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Lepton Flavor Violation in the MSSM: exact diagonalization vs. mass insertion expansion

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Lepton Flavor Violating (LFV) observables involving charged lepton decays are strongly constrained by the existing measurements. The forthcoming experiments will improve the precision even further, requiring equally precise and efficient calculations of these processes in New Physics models. In particular, LFV observables within the Minimal Supersymmetric Standard Model (MSSM) has been recently recalculated in ref. [1], using a new technique of a purely algebraic Mass Insertion expansion of the amplitudes [2, 3]. In this talk we present the most important results of this analysis. We list the updated bounds on the lepton flavor changing parameters of the MSSM derived from the measurements of the radiative decays $\ell \to \ell'\gamma$ and the 3-body decays $\ell \to 3\ell'$ and $\ell \to 2\ell'\ell''$. We also discuss the complementarity between the radiative and 3-body decays useful for eliminating "blind spots" in the parameter space and the non-decoupling effects in the LFV Higgs decays $h \to \ell\ell'$.

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1. New calculational technique: algebraic Mass Insertion expansion

Amplitudes describing flavor transitions within the New Physics models can be computed using two basic approaches:

- Calculations in the "interaction" ("symmetry") basis, using fields before the mass matrix diagonalization. Gauge interactions are flavor diagonal, flavor transitions originate from the off-diagonal entries of mass matrices, so-called "Mass Insertions" (MI).
  - Advantage: result exhibits direct dependence on original symmetry-related parameters.
  - Disadvantage: tedious and error prone diagrammatic calculations with MIs treated as interaction vertices, combinatorial complication quickly growing with MI order. Amplitude expressed as double infinite series, in loop order and the MI order.
- Calculation in the "mass eigenstates" basis, in terms of physical fields after mass matrix diagonalization.
  - Advantage: physical external states, more compact expressions, simpler diagrammatic calculations. At a given loop order exact formulae in terms of flavor changing parameters.
  - Disadvantage: complicated non-linear dependence of the results on initial symmetry-related parameters, various effects can be analysed only numerically.

As proven in ref. [2], transition from mass to interaction basis amplitudes can be done using purely algebraic techniques, using the so called-flavor "Flavor Expansion Theorem" (FET). It gives an effective prescription how to expand any flavor transition amplitude calculated in the mass basis to any order in Mass Insertions, with coefficients of the MI powers depending only on the diagonal elements of mass matrices in interaction basis, without the need of performing directly any diagrammatic calculations with MIs as the additional interaction vertices. The technique applies to amplitudes involving all types of particles, scalar, vector or fermionic. In addition, the algorithm based on FET has been fully automatised in the symbolic *Mathematica* package *MassToMI* [3], further facilitating its applications.

The FET technique can be used to analyse flavor physics within any New Physics model, but it was developed primarily as a generalisation of the calculations within the Minimal Supersymmetric Standard Model. Such calculations used the technically complicated diagrammatic form of MI expansion and often neglected some contributions. In ref. [1] FET was used to reanalyse systematically, without any simplifying assumptions, the LFV processes in the charged lepton sector of the MSSM. They are of particular importance, being:

- accurately measured – strong experimental bounds exist
- extremely suppressed in the SM – thus very sensitive to New Physics effects
- cleanly predicted theoretically, no QCD and hadronic uncertainties

In this case the FET method is particularly advantageous because LFV transitions are mediated by the chargino and neutralino exchanges (contrary to usually gluino-dominated diagrams with external quarks) and recovering proper decoupling properties of the amplitudes requires expanding

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Process	(I,J)	Δ_{LL}^{IJ}	Δ_{RR}^{IJ}	Δ_{LR}^{IJ}	Δ_{RL}^{IJ}	$\Delta_{LR}^{\prime IJ}$	$\Delta_{RL}^{\prime IJ}$
$\tan\beta = 2$							
$\mu \to e\gamma$	$(2,1)$	$8.4 \cdot 10^{-4}$	$5.0 \cdot 10^{-3}$	$8.4 \cdot 10^{-6}$	$8.3 \cdot 10^{-6}$	$4.1 \cdot 10^{-6}$	$4.1 \cdot 10^{-6}$
$\tau \to \mu\gamma$	$(3,2)$	$5.3 \cdot 10^{-1}$	$\mathscr{O}(1)$	$9.1 \cdot 10^{-2}$	$9.1 \cdot 10^{-2}$	$4.5 \cdot 10^{-2}$	$4.5 \cdot 10^{-2}$
$\tau \to e\gamma$	$(3,1)$	$4.6 \cdot 10^{-1}$	$\mathscr{O}(1)$	$7.8 \cdot 10^{-2}$	$7.8 \cdot 10^{-2}$	$3.9 \cdot 10^{-2}$	$3.8 \cdot 10^{-2}$
$\tan\beta = 20$							
$\mu \to e\gamma$	$(2,1)$	$1.0 \cdot 10^{-4}$	$4.5 \cdot 10^{-4}$	$7.5 \cdot 10^{-5}$	$7.4 \cdot 10^{-5}$	$3.7 \cdot 10^{-6}$	$3.7 \cdot 10^{-6}$
$\tau \to \mu\gamma$	$(3,2)$	$6.5 \cdot 10^{-2}$	$2.9 \cdot 10^{-1}$	$8.2 \cdot 10^{-1}$	$8.2 \cdot 10^{-1}$	$4.0 \cdot 10^{-2}$	$4.0 \cdot 10^{-2}$
$\tau \to e\gamma$	$(3,1)$	$5.7 \cdot 10^{-2}$	$2.5 \cdot 10^{-1}$	$7.0 \cdot 10^{-1}$	$7.0 \cdot 10^{-1}$	$3.4 \cdot 10^{-2}$	$3.4 \cdot 10^{-2}$

Table 1: Upper bounds on Δ-parameters assuming degenerated MSSM spectrum with average slepton and gaugino mass scale $M = 400$ GeV.

them at least to 3rd order in both the flavor violating slepton MIs and flavor conserving off-diagonal entries of the supersymmetric fermion mass matrices – technically a daunting task when using diagrammatic calculations. Using FET, it was possible to perform such expansions without assuming any constraints on the MSSM spectrum, to present compact analytical results for all processes and to demonstrate explicitly that all amplitudes (with the exception of some terms contributing to the Higgs boson LFV decays) decouple in the limit of heavy SUSY spectrum, scaling like $1/M_{SUSY}^2$.

2. Upper bounds on MSSM LFV parameters

Analysis of ref. [1] includes the following processes:

- radiative lepton decays $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$
- 3-body charged lepton decays $\mu \to 3e$, $\tau \to 3e$, $\tau \to 3\mu$, $\tau \to \mu ee$, $\tau \to \mu\mu e$
- $\mu \to e$ conversion in Nuclei
- LFV Higgs boson decays $h \to \mu e$, $h \to \tau e$, $h \to \tau\mu$

Comparing the results of theoretical calculations with the experimental upper bounds on the measured decay rates, one can obtain the upper limits on the maximal allowed size of the off-diagonal terms in the slepton mass matrices. The results have been as usual parametrized in terms of dimensionless slepton mass insertions in 3×3 LL, RR and LR sub-blocks of the full 6×6 slepton mass matrix (see e.g. refs. [4, 5]). They are defined as

$$\Delta_{XY}^{IJ} = \frac{(M_{XY}^2)^{IJ}}{((M_{XX}^2)^{II}(M_{YY}^2)^{JJ})^{1/2}} \tag{2.1}$$

where $X = L,R$ denotes slepton “chiralities” and $I,J = 1\ldots 3$ enumerate slepton flavor generation (alternatively one can use particle names $I,J = e,\mu,\tau$)

With current experimental accuracies, the best bounds on Δ-parameters turn out to be given by the radiative lepton decays. The example of such bounds for the fully degenerated SUSY spectrum, with all MSSM mass parameters set to $M = 400$ GeV, is presented in Table 1. For higher SUSY masses the bounds scale (weaken) with M^2.

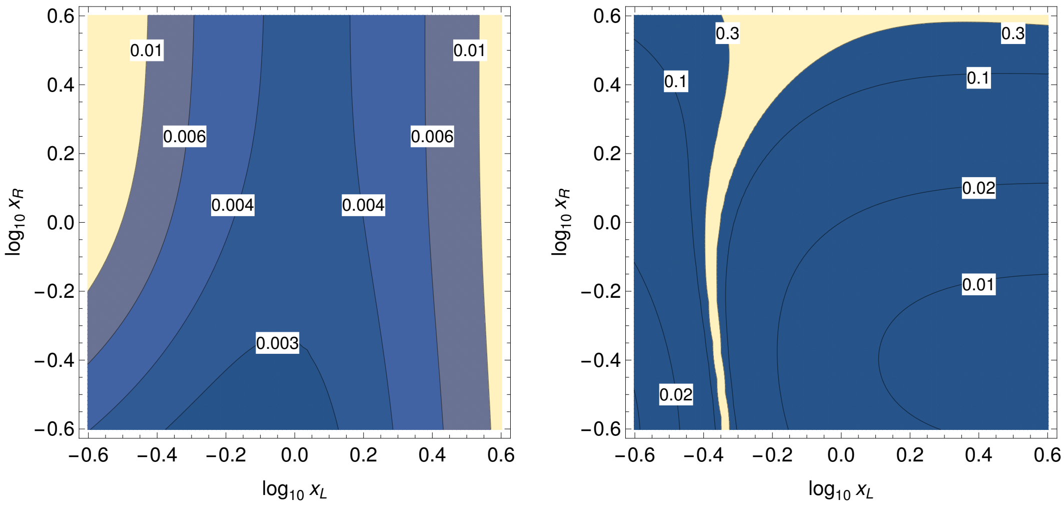

Figure 1: Upper bounds on $\Delta^{e\mu}_{LL}$ (left panel) and $\Delta^{e\mu}_{RR}$ (right panel) for $\tan\beta = 2$ and $M_2 = M_1 = \mu = M_{SUSY} =$ 800 GeV as a function of normalised slepton masses x_L, x_R (see eq (2.2)). All LFV parameters apart from $\Delta^{e\mu}_{LL}(\Delta^{e\mu}_{RR})$ are set to 0 in the left(right) panel, respectively.

Currently, the bounds obtained from the 3-body LFV charged lepton decays are typically 1-2 orders of magnitude weaker than from the radiative decays. However, it is important to note that for non-degenerate mass spectra "blind spots" exist in the limits derived from the radiative lepton decays – for some mass patterns they become weak or vanish entirely. This is illustrated in Fig. 1, where the bounds on $\Delta^{e\mu}_{LL}$, $\Delta^{e\mu}_{RR}$ from $\mu \to e\gamma$ measurement are shown for $\tan\beta = 2$ and $M_2 = M_1 = \mu = M_{SUSY} = 800$ GeV as a function of mass of sleptons of different chiralities, normalised to the SUSY scale:

$$x_L = \frac{m_{\tilde{\mu}_L}}{M_{SUSY}} = \frac{m_{\tilde{e}_L}}{M_{SUSY}} \qquad x_R = \frac{m_{\tilde{\mu}_R}}{M_{SUSY}} = \frac{m_{\tilde{e}_R}}{M_{SUSY}} \tag{2.2}$$

As can be seen from Fig. 1, bounds on $\Delta^{e\mu}_{LL}$ disappear for $m_{\tilde{e}_L} \approx m_{\tilde{\mu}_L} \ll M_2$ and for $m_{\tilde{e}_L} \approx m_{\tilde{\mu}_L} \gg M_2$ while bounds on $\Delta^{e\mu}_{RR}$ disappear for $m_{\tilde{e}_L} \approx m_{\tilde{\mu}_L} \approx 0.4 M_2$ (for heavier right slepton masses, $m_{\tilde{e}_R} \approx m_{\tilde{\mu}_R} \gtrsim 3M_2$ bounds on $\Delta^{e\mu}_{RR}$ disappear in even wider range $m_{\tilde{e}_L} \approx m_{\tilde{\mu}_L} \gtrsim -0.4$). As discussed in more details in ref. [1], MI expanded expressions for $\ell \to \ell'\gamma$ decay amplitude allow to find where such cancellations occur also analytically, not only through numerical scans over parameter space. In all such cases measurements of 3-body charged lepton decays become more constraining, as discussed in more details in the next Section.

3. Correlation between radiative and 3-body charged lepton decays

For most of the MSSM parameter choices, the amplitude of the 3-body charged lepton decays $\ell \to \ell'\ell''\ell'''$ is dominated by the photon penguin diagram, which defines also the speed of the radiative lepton decays. Thus, in a "photon penguin domination scenario" both decay rates are correlated. For the processes with the 3 leptons of identical flavor in the final state[1], $\ell \to 3\ell'$, the

[1] Similar relations for other final state compositions can be found e.g. in ref. [6].

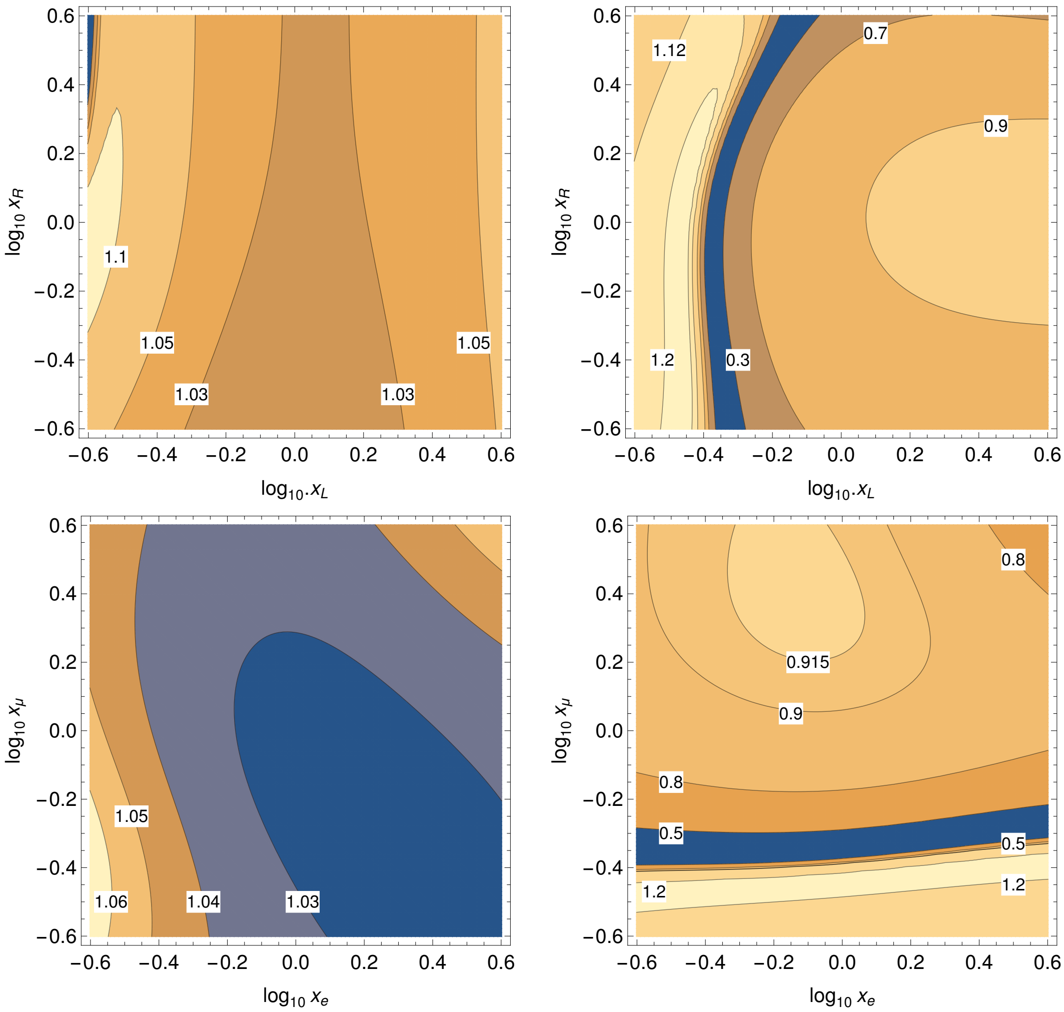

Figure 2: Ratio $R_{\mu e}$ for $\tan\beta = 2$ and $M_2 = M_1 = \mu = M_{SUSY} = 800$ GeV as a function of normalised slepton masses x_L, x_R (upper row) or x_e, x_μ (lower row). All LFV parameters apart from $\Delta^{e\mu}_{LL}(\Delta^{e\mu}_{RR})$ are set to 0 in the left(right) panels, respectively.

ratio $R_{\ell\ell'}$ defined in eq. (3.1) is close to 1:

$$R_{\ell\ell'} = \frac{\alpha_{em}}{3\pi}\left(\log\frac{m_\ell^2}{m_{\ell'}^2} - \frac{11}{4}\right)\frac{\mathrm{Br}(\ell\to\ell'\gamma)}{\mathrm{Br}(\ell\to 3\ell')} \approx 1 \tag{3.1}$$

The correlation breaks near "blind spots" in radiative decays, where the photon penguin contribution becomes comparable to or smaller than other terms. Such effect is illustrated in Fig. 2, where the ratio $R_{\mu e}$ is plotted as a function of the mass splitting between left and right slepton masses (upper panels) or between the smuon and selectron masses (lower panels, here we assume $x_e = m_{\tilde{e}_L}/M_{SUSY} = m_{\tilde{e}_R}/M_{SUSY}$ and $x_\mu = m_{\tilde{\mu}_L}/M_{SUSY} = m_{\tilde{\mu}_R}/M_{SUSY}$). Again, as in Fig 1, we set $\tan\beta = 2$ and all other SUSY mass parameters equal to $M = 800$ GeV.

As seen from Fig. 2, for some parameter choices (corresponding to cancellation areas in $\ell \to \ell'\gamma$ amplitude, like the one illustrated in right panel of Fig. 1) the ratio $R_{\ell\ell'}$ may be very small, and the constraints from the 3-body charged lepton decays become stronger than from the radiative decays.

It is worth noting that, contrary to simpler and more prone to cancellation structure of the $\ell \to \ell'\gamma$ decay amplitude, the expressions for the branching ratio for the 3-body decays are given by a sum of several positively defined contributions which cannot vanish simultaneously for any MSSM parameter setup. Thus limits based on such measurements never exhibit the "blind spots" appearing when analysing the radiative decays. Such an observation may be important for developing optimal future experiments measuring lepton flavor violating processes.

Using current 90%CL experimental bounds, $\mathrm{Br}(\mu \to e\gamma) \leq 5.7 \times 10^{-13}$ [7] and $\mathrm{Br}(\mu \to e^-e^+e^-) \leq 1.0 \times 10^{-12}$ [8], the generic constraints (excluding the possible cancellation effects) on the MSSM LFV parameters from the radiative lepton decays are 1-2 order of magnitude stronger than from the 3-body decays (see e.g. Table 4 and corresponding discussion in ref. [1]). If planned sensitivities of new projected experiments are achieved, $\mathrm{Br}(\mu \to e\gamma) \leq 6 \times 10^{-14}$ [9] and $\mathrm{Br}(\mu \to e^-e^+e^-) \leq 1.0 \times 10^{-16}$ [10, 11], the 3-body charged lepton decays become both stronger and safer source of the constraints on the LFV parameters – not only in the MSSM.

4. LFV Higgs boson decays

LFV Higgs boson decays are currently less constraining than radiative or 3-body decays. For example, the recent experimental upper limit for the $h \to \tau\mu$ decay published by the CMS Collaboration [12] reads

$$\mathrm{Br}(h \to \tau\mu)^{exp} \leq 2.5 \cdot 10^{-3} \tag{4.1}$$

while analysis of ref. [1] shows that for *any* MSSM parameter choice one always has

$$\mathrm{Br}(h \to \tau\mu)^{MSSM} \leq \mathcal{O}(10^{-4}) \, . \tag{4.2}$$

However, contrary to other LFV processes, some effects contributing to Higgs decays do not vanish (decouple) for heavy SUSY spectrum and the upper bound of eq. (4.2) can be reached for *any* SUSY scale. As a consequence, it is important to observe that for heavy SUSY particles and with improved experimental accuracy the Higgs boson decays may become the most constraining LFV observables.

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