PROCEEDINGS OF SCIENCE

Theory of rare K decays

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I review rare kaon decays. I introduce the flavor problem and possible solutions. Very rare kaon decays like $K \to \pi\nu\bar{\nu}$ are very important to this purpose: we study also $K \to \pi l^+ l^-$, $K \to \pi\pi ee$ where chiral dynamics is important to disentangle short distance effects. We discuss also the decays $K^0 \to \mu^+\mu^-$, which have received recently some attention due to the measurement by LHCB.

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1. Introduction and $K \to \pi\nu\bar{\nu}$

Rare kaon decays furnish challenging MFV probes and will severely constrain flavor physics motivated by New Physics (NP) [1]. SM predicts the $V-A \otimes V-A$ effective hamiltonian (Fig. 1)

$$\mathscr{H} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} (\underbrace{V_{cs}^* V_{cd}\, X_{NL}}_{\lambda x_c} + \underbrace{V_{ts}^* V_{td} X(x_t)}_{A^2\lambda^5\,(1-\rho-i\eta)x_t})\, \bar{s}_L \gamma_\mu d_L\, \bar{\nu}_L \gamma^\mu \nu_L, \tag{1.1}$$

$x_q = m_q^2/M_W^2$, θ_W the Weak angle and X's are the Inami-Lin functions with Wilson coefficients known at two-loop electroweak corrections; the main uncertainties are due to the strong corrections in the charm loop contribution. The structure in (1.1) leads to a pure CP violating contribution to $K_L \to \pi^0\nu\bar{\nu}$, induced only from the top loop contribution and thus proportional to $\Im m(\lambda_t)$ ($\lambda_t = V_{ts}^* V_{td}$) and free of hadronic uncertainties. This leads to the prediction

$$\mathscr{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = (2.9 \pm 0.2) \times 10^{-11} \qquad \mathscr{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (8.3 \pm 0.9) \times 10^{-11}.$$

where the parametric uncertainty due to the error on $|V_{cb}|$, ρ and η is shown.

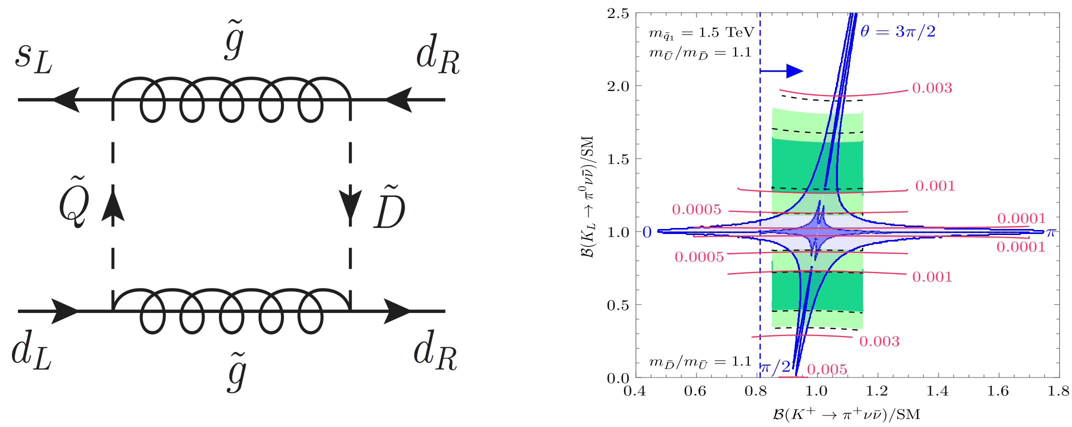

Figure 1: Impact of $K \to 2\pi$ isospin breaking terms ($\Im(A_2)$) (left) on $K \to \pi\nu\bar{\nu}$, typical effects, (right)

Typical BSM predict new flavor structures that might affect $K \to \pi\nu\bar{\nu}$ that now can be tested at NA62 and KOTO [2]; we describe two different BSM effects i) new flavor structures for ε' avoiding $\Delta S = 2$ constraints (Fig. 1) [3, 1] and ii) attempts to describe B-anomalies [4], typically induce large flavor effects at O(1) TeV [5]. i) the recent lattice results for $K \to 2\pi$ leave open the possibility of BSM for ε'; to isospin breaking terms in $\Im(A_2)$ have been studied [3] in Fig.1. We expect effects at most 10% in $K^+ \to \pi^+\nu\bar{\nu}$ while are more sizable for $K_L \to \pi^0\nu\bar{\nu}$. Theoretically addressing flavor in Randall Sundrum models is more challenging: we have studied the so called flavor anarchy scenario with 5D MFV and custodial symmetry; the only sources of flavor breaking are two 5D anarchic Yukawa matrices. These matrices also generate also the bulk masses, which are responsible for the resulting flavor hierarchy. The theory flows to a next to minimal flavor violation model where flavor violation is dominantly coming from the 3rd generation. We show that it is possible to find a range of parameters for bulk masses satisfying experimental flavor constraints, but also we explain the neutral B-anomalies, requiring NP flavor scale at O(1) TeV.

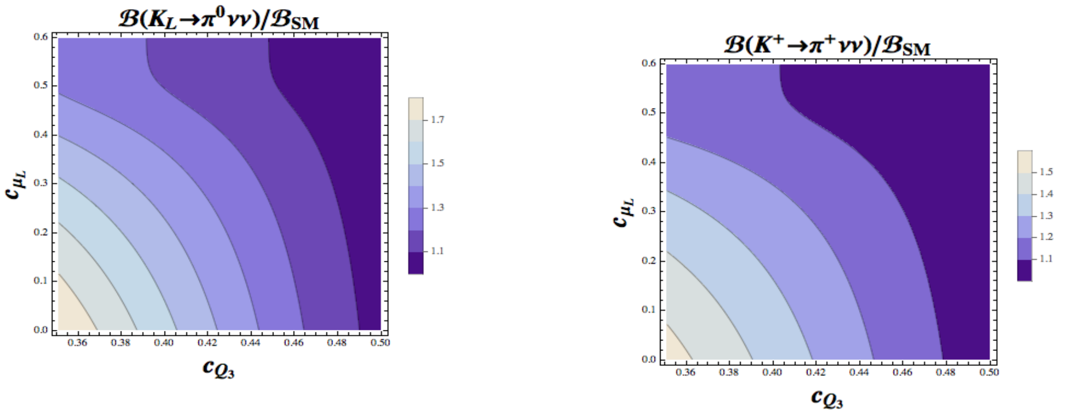

Figure 2: RS scenario to explain B-anomalies: $B(K \to \pi \nu \bar{\nu})$ ranges as a function of fermion profiles (c_i's)

Then we address $K \to \pi \nu \bar{\nu}-$decays: we show the TH predictions as a function f the bulk fermion masses in Fig.2 [5]. A natural issue is to test O(1) TeV physics at LHC; we are trying to apply the technique of Ref. [6] to this purpose.

2. $K_{L,S} \to \mu^+ \mu^-$

Recent $K_S \to \mu \overline{\mu}$ LHCB measurement is very interesting and unexpected

$$B(K_S \to \mu \overline{\mu})_{LHCB} < 9 \times 10^{-9} \text{ at } 90\ \% \text{ CL} \qquad B(K_S \to \mu \overline{\mu})_{SM} = (5.0 \pm 1.5) \times 10^{-12}. \tag{2.1}$$

It represents an important milestone since it has improved the previous limit, $< 3.2 \times 10^{-7}$ at 90 % CL, lasted 40 years. It is based on a production of 10^{13} K_S per fb^{-1} inside the LHCB acceptance and it is obtained using 1.0 fb^{-1} of pp collisions at $\sqrt{s} = 7$ TeV collected in 2011.

Two photon exchange generates the dominant contribution for both K_L and K_S decays to two muons [7]. The structure of weak and electromagnetic interactions entails a vanishing CP conserving short distance contribution to $K_S \to \mu^+ \mu^-$. Indeed the SM short diagrams (similar to $K \to \pi \nu \bar{\nu}$ in Fig. 2) lead to the SM effective hamiltonian similar to eq. (1.1).

The LD contributions to $K_S \to \mu^+ \mu^-$ Fig. (4) have been computed reliably in CHPT ($B = (5.0 \pm 1.5) \times 10^{-12}$). The relevant short distance contributions are

$$B(K_S \to \mu \overline{\mu})^{SD}_{SM} = 1 \times 10^{-5} |\Im(V^*_{ts} V_{td})|^2 \sim 10^{-13} \qquad \text{vs} \qquad B(K_S \to \mu \overline{\mu})_{NP} \leq 10^{-11} \tag{2.2}$$

We have shown that in some appealing susy scenario in Fig. (3) [8] large allowed new physics contributions (NP) can be substantially larger that SM SD contributions.

The short distance hamiltonian will contribute also to $K_L \to \mu \overline{\mu}$, through a CP conserving amplitude, $\Re(A_{\text{short}})$, that has to be disentangled from the large LD two-photon exchange contributions, $A_{\gamma\gamma}$: the absorptive LD contribution is much larger than SD, in the rate respectively 25 times larger than dispersive; total ($B_{\text{expt}} = (6.84 \pm 0.11) \times 10^{-9}$) To extract SD info the situation would be better if we would know the sign of $A_{\gamma\gamma}$, theoretically and experimentally unknown. While K_L-decays outside the LHCB fiducial volume the interference $A(K_L \to \mu \overline{\mu}^* A(K_S \to \mu \overline{\mu}$ may affect the LHCB K_S-rates: we can study the time interference $K_{S,L} \to \mu \mu$; this can be done by flavor tagging $K^0\bar{K}^0$, specifically by detecting the associated $\pi^\pm$ and (or) $K^\mp$, determining the impurity parameter $D = \frac{K^0-\bar{K}^0}{K^0+\bar{K}^0}$. Then interference term will affect the measured branching [7]:

$$\mathscr{B}(K_S^0 \to \mu^+\mu^-)_{\text{eff}} = \tau_S \left(\int_{t_{\min}}^{t_{\max}} dt e^{-\Gamma_S t}\varepsilon(t)\right)^{-1} \left[\int_{t_{\min}}^{t_{\max}} dt \left\{\Gamma(K_S^0 \to \mu^+\mu^-)e^{-\Gamma_S t} + \frac{D f_K^2 M_K^3 \beta_\mu}{8\pi} \mathrm{Re}\left[i\left(A_S A_L - \beta_\mu^2 B_S^* B_L\right) e^{-i\Delta M_K t}\right] e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\right\}\varepsilon(t)\right], \tag{2.3}$$

Then we are i) increasing the sensitivity to short distance and ii) possibly determining the sign $A_{L\gamma\gamma}$

$$\sum_{\text{spin}} \mathscr{A}(K_1 \to \mu^+\mu^-)^* \mathscr{A}(K_2 \to \mu^+\mu^-) \sim \underbrace{\mathrm{Im}[\lambda_t] y'_{7A}}_{SD} \left\{ \underbrace{A^\mu_{L\gamma\gamma}}_{LD} - 2\pi \sin^2\theta_W \left(\mathrm{Re}[\lambda_t] y'_{7A} + \mathrm{Re}[\lambda_c] y_c\right) \right\} \tag{2.4}$$

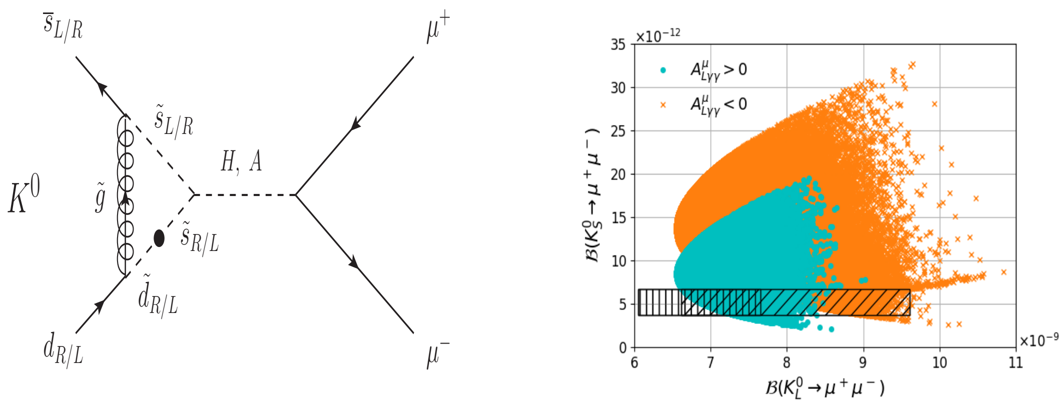

Figure 3: Susy scenario: $K_S \to \mu\mu$ diagram (left), theory predictions: in dashed area no interference effects are considered (right)

Experimentally, one can also access an *effective* branching ratio of $K_S^0 \to \mu^+\mu^-$ [7] which includes an interference contribution with $K_L^0 \to \mu^+\mu^-$ in the neutral kaon sample.

LHCB has also a beautiful kaon physics program [10, 11, 12]

	PDG	Prospects	
$K_S \to \mu\mu$	$< 9\times 10^{-9}$ at 90% CL	$(\text{LD})(5.0\pm 1.5)\cdot 10^{-12}$	$\text{NP} < 10^{-11}$
$K_L \to \mu\mu$	$(6.84\pm 0.11)\times 10^{-9}$	difficult : SD << LD	
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2\times 10^{-14}$	
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$	
$K_S \to eeee$	–	$\sim 10^{-10}$	
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$	

(2.5)

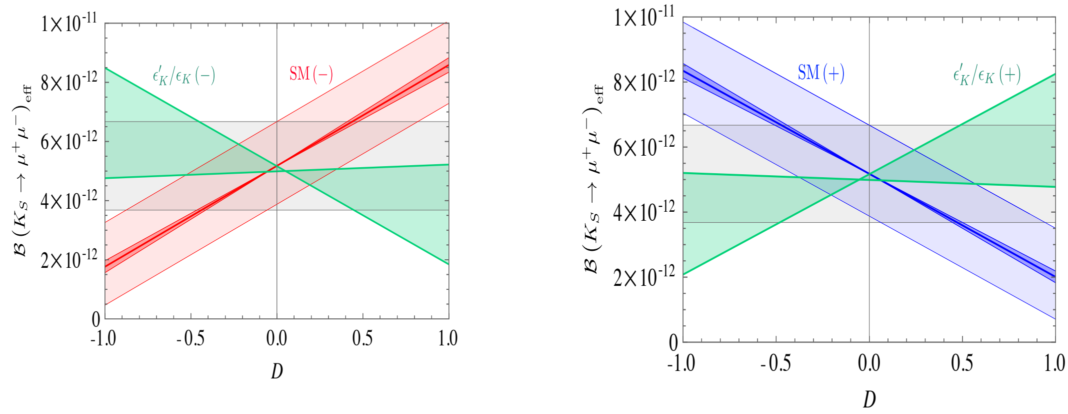

Figure 4: K_S LD diagram (left), interference effect from eq. 2.4 on $\mathscr{B}(K_S \to \mu^+\mu^-)$ depending on the $A_{L\gamma\gamma}$ sign: negative (center and in red SM while in green NP contributions) and positive (right and in blue SM while in green NP contributions).

3. The weak chiral lagrangian

In Ref. [9] we have studied how to determine the weak O(p^4) chiral countertems in

$$\mathscr{L}_{\Delta S=1} = \mathscr{L}^2_{\Delta S=1} + \mathscr{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma, K\to\pi l^+l^-} + \cdots .$$

Due to the accurate NA48/2 study of the decays $K^\pm \to \pi^\pm\pi^0\gamma$ and $K^\pm \to \pi^\pm\pi^0 e^+e^-$ the subset of CT's in the table can be determined

$$\begin{array}{llll} K^\pm \to \pi^\pm\gamma^* & N^r_{14} - N^r_{15} & a_+ = -0.578 \pm 0.016 & \text{NA48/2} \\ K_S \to \pi^0\gamma^* & 2N^r_{14} + N^r_{15} & a_S = (1.06^{+0.26}_{-0.21} \pm 0.07) & \text{NA48/1} \\ K^\pm \to \pi^\pm\pi^0\gamma & N^r_{14} - N^r_{15} - N^r_{16} - N_{17} & X_E = (-24 \pm 4 \pm 4)\ \text{GeV}^{-4} & \text{NA48/2} \\ K^+ \to \pi^+\gamma\gamma & N^r_{14} - N^r_{15} - 2N^r_{18} & \hat{c} = 1.56 \pm 0.23 \pm 0.11 & \text{NA48/2} \end{array} \qquad (3.1)$$

4. $K_{S,L} \to l^+l^-$, $K_{S,L} \to l^+l^-l^+l^-$ and $K_S \to \pi^+\pi^-l^+l^-$

Recent LHCB limit on $K_S \to \mu\overline{\mu}$ in the table is close to test interesting New Physics (NP) models. A high precision measurement can test the short distance (SD) SM but it requires to improve the long distance (LD) prediction with auxiliary channels [10]. $K_L \to \mu\mu$: the small ratio SD/LD $\sim \frac{1}{30}$ may obscure an experimental improvement on the rate. The situation would be a bit ameliorated if the sign for theoretical unknown sign of $A(K_L \to \gamma\gamma)$ would be known. Help to this ambiguity could come from the experimental study of $K_{S.L} \to l^+l^-l^+l^-$ [10] As shown in table these channels are at reach in a high intensity machine and they may also give LD distance info needed for a better control of $K_L \to \mu\mu$. These four body decays have also a peculiar feature, similarly to $K_{S,L} \to \pi^+\pi^-e^+e^-$, the two different helicity amplitudes interfere; then one can measure the sign $K_L \to\to \gamma^*\gamma^* \to l^+l^-l^+l^-$ by studying the time interference K_S K_L which it has a decay length $2\Gamma_S$ [10].

The interplay between LHCB and NA62 program is nicely shown in ref [12].

$K_S \to \pi^+\pi^- l^+ l^-$ Following the study of $K^+ \to \pi^+\pi^0 l^+ l^-$in Ref. [13] we have studied the decay $K_S \to \pi^+\pi^- l^+ l^-$ [9], that it has been studied by NA48/2 and it is a target of LHCB. One finds that the long-distance contributions to $K_S \to \pi^+\pi^-\gamma^*$ can be determined with remarkable accuracy, namely

$$BR(K_S \to \pi^+\pi^- e^+ e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}} . \tag{4.1}$$

This number is in excellent agreement with the PDG average [14]:

$$BR(K_S \to \pi^+\pi^- e^+ e^-)_{exp} = (4.79 \pm 0.15) \times 10^{-5} . \tag{4.2}$$

Similarly, one can predict that

$$BR(K_S \to \pi^+\pi^- \mu^+ \mu^-) = \underbrace{4.17 \cdot 10^{-14}}_{\text{Brems.}} + \underbrace{4.98 \cdot 10^{-15}}_{\text{Int.}} + \underbrace{2.17 \cdot 10^{-16}}_{\text{DE}} . \tag{4.3}$$

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