the cartesian variables $x_\pm$ and $y_\pm$ extracted from the model-independent fit to $B^\pm \to DK^\pm$ decays.

analogous $B^0 \to D^\mp\pi^\pm$ decay [7], which has a much higher branching fraction but much smaller interference effect than the B^0_s equivalent. The invariant mass of selected signal candidates is shown on the left-hand side of Fig. 4 which demonstrates the incredibly high statistics available with this channel. The unprecedentated size of the signal sample, $\sim$ 500K candidates, allows for simultaneous calibration of the flavour tagging parameters, important for identifying the flavour of the neutral B-meson at production. The general time-dependent decay rate for an initial state $B^0_{(s)}$ meson at $t = 0$ decaying to a final state f or $\bar{f}$ are given by,

$$\Gamma_{B^0{}_{(s)}\to f}(t) \propto e^{-\Gamma_{(s)}t}\left[C_f\cos(\Delta m_{(s)}t) - S_f\sin(\Delta m_{(s)}t)\ \underbrace{+\ \cosh\left(\frac{\Delta\Gamma_{(s)}t}{2}\right)}_{=1\text{ for }B^0}\ \underbrace{+A_f\sinh\left(\frac{\Delta\Gamma_{(s)}t}{2}\right)}_{=0\text{ for }B^0}\right], \tag{3.1}$$

$$\Gamma_{B^0{}_{(s)}\to \bar{f}}(t) \propto e^{-\Gamma_{(s)}t}\left[C_{\bar{f}}\cos(\Delta m_{(s)}t) - S_{\bar{f}}\sin(\Delta m_{(s)}t)\ \underbrace{+\ \cosh\left(\frac{\Delta\Gamma_{(s)}t}{2}\right)}_{=1\text{ for }B^0}\ \underbrace{+A_{\bar{f}}\sinh\left(\frac{\Delta\Gamma_{(s)}t}{2}\right)}_{=0\text{ for }B^0}\right], \tag{3.2}$$

where $\Gamma_{(s)}$ is the average width of the two flavour states of the $B^0{}_{(s)}$-meson and $\Delta\Gamma_{(s)}$ and $\Delta m_{(s)}$ are the width and mass differences between the two flavour states. The decay-time-dependent *CP*-asymmetry is fitted in order to obtain the parameters C_f, S_f, $S_{\bar{f}}$, A_f and $A_{\bar{f}}$ which depend on the ratio of the magnitude of the suppressed and favoured amplitudes, r_B, the strong phase difference, δ_B, and the weak phase difference, $(2\beta_{(s)}+\gamma)$. Note the overall weak phase includes a contribution from the mixing phase $\beta_{(s)}$ and the decay phase γ. In the case of the B^0-meson the width difference is negligbly small so the hyperbolic terms simplify. Furthermore the value of r_B is sufficiently small, ~ 0.02, such that the *CP*-coefficient C_f is assumed to be unity, $C_f = \frac{1-r_B^2}{1+r_B^2} = -C_{\bar{f}} \approx 1$. Consequently, in the case of the B^0 decay one is left with a decay-time-dependent asymmetry which depends on just two *CP*-coefficients,

$$S_f = -\frac{2r_B \sin[\delta_B - (2\beta + \gamma)]}{1 + r_B^2} \tag{3.3}$$

$$S_{\bar{f}} = \frac{2r_B \sin[\delta_B + (2\beta + \gamma)]}{1 + r_B^2} \tag{3.4}$$

The value of r_B is constrained by invoking flavour-symmetries and using branching ratio measurements of related processes performed by the BaBar and Belle collaborations [8, 9], whilst the value of β is constrained from the latest HFLAV fit [10]. The values of δ_B *vs.*γ are extracted in this analysis using the determined values of $S_f = 0.058 \pm 0.020\,(\text{stat}) \pm 0.11\,(\text{syst})$ and $S_{\bar{f}} = 0.038 \pm 0.020\,(\text{stat}) \pm 0.007\,(\text{syst})$. Their profile likelihood contour is shown on the right-hand side of Fig. 4.

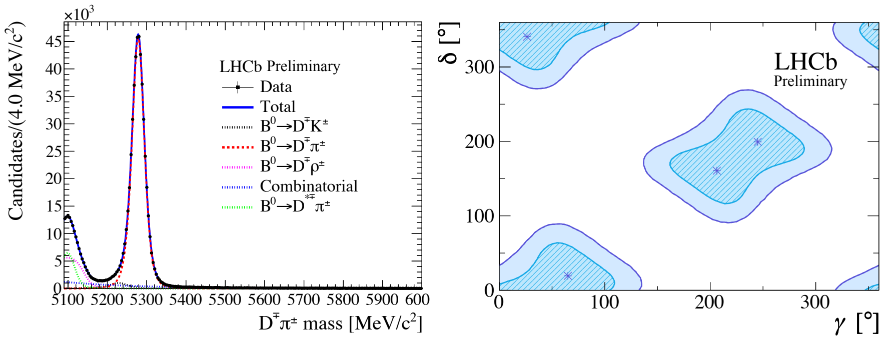

Figure 4: Left: Invariant mass of selected $B^0 \to D^{\mp}\pi^{\pm}$ candidates. Right: Two-dimensioanl profile likelihood contours for γ *vs.* δ_B from the TD $B^0 \to D^{\mp}\pi^{\pm}$ analysis.

4. Combination of LHCb γ measurements

The ultimate sensitivity to γ is only acheived when several modes are combined together. Although each given *B*-meson decay has two hadronic unknowns, r_B and δ_B, as well as the parameter of interest γ, these can be extracted in several different *D*-meson decay modes, provided it is accesible by both D^0 and $\bar{D}^0$. The extraction of γ with decays to *CP*-eigenstates, such as $D \to K^+K^-$ and $D \to \pi^+\pi^-$, is known as the GLW method [11, 12]. These have excellent sensitivity to γ but several ambigous solutions. The ADS method [13, 14] makes use of decays to doubly-Cabibbo-suppressed final states, such as $D \to K^+\pi^-$. These have poorer sensitivity but a single unambigous solution. The GGSZ method [15, 16] makes use of 3-body self-conjugate final states, such as those discussed in Sec. 2 with $D \to K^0_S hh$, and has both excellent sensitivity and a single solution. The LHCb collaboration has produced a new combination of the CKM angle γ for this conference presented in Ref. [17]. An overview of the various inputs used in this combination is provided in Table 1.

The overall LHCb combination yields a value of $\gamma = (74.0^{+5.0}_{-5.8})^\circ$ [17] which dominates the latest world average provided by HFLAV for the Moriond 2018 conference of $\gamma = (73.5^{+4.2}_{-5.1})^\circ$ [10], although this world average does not include the latest GGSZ measurement presented in Sec. 2. The confidence intervals for these combinations are shown in Fig. 5.

Table 1: List of the LHCb measurements used in the combination, where TD is time-dependent and the method acronyms refer to the authors of Refs. [11–16, 18–21].

B decay	D decay	Method	Ref.	Dataset[†]	Status since last combination [22]
$B^+ \to DK^+$	$D \to h^+h^-$	GLW	[23]	Run 1 & 2	Minor update
$B^+ \to DK^+$	$D \to h^+h^-$	ADS	[24]	Run 1	As before
$B^+ \to DK^+$	$D \to h^+\pi^-\pi^+\pi^-$	GLW/ADS	[24]	Run 1	As before
$B^+ \to DK^+$	$D \to h^+h^-\pi^0$	GLW/ADS	[25]	Run 1	As before
$B^+ \to DK^+$	$D \to K^0_S h^+h^-$	GGSZ	[5]	Run 1	As before
$B^+ \to DK^+$	$D \to K^0_S h^+h^-$	GGSZ	[3]	Run 2	New
$B^+ \to DK^+$	$D \to K^0_S K^+\pi^-$	GLS	[26]	Run 1	As before
$B^+ \to D^*K^+$	$D \to h^+h^-$	GLW	[23]	Run 1 & 2	Minor update
$B^+ \to DK^{*+}$	$D \to h^+h^-$	GLW/ADS	[27]	Run 1 & 2	Updated results
$B^+ \to DK^{*+}$	$D \to h^+\pi^-\pi^+\pi^-$	GLW/ADS	[27]	Run 1 & 2	New
$B^+ \to DK^+\pi^+\pi^-$	$D \to h^+h^-$	GLW/ADS	[28]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to K^+\pi^-$	ADS	[29]	Run 1	As before
$B^0 \to DK^+\pi^-$	$D \to h^+h^-$	GLW-Dalitz	[30]	Run 1	As before
$B^0 \to DK^{*0}$	$D \to K^0_S\pi^+\pi^-$	GGSZ	[31]	Run 1	As before
$B^0_s \to D_s^{\mp}K^{\pm}$	$D_s^+ \to h^+h^-\pi^+$	TD	[6]	Run 1	Updated results
$B^0 \to D^{\mp}\pi^{\pm}$	$D^+ \to K^+\pi^-\pi^+$	TD	[7]	Run 1	New

[†] Run 1 corresponds to an integrated luminosity of 3 fb^{-1} taken at centre-of-mass energies of 7 and 8 TeV. Run 2 corresponds to an integrated luminosity of 2 fb^{-1} taken at a centre-of-mass energy of 13 TeV.

5. Conclusion

The field of flavour physics is approaching an exciting time regarding measurements of the CKM angle γ. Given the incredibly small theoretical uncertainties involved, sensitivity to the angle is entirely driven by experimental constraints. LHCb is a world leader in this area and, with further data from Run 2 of the LHC still to be analysed and the expected gains with LHCb's future upgrades, this will continue in the future. The prospects are that within the next decade γ will transition to become *the standard candle* for precision SM CKM constraints. With the expected Belle-II dataset of 50 ab^{-1} and a possible 300 fb^{-1} or more available with LHCb's Phase II upgrade, the world precision for γ will reach $\sim 0.3^\circ$ which will be dominated by LHCb measurements.

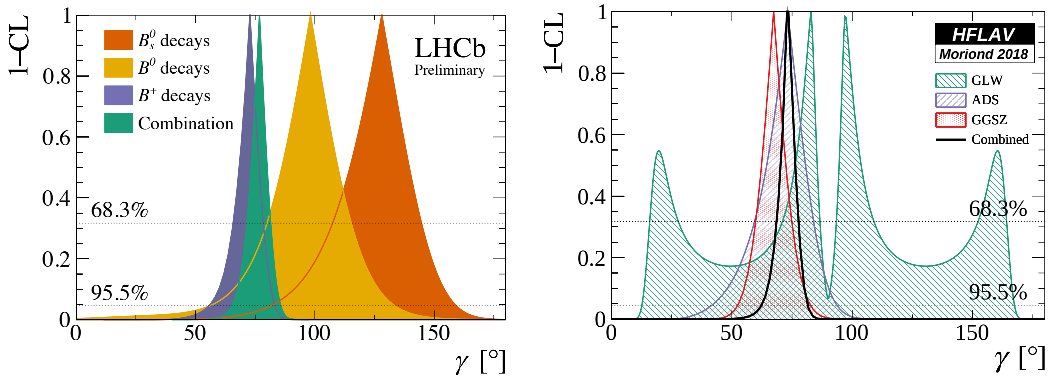

Figure 5: Left: The LHCb experiment combination of CKM angle γ split by initial B-meson type. Right: The world average for CKM angle γ produced by HFLAV split by analysis method.

This sub-degree precision will open up many interesting possibilties; allowing a precise tree-level benchmark for the SM, access to penguin free measurements of β and β_s, probes of NP sensitive scenarios with subtle differences in γ between charged and neutral initial B states as well as the ability to probe directly the tree-level Wilson coefficients, C_1 and C_2.

At this conference two new LHCb measurements were presented; a time-integrated determination of the CKM angle γ using $B^+ \to DK^+$ decays with $D \to K_S^0 h^+ h^-$ [3] and a time-dependent determination using $B^0 \to D^\mp \pi^\pm$ decays [7]. Furthermore, the latest LHCb combination [17], produced specifically for this conference, was shown. This details the most process single-experiment determination of γ to date and finds $\gamma = (74.0^{+5.0}_{-5.8})^\circ$. Future extrapolations suggest LHCb will continue to dominate global averages of the CKM angle γ in the future, reaching degree level precision in the next 5-10 years.

References

[1] J. Brod and J. Zupan, *The ultimate theoretical error on γ from $b \to dk$ decays*, Journal of High Energy Physics **2014** (2014) 51.

[2] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, *Determining γ using $B^\pm \to \mathrm{dk}^\pm$ with multibody d decays*, Phys. Rev. D **68** (2003) 054018.

[3] LHCb collaboration, R. Aaij *et al.*, *Measurement of the CKM angle γ using $B^\pm \to DK^\pm$ with $D \to K_S^0 \pi^+\pi^-$, $K_S^0 K^+ K^-$ decays*, `arXiv:1806.01202`, submitted to JHEP.

[4] J. Libby *et al.*, *New determination of the $D^0 \to K^-\pi^+\pi^0$ and $D^0 \to K^-\pi^+\pi^+\pi^-$ coherence factors and average strong-phase differences*, Phys. Lett. **B731** (2014) 197, `arXiv:1401.1904`.

[5] LHCb collaboration, R. Aaij *et al.*, *Measurement of the CKM angle γ using $B^\pm \to DK^\pm$ with $D \to K_S^0 \pi^+\pi^-$, $K_S^0 K^+ K^-$ decays*, JHEP **10** (2014) 097, `arXiv:1408.2748`.

[6] LHCb collaboration, R. Aaij *et al.*, *Measurement of CP asymmetry in $B_s^0 \to D_s^\mp K^\pm$ decays*, JHEP **03** (2018) 059, `arXiv:1712.07428`.

[7] LHCb collaboration, R. Aaij *et al.*, *Measurement of CP violation in $B^0 \to D^\pm \pi^\mp$ decays*, JHEP **06** (2018) 084, `arXiv:1805.03448`.

[8] BABAR Collaboration, B. Aubert *et al.*, *Measurement of the branching fractions of the rare decays $B^0 \to D_s^{(*)+}\pi^-$, $B^0 \to D_s^{(*)+}\rho^-$, and $B^0 \to D_s^{(*)-}K^{(*)+}$*, Phys. Rev. D **78** (2008) 032005.

[9] The Belle Collaboration, A. Das *et al.*, *Measurements of branching fractions for $B^0 \to D_s^+\pi^-$ and $\overline{B}^0 \to D_s^+K^-$*, Phys. Rev. D **82** (2010) 051103.

[10] HFLAV, Y. Amhis *et al.*, *Averages of b-hadron, c-hadron, and τ-lepton properties as of summer 2016*, Eur. Phys. J. **C77** (2017) 895, `arXiv:1612.07233`, updated results and plots available at `https://hflav.web.cern.ch`.

[11] M. Gronau and D. Wyler, *On determining a weak phase from CP asymmetries in charged B decays*, Phys. Lett. **B265** (1991) 172.

[12] M. Gronau and D. London, *How to determine all the angles of the unitarity triangle from $B^0 \to DK_S^0$ and $B_s^0 \to D\phi$*, Phys. Lett. **B253** (1991) 483.

[13] D. Atwood, I. Dunietz, and A. Soni, *Enhanced CP violation with $B \to KD^0$ ($\bar{D}^0$) modes and extraction of the CKM angle γ*, Phys. Rev. Lett. **78** (1997) 3257, `arXiv:hep-ph/9612433`.

[14] D. Atwood, I. Dunietz, and A. Soni, *Improved methods for observing CP violation in $B^\pm \to KD$ and measuring the CKM phase γ*, Phys. Rev. **D63** (2001) 036005, `arXiv:hep-ph/0008090`.

[15] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, *Determining γ using $B^\pm \to DK^\pm$ with multibody D decays*, Phys. Rev. **D68** (2003) 054018, `arXiv:hep-ph/0303187`.

[16] Y. Grossman, Z. Ligeti, and A. Soffer, *Measuring γ in $B^\pm \to K^\pm(KK^*)_D$ decays*, Phys. Rev. **D67** (2003) 071301, `arXiv:hep-ph/0210433`.

[17] LHCb collaboration, *Update of the LHCb combination of the CKM angle γ using $B \to DK$ decays*, LHCb-CONF-2018-002.

[18] A. Bondar, *Proceedings of BINP special analysis meeting on Dalitz analysis*, 24.–26. Sep, 2002. Unpublished.

[19] R. Fleischer, *New strategies to obtain insights into CP violation through $B_s^0 \to D_s^\pm K^\mp$, $D_s^{*\pm}K^\mp$, ... and $B^0 \to D^\pm\pi^\mp$, $D^\pm_s\pi^\mp$, ... decays*, Nucl. Phys. **B671** (2003) 459, `arXiv:hep-ph/0304027`.

[20] R. Aleksan, I. Dunietz, and B. Kayser, *Determining the CP violating phase γ*, Z. Phys. **C54** (1992) 653.

[21] I. Dunietz and R. G. Sachs, *Asymmetry between inclusive charmed and anticharmed modes in B^0, $\bar{B}^0$ decay as a measure of CP violation*, Phys. Rev. **D37** (1988) 3186.

[22] LHCb collaboration, *Measurement of the CKM angle γ from a combination of $B \to DK$ analyses*, LHCb-CONF-2017-004, Retired Conference note. Please do not cite it.

[23] LHCb collaboration, R. Aaij *et al.*, *Measurement of CP observables in $B^\pm \to D^{(*)}K^\pm$ and $B^\pm \to D^{(*)}\pi^\pm$ decays*, Phys. Lett. **B777** (2017) 16, `arXiv:1708.06370`.

[24] LHCb collaboration, R. Aaij *et al.*, *Measurement of CP observables in $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ with two- and four-body D decays*, Phys. Lett. **B760** (2016) 117, `arXiv:1603.08993`.

[25] LHCb collaboration, R. Aaij *et al.*, *A study of CP violation in $B^\mp \to Dh^\mp$ ($h = K, \pi$) with the modes $D \to K^\mp\pi^\pm\pi^0$, $D \to \pi^+\pi^-\pi^0$ and $D \to K^+K^-\pi^0$*, Phys. Rev. **D91** (2015) 112014, `arXiv:1504.05442`.

[26] LHCb collaboration, R. Aaij *et al.*, *A study of CP violation in $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ decays with $D \to K_S^0K^\pm\pi^\mp$ final states*, Phys. Lett. **B733** (2014) 36, `arXiv:1402.2982`.

[27] LHCb collaboration, R. Aaij *et al.*, *Measurement of CP observables in $B^{\pm} \to DK^{*\pm}$ decays using two- and four-body D-meson final states*, JHEP **11** (2017) 156, `arXiv:1709.05855`.

[28] LHCb collaboration, R. Aaij *et al.*, *Study of $B^{-} \to DK^{-}\pi^{+}\pi^{-}$ and $B^{-} \to D\pi^{-}\pi^{+}\pi^{-}$ decays and determination of the CKM angle γ*, Phys. Rev. **D92** (2015) 112005, `arXiv:1505.07044`.

[29] LHCb collaboration, R. Aaij *et al.*, *Measurement of CP violation parameters in $B^{0} \to DK^{*0}$ decays*, Phys. Rev. **D90** (2014) 112002, `arXiv:1407.8136`.

[30] LHCb collaboration, R. Aaij *et al.*, *Constraints on the unitarity triangle angle γ from Dalitz plot analysis of $B^{0} \to DK^{+}\pi^{-}$ decays*, Phys. Rev. **D93** (2016) 112018, Erratum ibid. **D94** (2016) 079902, `arXiv:1602.03455`.

[31] LHCb collaboration, R. Aaij *et al.*, *Measurement of the CKM angle γ using $B^{0} \to DK^{*0}$ with $D \to K^{0}_{S}\pi^{+}\pi^{-}$ decays*, JHEP **08** (2016) 137, `arXiv:1605.01082`.