PROCEEDINGS OF SCIENCE

BSM physics and the lattice

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We review some selected results from lattice field theories beyond QCD. In particular we focus on finding evidence for the existence of infrared fixed points, on the dynamics of theories just outside the conformal window, and on the recent interest for simulations of Composite Higgs models. We discuss the potential impact of lattice simulations on BSM model building.

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1. Introduction

The latest results at the LHC, including the ones presented at this conference, put stringent bounds on the size of deviations from Standard Model predictions, and the absence of new states at the TeV scale makes it very difficult to develop models of new physics beyond the Standard Model (BSM). Without a clear steering from experiments, a wide range of possibilities remains open, while an increasing amount of fine tuning seems to be necessary in order to explain the lack of sizeable deviations from the Standard Model. LHC experiments have accumulated an unprecedented amount of data; a careful scrutiny of these data is currently under way and will test the Standard Model to even more stringent levels of precision. Effective Field Theories (EFT) provide an agnostic approach to quantify the possible deviations from the Standard Model: any new physics is encoded in the low-energy constants (LECs) that appear in the effective lagrangian. Bounds on the LECs can be obtained from data, and ultimately should shed some light on the underlying new dynamics. While this approach is clearly unbiased, it is hampered by the large amount of LECs that appear when considering higher-dimensional operators that can be added to the Standard Model lagrangian. Signficant progress has been achieved using this kind of bottom-up approach, see e.g. Ref. [1] for a recent review.

While the parametrization provided by the EFT approach is the most general based on symmetry grounds, assuming a specific BSM dynamics implies constraints on the LECs, thereby making any new theory more predictive, and ultimately easier to be tested against experimental data.

Models that feature a strongly-interacting sector are the focus of this talk, as lattice field theory provides a tool to calculate beyond perturbation theory, and therefore make quantitative predictions of relevance for BSM phenomenology. Since the original technicolor proposal, there have been multiple incarnations of strongly-interacting dynamics beyond the Standard Model. Of particular interest here are the so-called Composite Higgs models [2, 3], where the hierarchy problem is solved by assuming that the Higgs boson is the pseudo Goldstone boson of some broken symmetry of the underlying theory, and models of the Higgs boson as dilaton [4], where the light Higgs is a remnant of a broken scale invariance in the IR sector of the theory, see e.g. Ref. [5] and references therein. The strong dynamics of the new sector is responsible for the phenomena that lead to light states in the spectrum: spontaneous breaking of flavor symmetry for the case of pseudo Goldstone bosons, and spontaneous (or anomalous) breaking of scale invariance for the case to the dilaton.

While for phenomenological purposes an EFT approach is perfectly suitable to describe the dynamics of the low-energy degrees of freedom, a UV-complete theory is necessary in order to perform lattice simulations. Having committed to a specific UV completion, it is possible to compute the LECs of the EFT from first principles, and therefore constrain these kind of models. The situation is analogous to the case of QCD. The low-energy interactions of pions are described by the chiral lagrangian, where the LECs are unknown parameters to be determined from experimental data. However, given the UV complete theory, i.e. quantum chromodynamics, it is possible to simulate the theory and compute the LECs. For BSM physics this is a numerically challenging programme, and requires a choice of the UV completion in order to be able to proceed. Unfortunately at this stage there are very few hints from experiments that would suggest a realistic UV completion. If we insist on the analogy with QCD, it is useful to keep in mind that the parton picture, and the symmetry pattern that ultimately led to QCD, emerged from the knowledge of a multitude of hadronic states. For the case of Composite Higgs models, the only state of the new theory that has been observed so far is the Higgs boson itself.

In the absence of experimental guidance, theorists have relied on mathematical consistency in order to identify theories of potential interest. Recent works presented a systematic classification of UV completions for Composite Higgs models [6]. Some models have been proposed and have already been studied numerically in Refs. [7, 8, 9], see also Refs. [10, 11] for recent reviews.

2. IR conformal theories

Approximate symmetry under scale transformations has been advocated as a potential explanation for the appearance of a light scalar state in the spectrum of a QFT. Surely the first step to study near-conformal dynamics must be a quantitative undestanding of theories that are genuinely scale-invariant. It is know that gauge theories with a sufficiently large number of fermion fields can develop a so-called infrared (IR) fixed point, i.e. at low energies the gauge coupling flows to a fixed point value, where the beta function vanishes [12, 13]. These examples feature a perturbative zero of the beta function at small values of the coupling. The fixed points are characterized as usual by the anomalous dimensions of the relevant operators; perturbative solutions usually yield small anomalous dimensions, which are not ideally suited for model building. However, a large landscape of gauge theories can be explored by varying the gauge group, the representation in which the fermionic matter is introduced, and the number of fermionic species. The range in the number of fermions for which a fixed point develops is often called the *conformal window*. Exploring this landscape, mapping the boundaries of the conformal window, may yield examples of theories that have a strongly-interacting fixed point [14]. As for the phenomenological aspects, deformations of theories with large anomalous dimensions could provide models of BSM dynamics [15].

Finding fixed points by numerical simulations is a difficult task, as the lattice reguarization breaks scale invariance in many ways:

- simulations being performed by definition in a finite volume means that the large-distance regime may not be reached on current lattices;
- results of simulations performed at finite values of the fermion mass need to be extrapolated to the massless limit;
- the lattice regularization introduces irrelevant operators in the action, whose effect can mask the conformal behaviour.

All these effects need to be taken into account, and are conveniently summarised in scaling laws that describe the approach of observables to the conformal limit [16, 17, 18, 19, 20]. In particular the scaling of the spectrum as a function of the fermion mass, and the density of eigenvalues of the Dirac operator, allow the extraction of the mass anomalous dimension, which is assumed to be the only relevant operator at the IR fixed point. The scaling laws for these quantities are

$$M_X \propto m^{1/(1+\gamma)}, \quad \rho(\lambda) \sim \lambda^{(3-\gamma)/(1+\gamma)}, \tag{2.1}$$

where M_X denotes all masses in the spectrum, m is the fermion mass, $\rho(\lambda)$ is the density of eigenvalues, and γ the mass anomalous dimension. While it is impossible to make justice to all the studies that have been performed recently, we show here two scaling plots, which illustrate the behaviours above, for an SU(2) gauge theory with two Dirac fermions in the adjoint representation. The figure on the left shows that the ratio $M_X/m^{1/(1+\gamma)}$ remains constant as a function of the fermion mass; this behaviour is characteristic of IR-conformal theories, and is clearly different from the behaviour in a chirally-broken theory like QCD. The plot on the right shows the behaviour of the eigenvalue density, see Ref. [20] for more details. These two measurements allow two independent determinations of γ, which yield a consistent value of $\gamma = 0.37 \pm 0.02$. The small value of the anomalous dimensions disfavours this theory for BSM model building. Similar results have

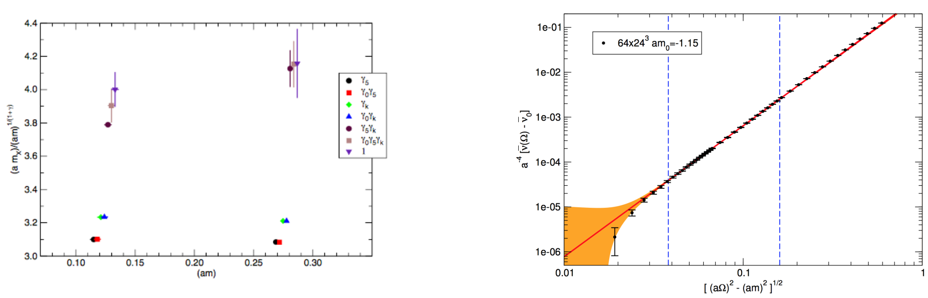

Figure 1: Scaling of the states in the spectrum as a function of the fermion mass (left), and density of eigenvalues of the Dirac operator (right). Both behaviours allow the extraction of the anomalous mass dimension γ. The plots are taken from Refs. [21], and [20] respectively.

been obtained for a number of theories, and it is impossible to summarise them all here. Recent results have appeared in Refs. [22, 23], and we refer the reader to review papers [10, 24] for a more extensive summary of results.

3. Under the sill of the conformal window

A possibility for model building is to consider theories that are confining, but 'close' (in the space of theories) to the point where an IR-conformal fixed point would develop. This can be achieved e.g. by tuning the number of fermionic species to be just below the value where the fixed point is found, so just under the sill of the conformal window discussed above. The theoretical expectation is that the 'proximity' to a fixed point is enough to yield a light scalar in the spectrum, i.e. a light Higgs as a composite particle of a strongly-interacting sector. Despite the fact that there is not a strong theoretical underpinning of these models, numerical studies have focussed on two theories that are potential candidates close to the conformal window, namely SU(3) with eight flavors in the fundamental representation of the color group [25, 26], and SU(3) with two flavors in the sextet representation [9].

Results for the spectrum of SU(3) gauge theory with eight flavors are shown in Fig. 2. The striking feature in these data is the existence of a 0^{++} state that remains degenerate with the pseudoscalar states, the latter being the pseudo Nambu Goldstone bosons of chiral symmetry. Both groups confirm the same pattern in the masses of the theory.

Results for the spectrum of the SU(3) theory with sextet fermions are shown in Fig. 3. Once again a light scalar state is observed, which remains lighter than its pseudoscalar counterpart. The

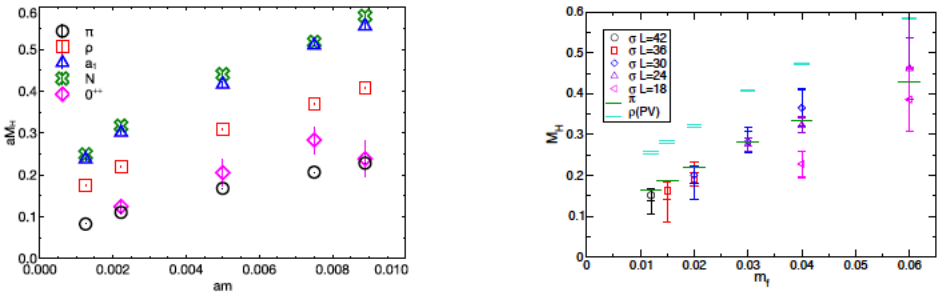

Figure 2: Data for the spectrum of SU(3) with eight flavors, as obtained in Ref. [25] (left), and Ref. [26] (right). In the plot on the left, the mass of several states is shown as a function of the fermion mass, while in the plot on the right the authors have focussed mostly on the mass of the ligt scalar for several values of the volume, and the pseudoscalar. It is reassuring to see that finite volume effects seems to be under control.

spectrum is computed for two different values of the bare gauge coupling, showing that lattice artefacts are reasonably under control.

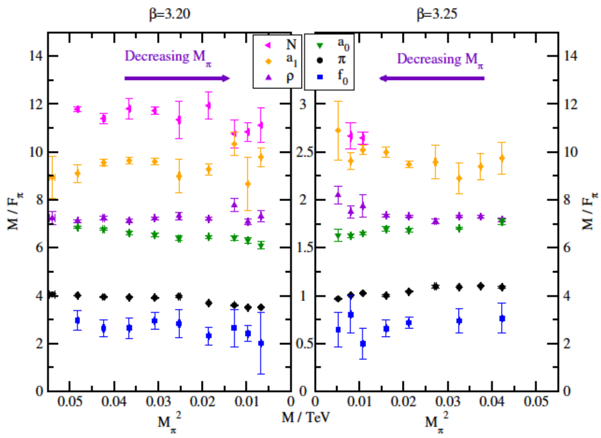

Figure 3: Data for the spectrum of SU(3) with two sextet flavors. Figure taken from Ref. [27]

Although these numerical results seem to suggest an interesting pattern, the actual dynamics that is responsible for the light scalar states is not fully understood.

4. A dilaton EFT

Assuming that the light scalar state observed in numerical simulations is a pseudo Nambu Goldstone boson (PNGB) associated to the breaking of scale symmetry, the dynamics of this *dilaton* can be encoded by an effective theory. Several proposals have been put forward, resulting in effective lagrangians, which encode the nonperturbative effects in the new sector via a finite number of LECs. Recent examples of this type of approach can be found in Refs. [28, 29, 30, 31, 32]. While we cannot review here the details of each approach, it is interesting to discuss an example.

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The effective lagrangian presented in Ref. [30] is given by

$$\begin{aligned}\mathscr{L} =& \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\,\left[\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right] \\ &+ \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\,\left[\Sigma+\Sigma^\dagger\right] - \kappa\chi^p\,, \end{aligned} \tag{4.1}$$

where $y = 3-\gamma$ is the scaling dimension of $\bar\psi\psi$, and the dilaton potential is characterised by the exponent p. The dilaton field is χ, and Σ is the usual non-linear representation of the light PNGB degrees of freedom. The effective lagrangian leads to the following scaling relations between the fermion mass m, the mass of the PNGBs M_π, the mass of the dilaton M_d, and their decay constants f_π, f_d:

$$\begin{aligned} M_\pi^2 F_\pi^{2-y} &= Cm\,, \\ M_\pi^2 &= BF_\pi^{p-2}\,, \\ M_d^2 &= \frac{yn_f}{2}\frac{f_\pi^2}{f_d^2}(p-y)BF_\pi^{p-2}\,. \end{aligned}$$

Fitting the lattice data shown in the Section above to these scaling relations allows the extraction of p and y. The results of the fit are surprisingly similar for the two theories,

$$\begin{cases} \mathrm{SU}(3), n_f = 8, \mathrm{fund}, & y = 2.1\pm 0.1, p = 4.3\pm 0.2, \frac{f_\pi^2}{f_d^2} = 0.08\pm 0.04\,, \\ \\ \mathrm{SU}(3), n_f = 2, \mathrm{sextet}, & y = 1.9\pm 0.1, p = 4.4\pm 0.3, \frac{f_\pi^2}{f_d^2} = 0.09\pm 0.06\,, \end{cases}$$

suggesting some underlying dynamical reason related to the proximity of the conformal window. A clear understanding of this behaviour is an interesting challenge in this field.

5. Composite Higgs

Composite Higgs (CH) models have a long history going back to Refs. [2, 3]; unlike technicolor models, the spectrum of CH models contains a Higgs boson, which is a bound state made of elementary constituents interacting strongly. The Higgs boson in these models is light because it is the pseudo Nambu Goldstone boson associated with the spontaneous breaking of some global symmetry in the strong sector. For a recent review on the topic, see e.g. Ref. [33], and the extensive references therein. Here we will focus on one specific realisation, the Ferretti model [34], which has been the focus of recent lattice studies. This model provides an explicit UV complete theory based on a SU(4) gauge group, and a matter content that produces a phenomenologically interesting pattern of global symmetries, while preserving gauge anomaly cancellation and asymptotic freedom, see Ref. [6] for an exhaustive study of possible candidate theories. The end result is a theory with fermions in multiple representations of the gauge group – namely five Weyl fermions in the two-index antisymmetric representation of SU(4), and three Dirac fermions in the fundamental representation of SU(4). Such theories can be discretised on a Euclidean spacetime lattice, and studied by numerical simulations.

Once again the phenomenology of this model is better studied using an effective lagrangian description. For instance the Higgs potential, which is entirely generated by the coupling to the Standard Model bosons and top quark, is described in terms of two constants

$$V(H) = \alpha \cos \frac{2H}{f} - \beta \sin^2 \frac{2H}{f} \,. \tag{5.1}$$

The LECs α and β are constrained by requiring that the potential has a symmetry breaking minimum, and by the experimental measurements of the ratio $\xi = v^2/f^2$ and of the mass of Higgs. The above conditions lead to the system of equations

$$\begin{aligned} \alpha + 2\beta &> 0 \,, \\ \xi = v^2/f^2 &= -\alpha/(2\beta) \,, \\ m_h^2/v^2 &= 8\,(2\beta - \alpha) \,, \end{aligned}$$

where v is the Higgs vev, f is the decay constant of the PNGB, and m_h is the Higgs mass. The current experimental bounds select the purple wedge in Fig.4 as the allowed region in the (α, β) plane.

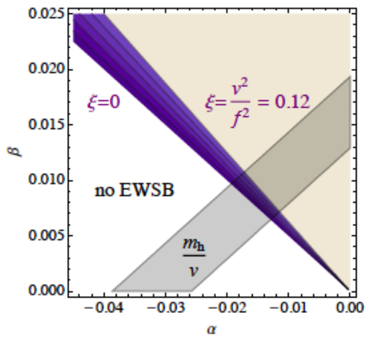

Figure 4: Allowed region in the (α, β) plane. The white area is excluded because EW symmetry is unbroken, while the yellow area corresponds to values of v^2/f^2 that are too large to be compatible with experimental results. The purple wedge is the region obtianed when v^2/f^2 spans from 0 to 0.12. The grey band is obtained from the current mesaurement of the Higgs mass. Figure from Ref. [35].

Given the explicit UV completion, the constants α and β can be computed from Monte Carlo simulations. They are obtained from field correlators in the strongly-interacting sector, as discussed in Refs. [36, 35, 37]. Even though these computations are very demanding from the numerical point of view, they would yield first principle results that would quantify the viability of this model. For example, values of α or β that are outside the intersection of the purple and grey regions would rule the model out.

Other quantities of interest, which can be accessed more easily by numerical simulations, are the spectrum of the theory, in particular the spectrum of the vector resonances that could in principle be detected as new states at collider experiments, and the baryonic spectrum, which describes the top partners in models of partial compositeness.

First simulations with fermions in multiple representations have been developed in recent years, for a theory that is close to the one proposed by Ferretti. At this stage most of the focus should be on understanding algorithmic issues, and mapping the space of bare parameters, in order to disentangle systematic errors from interesting physical effects. For first results see e.g. Refs. [38, 39, 40].

6. Outlook

As the LHC data put increasingly severe bounds on BSM models, lattice simulations can provide a quantitative insight in the dynamics of strongly interacting theories. These are interesting in their own right as theoretical studies, but their phenomenological potential can only be explored by working close to model building and experiment.

Composite Higgs models have a rich nonperturbative phenomenology that can be explored by Monte Carlo simulations. First results are starting to appear, which involve simulations with fermions in multiple representations of the gauge group, a new, uncharted territory for lattice field theories.

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