

Theory on $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ anomaly

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The observed excess in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ is one of the major anomalies in particle physics. After a short summary of experimental results and standard model predictions, we introduce a model-independent approach using an effective field theory in order to identify possible new physics scenarios. The allowed regions of Wilson coefficients are obtained with the most recent experimental data. We also discuss other observables, q^2 distributions and a tau polarization. We briefly mention future prospects of experiments.

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1. Introduction

The branching fractions of exclusive semitauonic B meson decays, $\bar{B} \rightarrow D \tau \bar{\nu}$ and $\bar{B} \rightarrow D^* \tau \bar{\nu}$ are predicted as $O(1)\%$ in the standard model (SM) and millions of these decay events had taken place at B factory experiments (The precise branching fractions are given below.) They are not rare in this sense. Their experimental observation, however, is not as easy as other semileptonic decays to an electron or a muon, $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}$ ($\ell = e, \mu$). The tau lepton in the decay products further decays in the detector resulting in two or more neutrinos in the final state, which make the event identification harder. So, the first observation was reported by Belle collaboration in 2007[1], in a rather late stage of the experiment. Data of BaBar and LHCb as well as Belle are available at present and the current experimental status is summarized below.

The processes of $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ are described by $b \rightarrow c \tau \bar{\nu}$ at the quark level, which is mediated by the W boson in the SM as shown in Fig. 1. The relevant leptonic coupling of the W boson is the same as $b \rightarrow c \ell \bar{\nu}$. This lepton flavor universality (LFU) may be broken in the physics beyond the SM. For instance, the charged Higgs boson in the two-Higgs-doublet model (2HDM) of type II or the minimal supersymmetric standard model (MSSM) has a coupling proportional to the charged lepton mass, m_τ or m_ℓ . As pointed out in Ref. [2], the charged Higgs contribution to the decay amplitude (see Fig. 1) is proportional to $m_b m_\tau \tan^2 \beta$, where $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets, and significantly enhanced for large $\tan \beta$ as expected in $SO(10)$ GUT.

In Sec. 2, we introduce observables, R 's, that quantify violation of LFU. A summary of the current experimental status and SM predictions are also given. In Sec. 3, an effective field theory for $b \rightarrow c \tau \bar{\nu}$ is introduced and a model-independent analysis with it is presented. Then, we discuss other observables than R 's in Sec. 4. Section 5 is devoted to our summary.

2. Testing lepton flavor (non)universality

The SM consists of three sectors: gauge, Higgs and Yukawa sectors. While the first two sectors follow the lepton flavor universality, the third one does not. Yukawa couplings give charged leptons of three flavors different masses, m_e , m_μ and m_τ , which lead to the lepton flavor nonuniversality (LFNU) in the SM.

It is convenient (and customary) to introduce the following ratios of branching fractions, $R(D)$ and $R(D^*)$,

$$R(D^{(*)}) := \frac{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell)}. \quad (2.1)$$

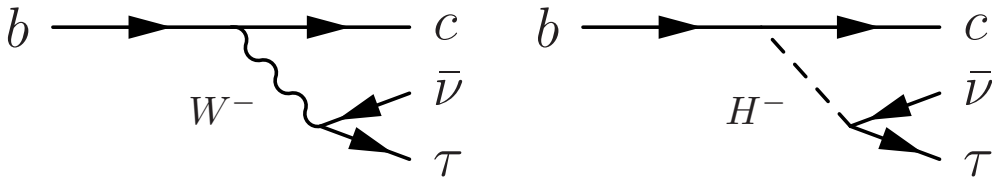


Figure 1: Feynman diagrams of the semitauonic b decay. Left: W boson exchange in the SM. Right: charged Higgs boson exchange in 2HDM.

In the SM, the dominant source of LFNU that deviates $R(D^{(*)})$ from unity is the tau mass as mentioned above and both the numerators and the denominators in these ratios are calculable in principle. Taking ratios provides us with two advantages for theoretical predictions[3]: one is the cancellation of the relevant Kobayashi-Maskawa matrix element, V_{cb} , and another is that uncertainties in form factors that describe the hadronic matrix elements tend to reduce though they still dominate the theoretical uncertainty.

The hadronic matrix element of the $b \rightarrow c$ left-handed charged current in the $B \rightarrow D$ transition is described by two form factors. One of them does not contribute to the normalization mode $\bar{B} \rightarrow D \ell \bar{\nu}_\ell$ and the other does to both the τ and ℓ modes. As for the $B \rightarrow D^*$ transition, the matrix element is represented by four form factors. One of them does not contribute to the ℓ mode and the others do to both the τ and ℓ modes.

The form factors that contribute to $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell$ are determined or constrained by the abundant experimental data. The remaining two form factors that appear only in the τ mode are estimated using theoretical methods such as heavy quark effective theory (HQET), QCD (light-cone) sum rule and lattice QCD. The HQET is a systematic expansion in terms of the inverse power of heavy quarks (bottom and charm) and the QCD coupling constant. In the heavy quark limit, all the relevant form factors are expressed by a single universal function, the Isgur-Wise function[4, 5], which could be extracted from the data of $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell$. Deviations from the heavy quark limit are estimated by QCD sum rule. For $B \rightarrow D$, lattice QCD also provides important information. (For details of form factors, see Ref. [6] and other references below.)

Using the form factors thus determined, several groups have presented predictions in the SM. Some of them are quoted here ¹:

$$\begin{aligned}
R(D) &= 0.302 \pm 0.015 \text{ (MT, RW, 2010, HQET)[7],} \\
&= 0.296 \pm 0.016 \text{ (Fajfer, Kamenik, Nišandžić, 2012, lattice)[8],} \\
&= 0.299 \pm 0.011 \text{ (Bailey et al., 2015, lattice)[9],} \\
&= 0.300 \pm 0.008 \text{ (Na et al., 2015, lattice)[10],} \\
&= 0.299 \pm 0.003 \text{ (Bigi, Gambino, 2016, combined)[11],} \\
&= 0.299 \pm 0.003 \text{ (Bernlochner et al., 2017, combined)[12],}
\end{aligned} \tag{2.2}$$

for the D mode, and

$$\begin{aligned}
R(D^*) &= 0.252 \pm 0.003 \text{ (Fajfer, Kamenik, Nišandžić, 2012, HQET)[8],} \\
&= 0.252 \pm 0.004 \text{ (MT, RW, 2013, HQET)[6],} \\
&= 0.257 \pm 0.003 \text{ (Bernlochner et al., 2017, combined)[12],} \\
&= 0.260 \pm 0.008 \text{ (Bigi, Gambino, Schacht, 2017, combined)[13],} \\
&= 0.259 \pm 0.006 \text{ (Jaiswal, Nandi, Patra, 2017, CLN)[14],} \\
&= 0.257 \pm 0.005 \text{ (Jaiswal, Nandi, Patra, 2017, BGL)[14],}
\end{aligned} \tag{2.3}$$

for the D^* mode, where CLN[15] and BGL[16] denote two different parametrizations of form factors. We observe that these results are consistent with each other within their uncertainties. The un-

¹This list and references in the present manuscript are not intended to be complete. They roughly follow those mentioned in the oral presentation.

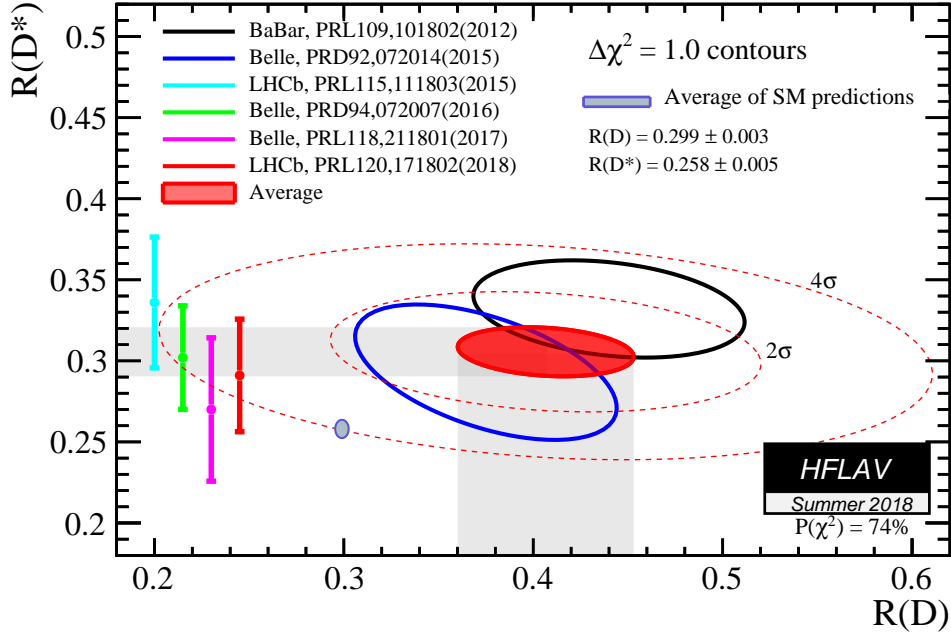


Figure 2: Present status of $R(D^{(*)})$ anomaly, taken from Ref. [18].

certainty in $R(D)$ decreases as theoretical methods are improved. On the other hand, that in $R(D^*)$ seems to have been underestimated in the past. The heavy flavor averaging group (HFLAV)[17] employs the recent results in Refs. [11, 12, 13, 14] and gives their averages as of the summer in 2018 [18]²

$$R(D) = 0.299 \pm 0.003 \text{ (SM)}, \quad (2.4)$$

and

$$R(D^*) = 0.258 \pm 0.005 \text{ (SM)}. \quad (2.5)$$

The current experimental data are also summarized by HFLAV as shown in Fig. 2. The above averages of the SM predictions are also shown. The averages of experimental values are

$$R(D) = 0.407 \pm 0.039 \pm 0.024 \text{ (BaBar, Belle)}, \quad (2.6)$$

and

$$R(D^*) = 0.306 \pm 0.013 \pm 0.007 \text{ (BaBar, Belle, LHCb)}. \quad (2.7)$$

Combining $R(D)$ and $R(D^*)$, the discrepancy between the SM predictions and the experimental data is evaluated by HFLAV to be about 3.8σ . This observed deviation of LFNU from the SM is often called “ $R(D^{(*)})$ anomaly”.

It is remarkable that the BaBar collaboration excluded the 2HDM of type II using their data[19]. As presented in Fig. 2, they observed excesses in both $R(D)$ and $R(D^*)$. BaBar found that one of the

²Summer 2018 HFLAV summary appeared after HQL2018. We employ it to make the present manuscript up-to-date.

excesses could be explained by choosing an appropriate value of the model parameter, $\tan\beta/m_H$, where m_H stands for the mass of the charged Higgs boson, but a simultaneous explanation of $R(D)$ and $R(D^*)$ was impossible.

In addition to the D and D^* modes, other decays of the same quark-level process are to be studied both experimentally and theoretically. Recently, LHCb collaboration reported their result on $B_c^- \rightarrow J/\psi \tau \bar{\nu}_\tau$ as[20]

$$R(J/\psi) := \frac{\mathcal{B}(B_c^- \rightarrow J/\psi \tau \bar{\nu}_\tau)}{\mathcal{B}(B_c^- \rightarrow J/\psi \ell \bar{\nu}_\ell)} = 0.71 \pm 0.17 \pm 0.18 \text{ (LHCb)}, \quad (2.8)$$

which is compared with the following SM predictions,

$$\begin{aligned} R(J/\psi) &= [0.279, 0.301] \text{ (Dutta, , Bhol, 2017)[21]}, \\ &= 0.283 \pm 0.048 \text{ (RW, 2017)[22]}. \end{aligned} \quad (2.9)$$

Though a tendency of excess like $R(D^{(*)})$ is seen, both the experimental and theoretical errors are larger than those in $R(D^{(*)})$. One of us (RW) estimates the significance of the excess about 1.7σ [22].

3. Effective field theory for $b \rightarrow c \tau \bar{\nu}$

As mentioned above, the charged Higgs boson in the 2HDM of type II (or MSSM) does not explain the current experimental data. Hence we have no prime candidate of new physics to explain the $R(D^{(*)})$ anomaly. It is sensible in this situation to take a model-independent approach employing an effective field theory.

We introduce the following effective Lagrangian for $b \rightarrow c \tau \bar{\nu}_\tau$, which contains all possible four-fermion operators of dimension 6 assuming left-handed neutrinos[6],

$$-\mathcal{L}_{\text{eff}} = 2\sqrt{2}G_F V_{cb} [(1 + C_{V_1})\mathcal{O}_{V_1} + C_{V_2}\mathcal{O}_{V_2} + C_{S_1}\mathcal{O}_{S_1} + C_{S_2}\mathcal{O}_{S_2} + C_T\mathcal{O}_T], \quad (3.1)$$

where the four-fermion operators are defined by

$$\mathcal{O}_{V_1} = \bar{c}_L \gamma^\mu b_L \bar{\tau}_L \gamma_\mu \nu_{L\tau}, \quad (3.2)$$

$$\mathcal{O}_{V_2} = \bar{c}_R \gamma^\mu b_R \bar{\tau}_L \gamma_\mu \nu_{L\tau}, \quad (3.3)$$

$$\mathcal{O}_{S_1} = \bar{c}_L b_R \bar{\tau}_R \nu_{L\tau}, \quad (3.4)$$

$$\mathcal{O}_{S_2} = \bar{c}_R b_L \bar{\tau}_R \nu_{L\tau}, \quad (3.5)$$

$$\mathcal{O}_T = \bar{c}_R \sigma^{\mu\nu} b_L \bar{\tau}_R \sigma_{\mu\nu} \nu_{L\tau}, \quad (3.6)$$

and C_X ($X = V_{1,2}, S_{1,2}, T$) denotes the Wilson coefficient of \mathcal{O}_X that represents potential new physics contributions. We note that C_X 's are complex numbers in general.

The data of $R(D^{(*)})$ and $R(J/\psi)$ constrain the contributions of these operators and we obtain allowed (or excluded) regions of the Wilson coefficients. The charged Higgs boson of the 2HDM of type II, as depicted in Fig. 1, gives the S_1 operator and no allowed region of C_{S_1} remains at 95% confidence level (CL). We present the current constraint on each Wilson coefficient except C_{S_1} in

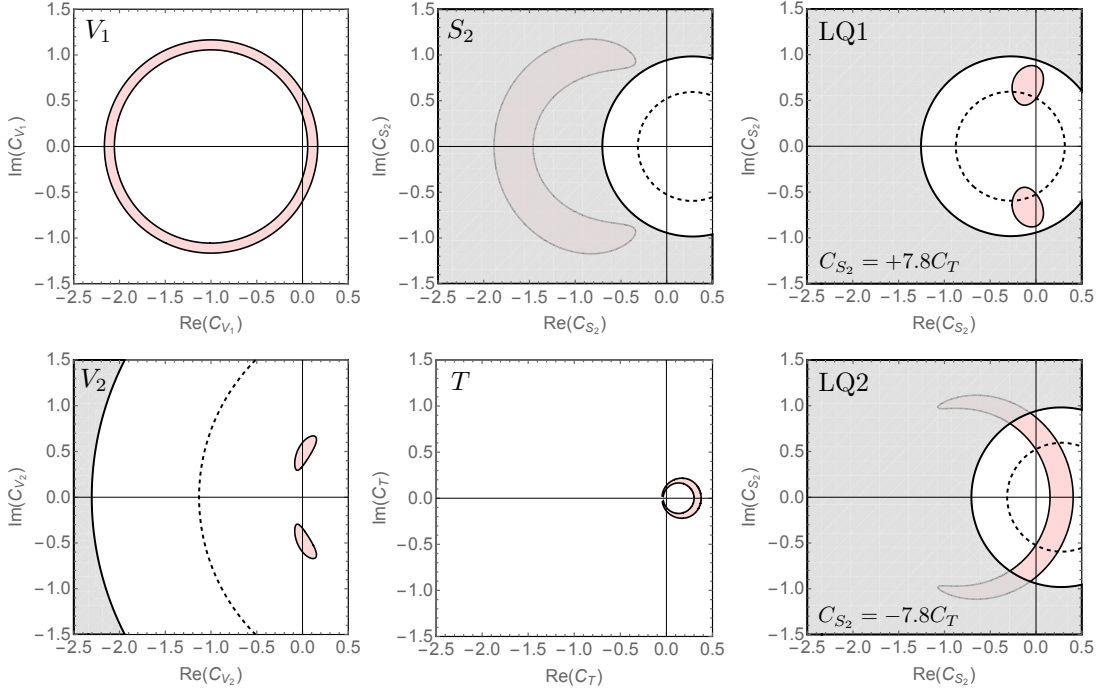


Figure 3: Current experimental constraints on Wilson coefficients. The red regions represent the allowed regions at 95% CL with $R(D^{*})$ and $R(J/\psi)$ given in the main text. The gray regions are excluded by the constraint $\mathcal{B}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 0.3$. The dotted lines correspond to $\mathcal{B}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 0.1$. The constraints on two leptoquark scenarios of $C_{S_2} = \pm 7.8C_T$ are also shown.

Fig. 3 assuming that coefficients other than the one examined are vanishing. The red regions are allowed at 95% CL.

The gray regions are those excluded by the pure taonic decay of B_c , $B_c^- \rightarrow \tau \bar{\nu}$ [23]. The lifetime (or the total width) is measured by CDF, D0 and LHCb as $\tau_{B_c} = 0.507 \pm 0.009$ ps[24], while a theoretical estimation in the SM with the operator product expansion gives $\tau_{B_c}^{\text{OPE}} = 0.52_{-0.12}^{+0.18}$ ps[25]. New physics is limited so that its contribution added to the SM one must not make the B_c lifetime shorter than observed. Thus a bound $\mathcal{B}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 0.3$ is obtained in Ref. [23]. The gray regions in Fig. 3 correspond to this constraint. Furthermore, the search of $B_{u,c}^- \rightarrow \tau \bar{\nu}$ at LEP1 gives a bound $\mathcal{B}(B_c^- \rightarrow \tau \bar{\nu}) \lesssim 0.1$ [26]. The dotted lines in Fig. 3 represent this bound. We note that the tensor operator, O_T , does not contribute to $B_c^- \rightarrow \tau \bar{\nu}$. We observe that the S_2 scenario is disfavored by the constraint of $B_c^- \rightarrow \tau \bar{\nu}$.

In Fig. 3, we also present constraints on two interesting leptoquark (LQ) scenarios denoted by LQ1 and LQ2[6, 27]. The operators O_{S_2} and O_T are simultaneously induced by the exchange of a scalar leptoquark in these scenarios. The $SU(3) \times SU(2) \times U(1)$ quantum numbers of the leptoquark are $(3, 2, 7/6)$ for LQ1 and $(3^*, 1, 1/3)$ for LQ2. The Wilson coefficients are related as $C_{S_2} = +4C_T$ in the LQ1 scenario and as $C_{S_2} = -4C_T$ in the LQ2 scenario at the mass scale of the leptoquark. Using the renormalization group equation, one obtains $C_{S_2} \simeq \pm 7.8C_T$ at the bottom mass scale[28]. It is interesting that the combinations of S_2 and T operators in the LQ scenarios are allowed although the S_2 operator alone is excluded by $B_c^- \rightarrow \tau \bar{\nu}$.

4. Other observables and future prospect

In addition to $R(D^{(*)})$, the BaBar collaboration reported q^2 distributions in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ [29], where q^2 is the squared momentum transfer (or equivalently the invariant mass of the leptonic system). In Ref. [30], theoretical predictions of the q^2 distributions in the SM and the new physics scenarios, $V_{1,2}$, S_2 , T and two LQ's, are compared with the BaBar q^2 data. It turns out that the SM and the $V_{1,2}$ scenarios are allowed, while the S_2 scenario and the T scenario are both disfavored. Interestingly, the LQ1 and LQ2 scenarios, which are combinations of S_2 and T , are allowed.

The following ratios of q^2 distributions are also introduced in Ref. [30]:

$$R_D(q^2) := \frac{d\mathcal{B}(\bar{B} \rightarrow D \tau \bar{\nu})/dq^2}{d\mathcal{B}(\bar{B} \rightarrow D \ell \bar{\nu})/dq^2} \frac{\lambda_D(q^2)}{(m_B^2 - m_D^2)^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^{-2}, \quad (4.1)$$

and

$$R_{D^*}(q^2) := \frac{d\mathcal{B}(\bar{B} \rightarrow D^* \tau \bar{\nu})/dq^2}{d\mathcal{B}(\bar{B} \rightarrow D^* \ell \bar{\nu})/dq^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^{-2}, \quad (4.2)$$

where $\lambda_D(q^2) := \{(m_B - m_D)^2 - q^2\}\{(m_B + m_D)^2 - q^2\}$. These q^2 -dependent ratios are also predictable with smaller theoretical uncertainties than the q^2 distributions, and are expected to be useful in narrowing down new physics candidates. It is shown that most of the allowed scenarios can be discriminated with 6 ab^{-1} or less at the SuperKEKB/Belle II experiment. Hence, the study of q^2 distribution is a beneficial subject at an early stage of Belle II.

The longitudinal polarization of the tau lepton is also a useful observable. It is defined by

$$P_\tau := \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-}, \quad (4.3)$$

where Γ^{λ_τ} ($\lambda_\tau = \pm$) denotes the decay rate of a given tau helicity λ_τ . We note that the tau helicity is not a Lorentz invariant quantity and depends on the frame. In the present context, it is defined in the $\tau \bar{\nu}$ rest frame. One obtains information on the tau helicity or polarization from the decay distribution.

Since the tau polarization is also a ratio of partial rates, the uncertainty in the theoretical calculation is supposed to be small. We evaluated it in the SM as $P_\tau(D^*) = -0.497 \pm 0.013$ [6]. Recently, a more conservative theoretical calculation is reported as $P_\tau(D^*) = -0.47 \pm 0.04$ [13]. These SM predictions are compared to the result of Belle, $P_\tau(D^*) = -0.38 \pm 0.51_{-0.16}^{+0.21}$ [31, 32], which is the first measurement of the tau polarization in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$. The SM predictions and the experimental value are consistent within their errors, and the theoretical uncertainty is much smaller than the experimental one. In new physics scenarios, it is predicted as -0.50, -0.50, +0.14, -0.41 and -0.50 for the best fit values of the V_1 , V_2 , T , LQ1 and LQ2 scenarios respectively[22]³. We conclude that all of these scenarios are consistent with the measured value of $P_\tau(D^*)$ because of the large experimental error.

Belle II plans to accumulate data of 50 ab^{-1} during its period of operation. Expected constraints on C_X 's from $R(D^{(*)})$ and $R_{D^{(*)}}(q^2)$ data with 40 ab^{-1} are estimated in Ref. [30] assuming that the central values of measured $R(D)$ and $R(D^*)$ are those of the SM. In term of the mass scale of new physics defined by $M_{\text{NP}} := (2\sqrt{2}G_F|V_{cb}||C_X|)^{-1/2}$, the expected reach is roughly from 5 to 10 TeV depending on new physics scenarios.

³The uncertainties in these predictions are much smaller than the present experimental error.

5. Summary

The observed 3.8σ excess of $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ over the SM predictions suggests new physics in $b \rightarrow c \tau \bar{\nu}$, since the SM calculations of $R(D^{(*)})$ are fairly robust. The charged Higgs boson in the 2HDM of type II, which introduces additional LFNU and is the archetype of new physics in this channel, fails to explain this $R(D^{(*)})$ anomaly.

A model-independent approach is employed in order to search for possible new physics candidates. The effective Lagrangian that consists of all possible four-fermion operators of dimension 6 is introduced. The Wilson coefficients of these operators quantify possible new physics contributions. Comparing the theoretical predictions and the experimental constraints of $R(D^{(*)})$, $R(J/\psi)$ and $\mathcal{B}(B_c^- \rightarrow \tau \bar{\nu})$ as shown in Fig. 3, the allowed new physics scenarios, $V_{1,2}$, T LQ1 and LQ2, are identified.

Then, the q^2 distributions in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ and the q^2 -dependent ratios, $R_{D^{(*)}}(q^2)$, are discussed. It turns out that the BaBar data of the q^2 distributions disfavors the T scenario as well as the S_2 scenario. The possible scenarios will be further constrained at Belle II with a rather small integrated luminosity of about 6 ab^{-1} by measuring $R_{D^{(*)}}(q^2)$. Among several other observables, the longitudinal tau polarization, P_τ is also discussed. The measured value of $P_\tau(D^*)$ is consistent with the SM prediction though the current experimental uncertainty is large.

The present situation implies that further studies in both theoretical and experimental aspects are required. Further reduction of theoretical uncertainties in the form factors are desired. Contributions of lattice QCD is important in this respect. The study of possible flavor structure of new physics is also important. The charmless mode, $b \rightarrow u \tau \bar{\nu}$, may give us a hint[33]. Needless to say, there are also lots of theoretical works that consider new particle search at LHC and/or other anomalies such as P'_5 and LFNU in $b \rightarrow s \ell \bar{\ell}$ connecting to the $R(D^{(*)})$ anomaly. See, for example, [34, 35, 36, 37, 38]. In the experimental aspect, as mentioned above, Belle II will provide us with valuable data that are mainly related to decay distributions in $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$. LHCb will also study the same decay modes as well as other modes including $B_{s,c}$ or Λ_b .

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