

$K\pi$ scattering and the $K^*(892)$ resonance in 2+1 flavor QCD

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In this project, we will compute the form factors relevant for $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$ decays. To map the finite-volume matrix elements computed on the lattice to the infinite-volume $B \rightarrow K\pi$ matrix elements, the $K\pi$ scattering amplitude needs to be determined using Lüscher's method. Here we present preliminary results from our calculations with 2 + 1 flavors of dynamical clover fermions. We extract the P -wave scattering phase shifts and determine the K^* resonance mass and the $K^*K\pi$ coupling for two different ensembles with pion masses of 317(2) and 178(2) MeV.

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1. Introduction

Processes of the type $b \rightarrow s\ell^+\ell^-$ occur at one-loop order and higher in the Standard Model and hence are suppressed. It is for this reason that these flavor-changing neutral-current decays are very important in the search for physics beyond the Standard Model. The effective Hamiltonian that describes $b \rightarrow s\ell^+\ell^-$ decays at low energies [1] has the operators

$$O_{7(\gamma')} = \frac{m_b}{e} \bar{s} \sigma^{\mu\nu} P_{R(L)} b F_{\mu\nu}, \quad O_{9(\gamma')} = \bar{s} \gamma_\mu P_{L(R)} b \bar{\ell} \gamma^\mu \ell, \quad O_{10(\gamma')} = \bar{s} \gamma_\mu P_{L(R)} b \bar{\ell} \gamma^\mu \gamma_5 \ell, \quad (1.1)$$

as well as four-quark and gluonic operators. The coefficients C_i corresponding to these operators encode the short-distance physics and can be computed perturbatively in the Standard Model or in various new physics models. Global fits of experimental data from mesonic $b \rightarrow s\ell^+\ell^-$ transitions show deviations from Standard-Model predictions, in particular in the Wilson coefficient $C_9^{(\mu)}$ for the muonic final states [2, 3]. One process that contributes significantly to this tension is $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$. The hadron $K^*(892)$ in this process is unstable under the strong interaction, with a decay width of approximately 50 MeV. The $B \rightarrow K^*$ form factors have been previously studied on the lattice in the single-hadron approach [4, 5], in which only a quark-antiquark interpolating field is used for the K^* and the analysis is performed as if the K^* were stable. This approach has uncontrolled systematic uncertainties, which are expected to become more severe as the quark masses are lowered toward their physical values and the K^* becomes broader.

To eliminate these systematic uncertainties, in this project we will compute the $B \rightarrow K\pi$ matrix elements of the relevant $b \rightarrow s$ currents, using the Briceño-Hansen-Walker-Loud generalization [6] of the Lellouch-Lüscher formalism [7] to map the finite-volume matrix elements computed on the lattice to the desired infinite-volume $B \rightarrow K\pi$ matrix elements. The formalism requires a determination of the $K\pi$ scattering amplitude on the same lattice using Lüscher's method [8, 9]. In this contribution, we present our preliminary results for this scattering amplitude. Previous lattice studies of $K\pi$ scattering can be found in Refs. [10, 11, 12, 13, 14, 15].

2. Calculating the scattering amplitude using lattice QCD

Lüscher's method [8, 9] utilizes the energy shifts caused by the interactions of the two-hadron system in the finite lattice volume to extract the infinite-volume scattering amplitude. The cubic box with periodic boundary conditions in the spatial dimensions leads to a quantization of the momenta and a purely discrete energy spectrum. In addition, the cubic box breaks the $SO(3)$ rotational symmetry, reducing it to O_h , or to the relevant Little Group $LG^{\vec{P}}$ at nonzero total momentum \vec{P} .

The operators that we use to extract the energy spectrum with the quantum numbers of the K^* must have $I = 1/2$ and must be projected to irreps of $LG^{\vec{P}}$ that contain the P wave (see Table 1). We use operators with quark-antiquark and K - π structure,

$$O_{\bar{q}q,i}(\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x) \gamma_i u(x), \quad O_{\bar{q}q,0i}(\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{s}(x) \gamma_0 \gamma_i u(x),$$

$$O_{K\pi}(\vec{p}_1, \vec{p}_2) = \sqrt{\frac{2}{3}} \pi^+(\vec{p}_1) K^0(\vec{p}_2) - \sqrt{\frac{1}{3}} \pi^0(\vec{p}_2) K^+(\vec{p}_1), \quad (2.1)$$

$\frac{L}{2\pi}\vec{P}$	Little Group $LG^{\vec{P}}$	Irrep $\Lambda^{\vec{P},r}$	Spin content	Dimension
(0, 0, 0)	O_h	A_{1g}	$J = 0, 4, \dots$	1
(0, 0, 0)	O_h	T_{1u}	$J = 1, 3, \dots$	3
(0, 0, 1)	C_{4v}	A_1	$J = 0, 1, \dots$	1
(0, 0, 1)	C_{4v}	E	$J = 1, 2, \dots$	2
(0, 1, 1)	C_{2v}	A_1	$J = 0, 1, \dots$	1
(0, 1, 1)	C_{2v}	B_3	$J = 1, 2, \dots$	1
(0, 1, 1)	C_{2v}	B_2	$J = 1, 2, \dots$	1
(1, 1, 1)	C_{3v}	A_1	$J = 0, 1, \dots$	1
(1, 1, 1)	C_{3v}	E	$J = 1, 2, \dots$	2

Table 1: List of the total momenta \vec{P} that we use, along with their corresponding little groups, irreducible representations, and spin content [16]. Since we are interested in $J = 1$, we only consider the highlighted irreps.

where, for example, $K^+(\vec{p}_1) = \sum_{\vec{x}} e^{i\vec{p}_1 \cdot \vec{x}} \bar{s}(x) \gamma_5 u(x)$. We project these operators to the required irreps using the following procedure, which is based on the characters $\chi(R)$:

$$\begin{aligned}
 O_{K\pi, \Lambda}^{\vec{P}} &= \frac{\dim(\Lambda)}{\text{order}(LG^{\vec{P}})} \sum_{R \in LG^{\vec{P}}, \vec{m} \in \mathbb{Z}^3} \chi(R) O_{K\pi} \left(\vec{P}/2 + R(\vec{P}/2 + \frac{2\pi}{L}\vec{m}), \vec{P}/2 - R(\vec{P}/2 + \frac{2\pi}{L}\vec{m}) \right), \\
 O_{\bar{q}q, \Lambda}^{\vec{P}} &= \frac{\dim(\Lambda)}{\text{order}(LG^{\vec{P}})} \sum_{R \in LG^{\vec{P}}} \chi(R) R O_{\bar{q}q}(\vec{P}).
 \end{aligned} \tag{2.2}$$

We then use these operators to construct a correlation matrix, $C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle$, for each of the irreducible representations. There exist several methods to determine the spectrum from these correlation matrices; we use the generalized eigenvalue problem (GEVP) [17, 18], where we solve the equation

$$C_{ij}(t) u_j^n(t) = \lambda_n(t, t_0) C_{ij}(t_0) u_j^n(t) \tag{2.3}$$

for the eigenvalues $\lambda_n(t, t_0)$ and eigenvectors $u_j^n(t)$. For large t , the eigenvalues satisfy $\lambda_n(t, t_0) \propto e^{-E_n t}$, and we perform single-exponential fits to $\lambda_n(t, t_0)$ to extract E_n . Once we obtain the finite-volume spectrum, we map it to the infinite-volume phase shifts using the Lüscher quantization condition [8]

$$\det \left[e^{2i\delta} (M^{\vec{P}} - i) - (M^{\vec{P}} + i) \right] = 0. \tag{2.4}$$

Here, $[e^{2i\delta}]_{lm, l'm'} \equiv e^{2i\delta_l} \delta_{ll'} \delta_{mm'}$ corresponds to the scattering matrix, where δ_l is the scattering phase shift for angular momentum l , and M is a known matrix of finite-volume functions depending on the the total momentum \vec{P} , the scattering momentum, and the box size. When neglecting D -wave and higher contributions, it takes the form

$$M^{\vec{P}} = \begin{matrix} & 00 & 10 & 11 & 1-1 \\ \begin{matrix} 00 \\ 10 \\ 11 \\ 1-1 \end{matrix} & \begin{pmatrix} w_{00}^{\vec{P}} & i\sqrt{3}w_{10}^{\vec{P}} & i\sqrt{3}w_{11}^{\vec{P}} & i\sqrt{3}w_{1-1}^{\vec{P}} \\ -i\sqrt{3}w_{10}^{\vec{P}} & w_{00}^{\vec{P}} + 2w_{20}^{\vec{P}} & \sqrt{3}w_{21}^{\vec{P}} & \sqrt{3}w_{2-1}^{\vec{P}} \\ i\sqrt{3}w_{1-1}^{\vec{P}} & -\sqrt{3}w_{2-1}^{\vec{P}} & w_{00}^{\vec{P}} - w_{20}^{\vec{P}} & -\sqrt{6}w_{2-2}^{\vec{P}} \\ i\sqrt{3}w_{11}^{\vec{P}} & -\sqrt{3}w_{21}^{\vec{P}} & -\sqrt{6}w_{22}^{\vec{P}} & w_{00}^{\vec{P}} - w_{20}^{\vec{P}} \end{pmatrix} \end{matrix}, \quad (2.5)$$

where

$$w_{lm}^{\vec{P}} \equiv \frac{1}{\pi^{3/2} \sqrt{2l+1} \gamma q^{l+1}} Z_{lm}^{\vec{P}}(1; q^2), \quad (2.6)$$

with the relativistic Lorentz factor γ , the generalized zeta function Z_{lm} , and the dimensionless scattering momentum $q = \frac{L}{2\pi}k$. The scattering momentum k of the n -th energy level is determined by solving

$$\sqrt{s_n^{\vec{P},\Lambda}} = \sqrt{m_\pi^2 + (k_n^{\vec{P},\Lambda})^2} + \sqrt{m_K^2 + (k_n^{\vec{P},\Lambda})^2}, \quad (2.7)$$

where the center-of-mass energy $s_n^{\vec{P},\Lambda}$ is related to the energy $E_n^{\vec{P},\Lambda}$ on the lattice via $\sqrt{s_n^{\vec{P},\Lambda}} = \sqrt{(E_n^{\vec{P},\Lambda})^2 - (\vec{P})^2}$. The matrix $M^{\vec{P}}$ simplifies to a block-diagonal form once the specific symmetries belonging to the system with total momentum \vec{P} are taken into account. The quantization condition (2.4) then becomes a product of quantization conditions, each belonging to its own irreducible representation Λ .

The $K\pi$ system has no parity symmetry at non-zero momentum [12]; consequently, certain irreps will have contributions from S -wave and P -wave scattering states at the same time. This becomes a challenging technical problem, because the S -wave phase shift is non-negligible in the region of interest. In this first analysis, we therefore limit ourselves to the irreducible representations that do not contain $l = 0$, as indicated in Table 1.

3. Parameters of the lattice gauge-field ensembles

We use two ensembles with $N_f = 2 + 1$ dynamical quark flavors, utilizing a clover-improved Wilson action and the tadpole-improved tree-level Symanzik gauge action. The gauge links in the fermion action are smeared using one level of stout smearing to avoid instabilities in the hybrid-Monte-Carlo evolution. The parameters of the lattice gauge ensembles are given in Table 2.

Label	$N_L^3 \times N_t$	a (fm)	L (fm)	m_π (MeV)	m_K (MeV)	N_{meas}
C13	$32^3 \times 96$	0.11403(77)	3.65(2)	317(2)	527(4)	7060
D6	$48^3 \times 96$	0.08766(79)	4.21(4)	178(2)	514(5)	5248

Table 2: Parameters of the two ensembles and numbers of measurements analyzed.

4. Preliminary results

Figure 1 (left) shows an example of an effective mass plot from the eigenvalues $\lambda_n(t, t_0)$ obtained with the GEVP method, for the E irrep with $\vec{P}^2 = (\frac{2\pi}{L})^2$. In Fig. 1 (right) the stability of the single-exponential fits to $\lambda_n(t, t_0)$ is demonstrated by varying the t_{\min} at which the fits start. We choose the t_{\min} at which the energies (or equivalently, the scattering momenta) reach stable plateaus.

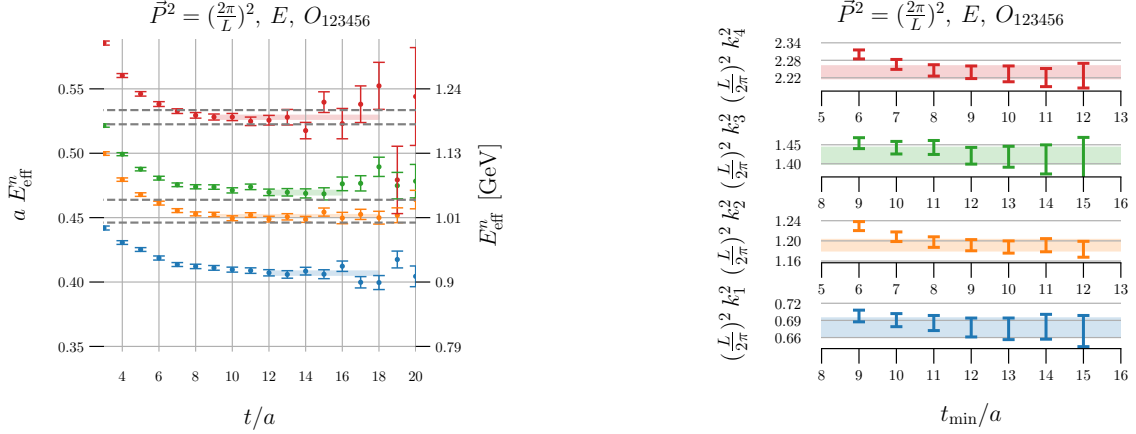


Figure 1: Left: effective-mass plot of the GEVP eigenvalues for irrep E , which has a total momentum squared of $\vec{P}^2 = (\frac{2\pi}{L})^2$. Right: the squared dimensionless scattering momenta obtained from the fitted energies, plotted as a function of t_{\min} . The data shown here are from the D6 ensemble.

The preliminary phase shift fit results obtained using Lüscher’s method are shown in Figs. 2 and 3 for ensembles C13 and D6, respectively. Also shown in the figures are fits of the energy-dependence using Breit-Wigner parametrizations of the form

$$\delta_{l=1} = \arccot \left(\frac{6\pi\sqrt{s}}{g_{K^*}^2 k^3} (m_{K^*}^2 - s) \right). \quad (4.1)$$

For both ensembles, we truncated the data before reaching the $K\eta$ threshold in order to avoid any inelastic effects. For the D6 ensemble, the $K\pi\pi$ threshold is located near the resonance, as shown in Fig. 3. However, the coupling of this three-meson channel with the $K\pi$ channel is not observed in experiment [19], and it is therefore reasonable to assume that it does not significantly affect our elastic scattering results. The values of g_{K^*} and m_{K^*} obtained from our fits, also displayed in Figs. 2 and 3, are consistent with previous lattice calculations at similar pion masses [12, 13, 14, 15].

5. Outlook

In the near future we plan to add a third ensemble of lattice gauge configurations, which will allow us to study the dependence on both m_π and a . Furthermore, we will include the irreducible representations in which the S and P waves mix, and we will include the coupling to the $K\eta$ channel. After the determinations of the scattering amplitudes are completed, we will proceed with the analysis of the transition matrix elements for the process $B \rightarrow K\pi\ell^+\ell^-$, which is the main goal of this project.

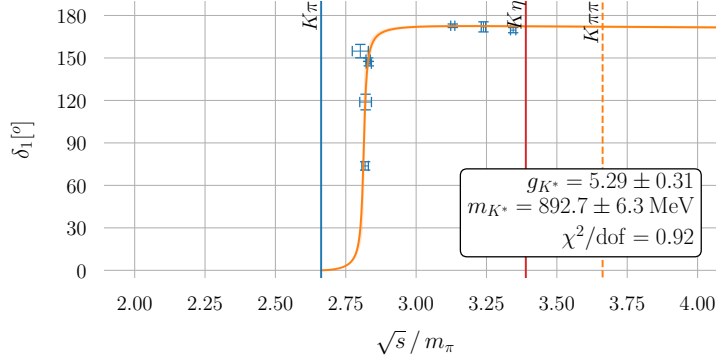


Figure 2: Phase shift points obtained from the C13 ensemble, along with a Breit-Wigner fit using Eq. (4.1). The parameters obtained from the fit are given in the bottom-right corner. The relevant thresholds are indicated with vertical lines.

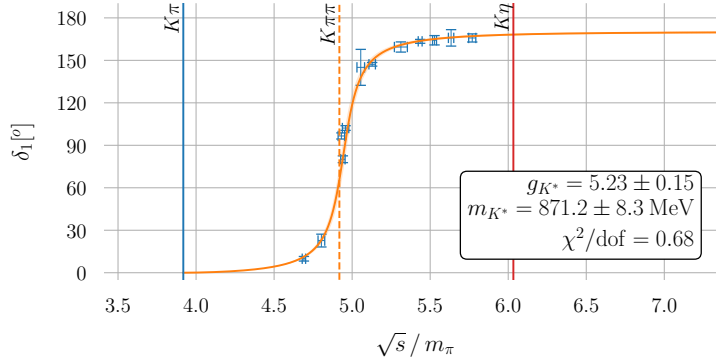


Figure 3: Like Fig. 2, but for the D6 ensemble.

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