

Scattering phase shift determinations from a two-scalar field theory

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A field theory involving two interacting scalar fields, previously studied by Rummukainen and Gottlieb, is revisited. Our study is not restricted to the limit of large quartic couplings, and a Symanzik-improved action is used so that continuum dispersion relations work well. The Metropolis method, combined with a local microcanonical updating algorithm, is employed in our Monte Carlo calculations. Isotropic lattices ranging from $16^3 \times 48$ to $53^3 \times 48$ are used, and scattering phase shifts are determined using a Lüscher analysis with multiple partial waves.

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1. Introduction

A theory of two real scalar fields, ϕ and ρ , described by the Lagrangian,

$$\mathcal{L} = -\frac{1}{2}\phi(\partial^2 + M_\phi^2)\phi - \frac{1}{2}\rho(\partial^2 + M_\rho^2)\rho - \frac{\lambda_\phi}{4!}\phi^4 - \frac{\lambda_\rho}{4!}\rho^4 - \frac{1}{2}g\phi^2\rho, \quad (1.1)$$

is studied here. Parameters in the above Lagrangian are chosen such that the ρ particle may decay into two ϕ particles in infinite volume. We also ensure that spontaneous symmetry breaking does not occur. We formulate this theory on a space-time lattice and study ϕ - ϕ scattering to extract resonance parameters of the theory using the box matrix formalism developed in Ref. [1], which is an implementation of the so-called Lüscher method[2, 3]. This model was previously studied in 1995 by Rummukainen and Gottlieb [4] in the Ising limit. Here, we do not take the large limit in the quartic self-couplings, and we avoid using unphysical lattice dispersion relations by making use of a tree-level Symanzik-improved action.

2. Choice of model parameters

We require the physical ρ mass to be greater than twice the physical ϕ mass, so that the decay is kinematically allowed, and less than three times the ϕ mass, as required for the Lüscher analysis. Quartic interactions in Eq. (1.1) are included so that our action is bounded from below in Euclidean space-time, but we wish to keep these couplings as small as possible to reduce mass renormalizations. We also require that our theory features no spontaneous symmetry breaking, so we design the action to have a unique minimum at $\phi = \rho = 0$. To reduce finite-volume effects, we impose the condition $m_\phi L > 4$, where m_ϕ is the measured ϕ mass and L is the spatial extent of the lattice. Then we choose $a_t M_\phi$ according to this constraint and our choice of lattice size, where a_t is the temporal lattice spacing. Finally, we need to pick a value for the 1-to-2 coupling, g , large enough to produce significant interaction energies. Table 1 gives the run parameters we chose to satisfy the above conditions.

$a_t M_\phi$	$a_t M_\rho$	$\frac{g}{M_\phi}$	λ_ϕ	λ_ρ
0.1	0.31	1	$\frac{g^2}{4M_\phi^2}$	$\frac{g^2}{M_\phi^2}$

Table 1: Model parameters

3. The need for an improved action

If we use a naive discretization scheme involving only finite differences in the derivative terms of the action, then the portion of the action including only the ϕ field can be written as,

$$S_\phi = a_s^{D-1} a_t \sum_x \sum_\mu \left(\frac{(\phi(x + a_\mu \hat{\mu}) - \phi(x))^2}{2a_\mu} + \frac{1}{2}M_\phi^2 \phi(x)^2 + \frac{\lambda_\phi}{4!} \phi(x)^4 \right), \quad (3.1)$$

where a_s denotes the spatial lattice spacing. In our work, we use an isotropic lattice. We find that using this scheme produces overly-large discretization errors when performing boosts with the

continuum dispersion relation, as shown in Fig. 1. Let $E_\phi^{\mathbf{P}}$ denote the energy of the ϕ particle having momentum $\mathbf{P} = \frac{2\pi\mathbf{d}}{L}$, where \mathbf{d} is a vector of integers and $L = a_s n_s$ is the length of the isotropic L^3 lattice. We determine $E_\phi^{\mathbf{P}}$ from the exponential fall-off of appropriate temporal correlation functions estimated by the Monte Carlo method in the standard way. The difference between the ϕ mass measured in the moving frame and its mass in the rest frame is $\Delta m_\phi = [(E_\phi^{\mathbf{P}})^2 - \mathbf{P}^2]^{1/2} - E_\phi^0$. In the continuum limit, this quantity should vanish. Any deviation from zero is a measure of discretization errors. One sees sizeable discretization errors in Fig. 1.

Ref. [4] handled this problem by making use of ‘‘lattice’’ dispersion relations. We, instead, decided to employ a tree-level Symanzik-improved action and found that with such an action, continuum energy-momentum dispersion relations worked well. For a lattice spacing a_s , the discretization error in the finite-difference approximation of the action is $\mathcal{O}(a_\mu^2)$. We can reduce this error down to $\mathcal{O}(a_\mu^4)$ by employing a tree-level Symanzik improvement, which is given below for the portion of the action containing only the ϕ terms (the ρ terms are similar):

$$S_\phi^I = a_s^{D-1} a_t \sum_x \left\{ \frac{1}{2a_\mu^2} \sum_\mu \left(-\frac{4}{3} \phi(x+a_\mu)\phi(x) - \frac{4}{3} \phi(x-a_\mu)\phi(x) \right. \right. \\ \left. \left. + \frac{1}{12} \phi(x+2a_\mu)\phi(x) + \frac{1}{12} \phi(x-2a_\mu)\phi(x) + \frac{5}{2} \phi(x)^2 \right) \right. \\ \left. + \frac{1}{2} M_\phi^2 \phi(x)^2 + \frac{\lambda_\phi}{4!} \phi(x)^4 \right\}. \quad (3.2)$$

Fig. 2 shows Δm_ϕ using the Symanzik improved action, and demonstrates the ameliorative effect that the improved action has on the discretization errors in the boosts.

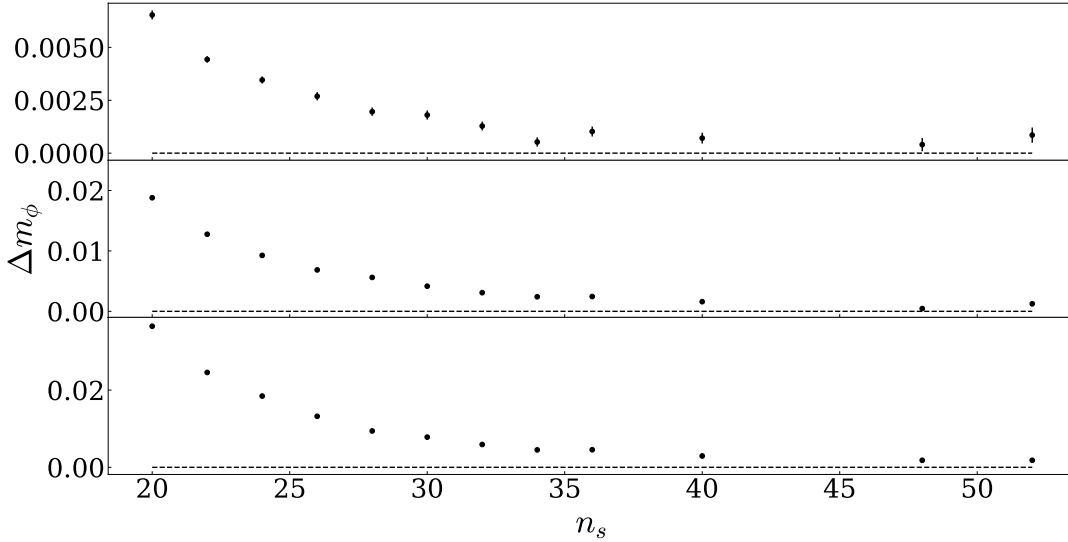


Figure 1: Differences between moving-frame and rest-frame measurements of the ϕ mass, using the unimproved action.

4. Phase shift determination and resonance parameter extraction

A large portion of the finite-volume spectrum was determined, but for the purpose of the

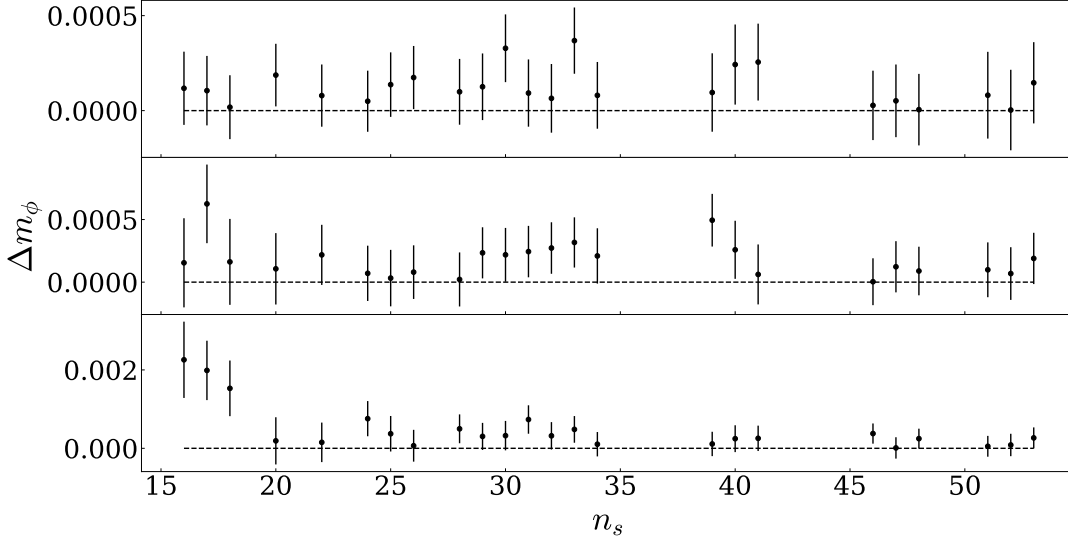


Figure 2: Differences between moving-frame and rest-frame measurements of the ϕ mass, using the Symanzik improved action.

Lüscher analysis, we are only interested in energies which differ significantly from their noninteracting values and are below the $3m_\phi$ threshold. These energies manifest as avoided level crossings, and Figs. 3-5 show examples of these avoided level crossings in the center-of-momentum frame.

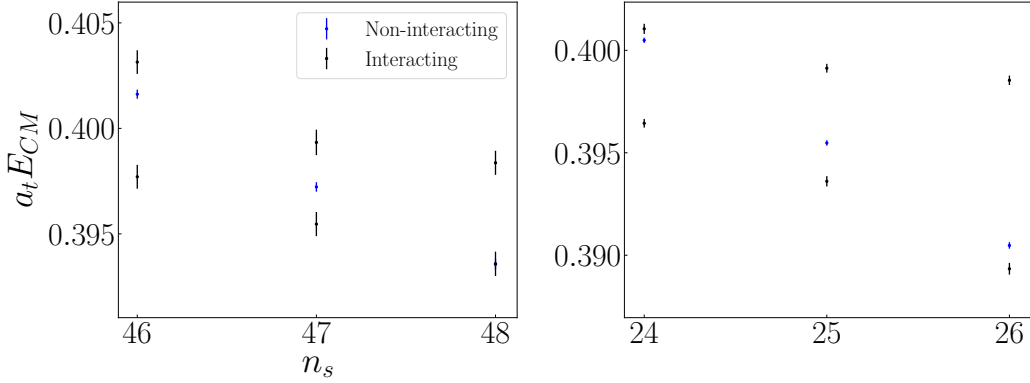
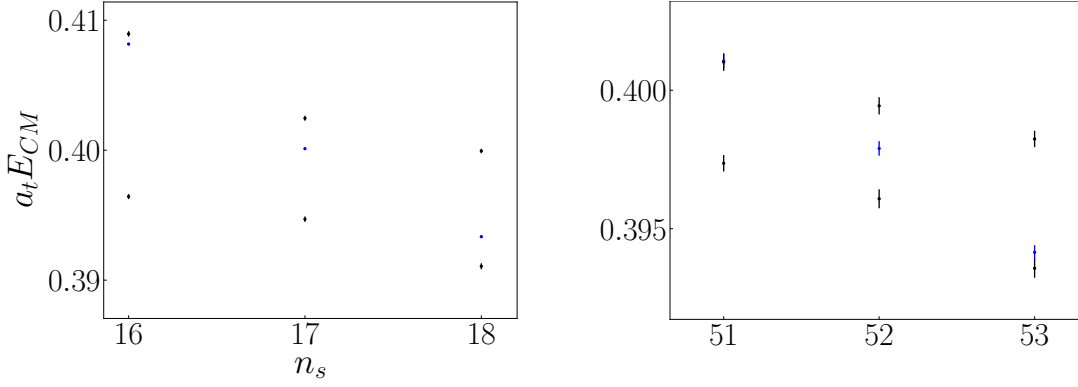
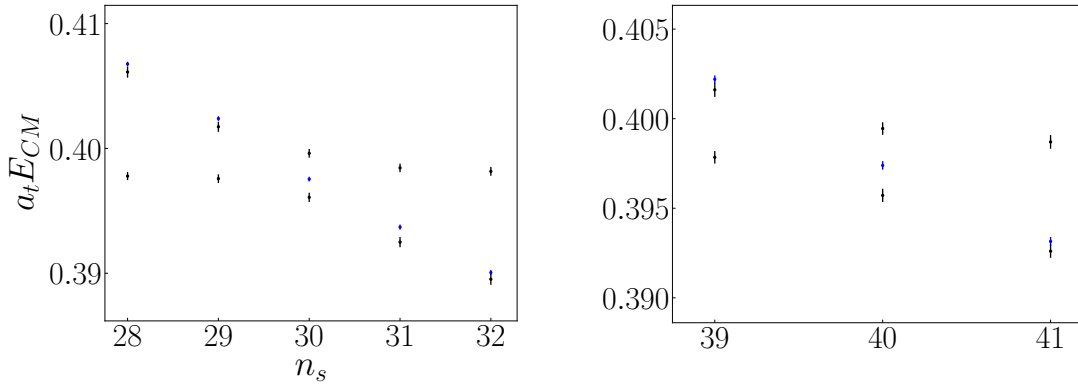


Figure 3: Center-of-momentum energies near avoided level crossings in the (left) $\mathbf{d}^2 = 0$ channel and the (right) $\mathbf{d}^2 = 2$ channel.

Using the box matrix formalism developed in Ref. [1] and applied in works such as [5, 6, 7], we write the quantization condition which relates our finite-volume spectrum to the infinite-volume K -matrix as,

$$\det [\tilde{K}^{-1}(E_{CM}) - B^{\Lambda, \mathbf{d}}(E_{CM})] = 0, \quad (4.1)$$

where E_{CM} refers to a particular lab-frame energy boosted to the center-of-momentum frame. \tilde{K} is related to the standard K -matrix as described below. The so-called box matrix B is Hermitian and block-diagonal in the octahedral irrep, Λ , and in total momentum class, \mathbf{d} . For spinless scattering, \tilde{K} may be indexed by only angular momentum l , but must be truncated by some maximum l_{\max} . In

Figure 4: Avoided level crossings in the $\mathbf{d}^2 = 1$ channel, boosted to the center-of-momentum frame.Figure 5: Avoided level crossings in the $\mathbf{d}^2 = 3$ channel, boosted to the center-of-momentum frame.

this work, we consider $l_{\max} = 0$ and investigate the systematic errors introduced by this truncation by also considering $l_{\max} = 2$. Note that $l = 1$ is excluded by parity. It should be noted that in Eq. (4.1), each energy determination provides a single relation between that energy and the entire \tilde{K} -matrix. When truncating \tilde{K} down to a single angular momentum, calculating $B(E_{\text{CM}})$ (using freely developed software made available in Ref. [1]) tells us \tilde{K}^{-1} directly. For higher- l truncations, however, we must parametrize \tilde{K} and use many calculations of the energies and $B(E_{\text{CM}})$ to fit to the entire \tilde{K} -matrix.

Define the following center-of-mass frame kinematic variables:

$$E_{\text{CM}} = \sqrt{E^2 - \mathbf{P}_{\text{tot}}^2}, \quad q_{\text{CM}} = \sqrt{\left(\frac{E_{\text{CM}}}{2}\right)^2 - m_\phi^2}, \quad (4.2)$$

where E is an energy measured in the lab-frame, $\mathbf{P}_{\text{tot}} = \frac{2\pi\mathbf{d}}{L}$ is the total lab-frame momentum associated with that energy, and m_ϕ is the ϕ mass measured in the $\mathbf{P}_{\text{tot}} = 0$ channel. With these definitions, we can now define \tilde{K}^{-1} in terms of the usual K -matrix and scattering phase-shift δ_l ,

$$\tilde{K}_l^{-1} = \left(\frac{q_{\text{CM}}}{m_\phi}\right)^{2l+1} K_l^{-1} = \left(\frac{q_{\text{CM}}}{m_\phi}\right)^{2l+1} \cot \delta_l(E_{\text{CM}}). \quad (4.3)$$

We choose to parametrize $\tilde{K}_{l=0}^{-1}$ by a Breit-Wigner,

$$\tilde{K}_0^{-1} = \frac{1}{2} \sqrt{\frac{E_{CM}^2}{m_\phi^2} - 4} \left(\frac{m_\rho^2 - E_{CM}^2}{m_\rho \Gamma_\rho} \right), \quad (4.4)$$

where the resonance mass m_ρ and its decay width Γ_ρ are parameters we fit to. Γ_ρ is related to the tri-field coupling as $\Gamma_\rho = \frac{g^2}{32\pi m_\rho^2} \sqrt{m_\rho^2 - 4m_\phi^2}$. When we include $l = 2$ in our truncation, we choose to parametrize it by the first term in an effective range expansion,

$$\tilde{K}_2^{-1} = -\frac{1}{m_\phi^5 a_2}. \quad (4.5)$$

Fig. 6 shows calculations of $\frac{q_{CM}}{m_\phi} \cot \delta_0$ along with a curve based on a fit to an $l_{\max} = 0$ \tilde{K} -matrix. Fig. 7 shows a similar fit, but this time including the $l = 2$ partial wave in our fit. We find

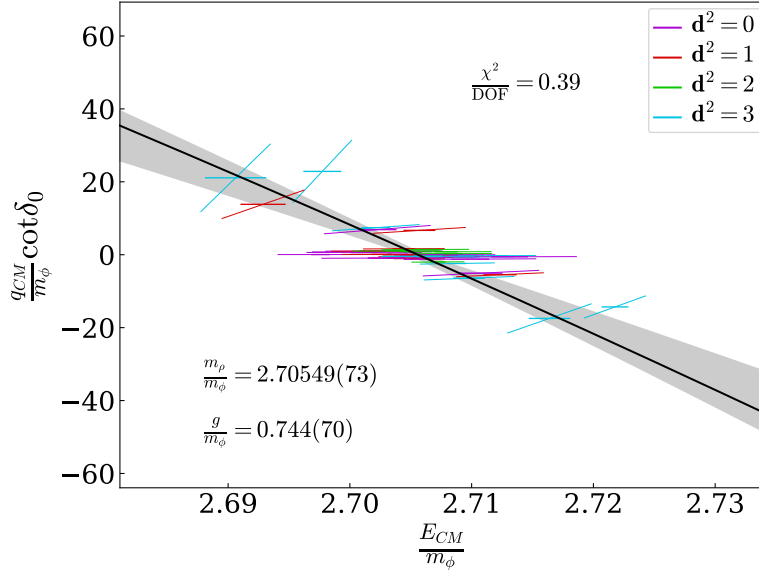


Figure 6: Fit to \tilde{K}^{-1} -matrix, truncated to $l_{\max} = 0$.

that not only is the $l = 2$ scattering length within error of zero, but also that including the $l = 2$ partial wave in our fit does not significantly change the determination of the resonance parameters m_ρ and Γ_ρ . We therefore determine that it is justified to truncate our \tilde{K} -matrix to $l_{\max} = 0$. Our determination of the fit parameters is still ongoing and will be published in the near future.

Acknowledgments

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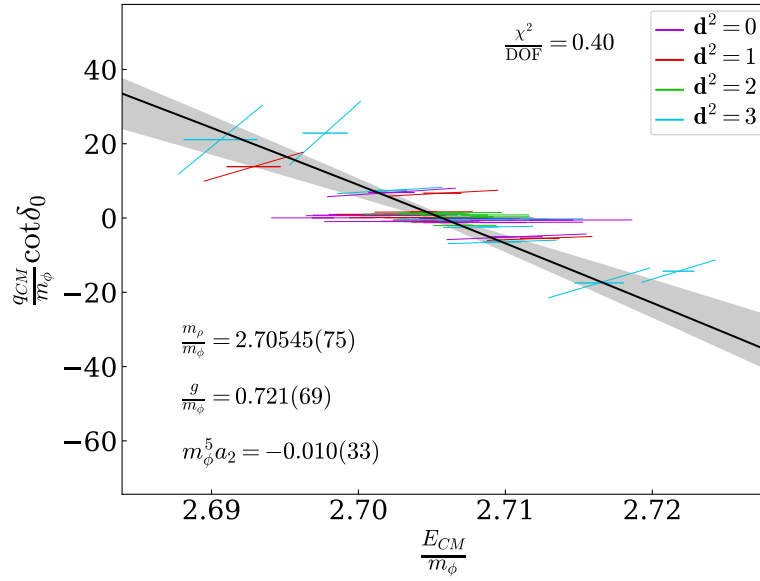


Figure 7: Fit to \tilde{K}^{-1} -matrix, truncated to $l_{\max} = 2$.

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