PROCEEDINGS OF SCIENCE

Hadron-Hadron Interactions from $N_f = 2+1+1$ Lattice QCD: $\pi - K$ scattering length

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In this proceeding we perform a separate chiral and continuum extrapolation of the scattering length for the $\pi - K$ scattering in the maximal isospin channel $I_3 = \frac{3}{2}$. In order to verify our approach, we extrapolate separately the meson parameters (pion mass and decay constants, eta mass) to the physical point. Our approach uses a minimal set of input parameters in the extrapolation and is an alternative to the combined chiral and continuum extrapolation. We compare the extrapolated scattering lengths with our already published results in [1].

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1. Introduction

The standard model is very successful in describing the ground state properties of light hadrons. Recent results with physical pion mass [2] and with continuum extrapolation [3, 4] show a very good agreement with the experiments. For a recent review on the subject see [5]. In this contribution we extract informations about the interaction in hadronic systems.

In particular we are investigating scatterings for small momenta, below the inelastic threshold. The quanitites of primary interests are the scattering phase shift and the scattering length, which can be related to experiments. Most of the lattice results are concerned with meson-meson interaction, in particular with pion-pion scattering. Besides lattice calculations, effective field theories, such as chiral perturbation theory (chPT) provide a good description due to the pseudo Goldstone boson nature of the pions under chiral symmetry. For example in the s-wave isospin=2 channel the pion mass dependence of the $\pi-\pi$ scattering length was found to be well described by chiral perturbation theory up to 400MeV pion mass [6].

On the contrary, when one or both particles in the interaction have non-zero strangeness the convergence of chPT is unclear and there are much less lattice results available. In this contribution we try to partly fill this gap by presenting our results for the $\pi-K$ s-wave scattering length in the $I=3/2$ isospin-channel ($a_0^{3/2}$).

On this conference there is another contribution that focuses on the $I=1/2$ $s-$ and $p-$wave kaon-pion scattering amplitudes [7]. Also the scattering length in the maximal isospin channel was recently determined at the physical point for the first time with a lattice spacing of $a \simeq 0.114$fm [8]. There are several results employing chiral extrapolations as well [9, 10, 11], however without taking attempts to continuum extrapolation.

In [1] we published our results with three different lattice spacing using combined chiral and continuum extrapolations. Using this approach we were not able to identify lattice artefacts, within our statistical accuracy. In this paper we perform an alternative controlled and separate chiral and continuum extrapolation of the scattering length, which could shed the light on the possible lattice spacing dependence of the scattering length. In order to control our extrapolations we use as an input parameter only the physical value of $\mu_{\pi K}/f_\pi$ and $M_K^2-0.5M_\pi^2$, where $\mu_{\pi K}$ is the reduced mass of the pion-kaon system, f_π is the pion-decay constant, M_π the pion mass and M_K the kaon mass. We reproduce all the other physical input parameters used in a combined chiral and continuum extrapolation: the physical value of the pion mass, the pion decay constant, the kaon mass and the eta meson mass (M_η). In particular we explicitly check that our extrapolations reproduce the physical value of

1. $r_0 f_\pi$ at the physical value of $\mu_{\pi K}/f_\pi$,
2. $r_0 f_\pi$ at the physical value of $(r_0 M_\pi)^2$,
3. $(r_0 M_\pi)^2$ at the physical value of $\mu_{\pi K}/f_\pi$,
4. $(r_0 M_\eta)^2$ at the physical value of $\mu_{\pi K}/f_\pi$,
5. $(r_0 M_\eta)^2$ at the physical value of $(r_0 M_\pi)^2$

in units of the appropriate powers of the Sommer-scale parameter (r_0).

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2. Numerical setup

For completeness we summarize here the details of our numerical simulations. For additional details we refer to [1]. In this contribution we use the ensembles provided by the European Twisted Mass Collaboration featuring 2+1+1 dynamical quark flavors [12]. In order to avoid the complicated flavor-parity mixing in the unitary non-degenerate strange-charm sector, we adopt a mixed action ansatz with so-called Osterwalder-Seiler (OS) [13] valence quarks, while keeping order $\mathcal{O}(a)$ improvement intact. In total we use 11 ensembles of gauge configurations with pion masses ranging from 260MeV to 490MeV distributed over three different lattice spacings. We show the summary of our simulation ensembles on Fig. 1. Note that we use partial quenching and simulate using three valence strange quark masses (whose range includes the physical strange quark mass) for each light quark mass. Referring to Fig 1 this means, that the simulations with the same shape at a given pion mass differ only by the valence strange quark mass.

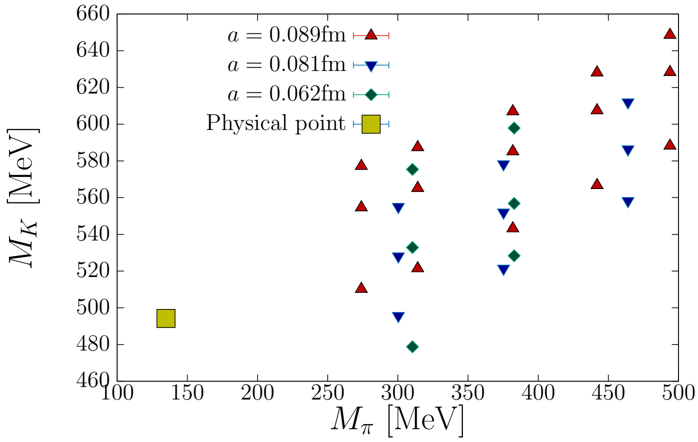

Figure 1: Summary of the simulated pion and kaon masses used in this work.

In order to compute the scattering length we use the Lüscher method which connects the finite volume interacting energy spectrum with the infinite volume scattering length [14]. In our case the relevant energy difference is $\Delta E_L = E_{\pi K} - M_K - M_\pi$, where $E_{\pi K}$ is the ground state energy of the interacting system. Using the finite range expansion one arrives at the formula relating the energy difference to the scattering length (a_0)

$$\Delta E_L = -\frac{2\pi a_0}{\mu_{\pi K} L^3}\left(1 + c_1 \frac{a_0}{L} + c_2 \frac{a_0^2}{L^2}\right) + \mathcal{O}\left(L^{-6}\right), \tag{2.1}$$

where $c_1 = 2.837297, c_2 = 6.375183$. We apply stochastic LapH smearing [15] to determine the correlators necessary for extracting the interaction energy.

3. Extrapolation methods

We obtain the scattering lengths ($a_0^{3/2}$) in our finite volume lattice simulations using the Lüschers formula with the computed energy difference ΔE_L as an input. In determining ΔE_L we have to compute the interaction energy of the pion kaon system using the appropriate four point correlation functions. The way we extract the ground state energy from this correlation function is explained in [1]. In particular we have to take care of the so-called thermal states, that distort the signal.

Using our results for the scattering length at pion masses larger than the physical one we now extrapolate to the physical point in the continuum limit. We determined the scattering length for several points in the three dimensional parameter space of the lattice spacing, light and strange quark mass. Thus the non-trivial task is to extrapolate along some path to the physical point. At this point it is important to stress that we are doing the inter-extrapolations not in terms of quark masses but entirely in terms of meson masses in practice.

We start with fixing the kaon mass relative to the pion mass using a combination of M_K and M_π, which in leading order(LO) of chPT is proportional to the strange quark mass. Therefore our fixing condition is

$$\left(\frac{r_0}{a}\right)^2 \left((aM_K)^2_{\text{fix}} - 0.5(aM_\pi)^2\right) = ((r_0M_K)_{\text{phys}})^2 - 0.5((r_0M_\pi)_{\text{phys}})^2, \tag{3.1}$$

where we have used the physical value of the right hand side $(r_0M_K)_{\text{phys}})^2 - 0.5((r_0M_\pi)_{\text{phys}})^2 = 1.33(7)$. We obtained the lattice spacing from ref. [17]. The other strange quark dependent observables relevant for the chPT expression of the scattering length (Eq. 3.2) will be interpolated linearly in terms of the kaon mass squared to $(aM_K)^2_{\text{fix}}$.

In order to extrapolate in the pion mass we use the next-to leading order(NLO) chPT expression from ref. [18]

$$\mu_{\pi K} a_0^{3/2} = \frac{\mu_{\pi K}^2}{4\pi f_\pi^2}\left\{-1 + \frac{32 M_\pi M_K}{f_\pi^2} L_{\pi K}(\Lambda_\chi) - \frac{16 M_\pi^2}{f_\pi^2} L_5(\Lambda_\chi) + \frac{1}{16\pi^2 f_\pi^2}\chi^{3/2}_{\text{NLO}}(\Lambda_\chi, M_\pi, M_K, M_\eta)\right\}, \tag{3.2}$$

where we choose the pion decay constant in lattice units as the renormalization scale $a\Lambda_\chi = af_\pi$ and $L_5(\Lambda_\chi)$, $L_{\pi K}(\Lambda_\chi)$ are the two LEC-s to be determined. At leading order this formula reduces to a quadratic function in terms of $\mu_{\pi K}/f_\pi$. Therefore it is useful to do the extrapolation not in terms of $(r_0M_\pi)^2$ but in terms of $\mu_{\pi K}/f_\pi$, because towards the chiral limit the corrections to the expected behavior becomes smaller and smaller. To do so we have to express M_π, M_η and f_π in terms of $\mu_{\pi K}/f_\pi$ using the techniques from ref. [17] for each lattice spacing. Our strategy is the following. We express first af_π as a function of $(aM_\pi)^2$, which enables us to express $\mu_{\pi K}/f_\pi$ as a function of $(aM_\pi)^2$, whose inverse we are looking for. For small pion masses one can use the Taylor-expansion of af_π as a function of $(aM_\pi)^2$ around the physical point:

$$af_\pi = c_1(a) + c_2(a)\cdot(aM_\pi)^2 + c_3(a)\cdot(aM_\pi)^4. \tag{3.3}$$

We show our results for the pion decay constant in the left panel of Fig. 2. For the intermediate lattice spacing ensembles we have not included the largest light quark mass data in the fit, which was seen to be outside the validity of our second order expansion. From the same fits we also obtain the pion mass in terms of $\mu_{\pi K}/f_\pi$, which we show in the right panel of Fig. 2.

The remaining observable to express in terms of $\mu_{\pi K} f_\pi$ is the η mass[1]. We do the extrapolations based on [19]. In particular we assume

$$(r_0M_\eta)^2 = b_1(a) + b_2\cdot(r_0M_\pi)^2. \tag{3.4}$$

[1]The kaon mass is already expressed in terms of the pion mass due to the strange quark mass fixing.

Note that here we perform a combined fit with parameter b_2 independent of the lattice spacing. We are currently performing simulation at an even smaller pion mass for the smallest lattice spacing ensemble, which will enable a more stable fit also for this observable. We show our fit results for the eta meson mass in Fig. 3.

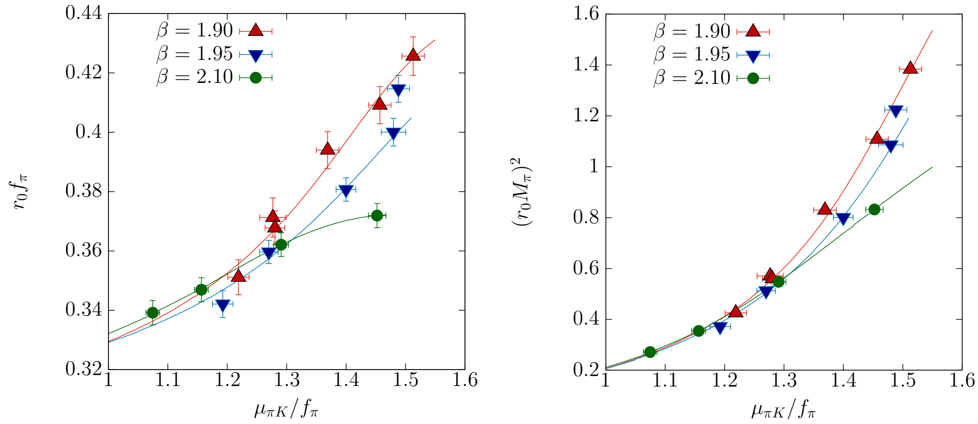

Figure 2: Left (Right) the pion decay constant (pion mass squared) as a function of the dimensionless parameter $\frac{\mu_{\pi K}}{f_\pi}$.

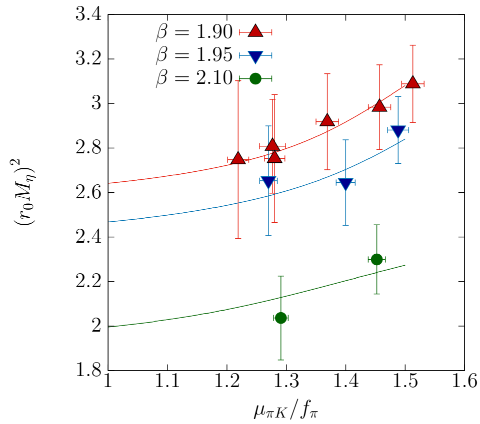

Figure 3: The eta mass squared as a function of the dimensionless parameter $\frac{\mu_{\pi K}}{f_\pi}$.

4. Results

In order to apply the above extrapolations in determining the scattering length at the physical point, we have to show that the extrapolations correctly reproduce the physical values of the meson parameters appearing in Eq. 3.2. We performed the continuum extrapolation of the pion mass, decay constant and eta mass at the fixed physical value of $\mu_{\pi K}/f_\pi$ and $(r_0 M_\pi)^2$. We summarize our results in Table 1. Note that we have one sigma discrepancy only for the eta mass, which can be attributed to the non-physical value of the strange quark mass [19].

We performed the chiral extrapolation (Eq. 3.2) using the lattice $\mu_{\pi K} a_0$ data with $L_{\pi K}$ as a fit parameter separately for each lattice spacing. Our fit is not stable enough to determine L_5 along

O	Fixed parameter	This work	Physical value	Relative deviation
$r_0 f_\pi$	$\mu_{\pi K}/f_\pi$	0.320(15)	0.310(8)	0.013(23)
$r_0 f_\pi$	$(r_0 M_\pi)^2$	0.318(13)	0.310(8)	0.011(20)
$(r_0 M_\pi)^2$	$\mu_{\pi K}/f_\pi$	0.113(13)	0.103(5)	0.100(134)
$(r_0 M_\eta)^2$	$\mu_{\pi K}/f_\pi$	1.3(3)	1.70(8)	0.21(18)
$(r_0 M_\eta)^2$	$(r_0 M_\pi)^2$	1.3(3)	1.70(8)	0.21(18)

Table 1: Summary of the chiral and continuum extrapolation of the meson parameters appearing in Eq. 3.2

with $L_{\pi K}$, therefore we use a prior for that from ref. [20], the same for all lattice spacings. At the end we extrapolate to the continuum at the physical value of $\mu_{\pi K}/f_\pi$. We show our results for the chiral and continuum extrapolations in Fig. 4.

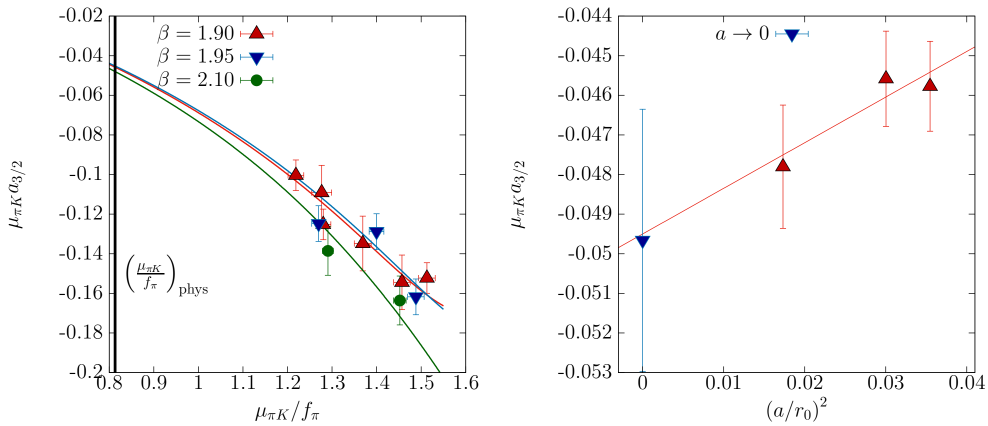

Figure 4: Left: The chiral extrapolation in terms of $\mu_{\pi K}/f_\pi$ towards the physical point for each lattice spacing. Right: The continuum extrapolation at the physical point

5. Conclusion

In this contribution we have determined the scattering length for the πK system at the maximal isospin channel in the s-wave. Our final result for the scattering length: $\mu_{\pi K} a_{3/2} = -0.049(3)$. We compare our results with the recent works on Fig. 5. With the analysis presented in the paper we get compatible results with our published work [1]. With our approach we are also able to to estimate the lattice artefacts to the scattering length, which seem to be a few percent effect (see in the left panel of Fig. 4). In order to further check our extrapolations, we currently perform calculations on an even smaller pion mass at the finest lattice spacing. Our plan for the future is to complete a combined chiral analysis of $\pi\pi$ $I = 2$,πK $I = 3/2$ and KK $I = 1$ scattering lengths.

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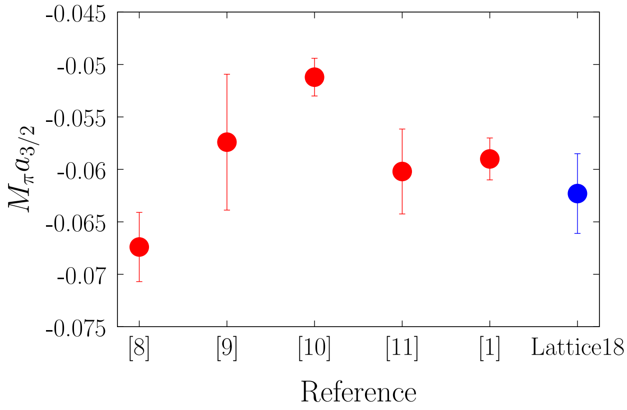

Figure 5: Our final estimation for $I = 3/2$ scattering length at the physical point. The blue point refers to this work.

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