

Baryonic states in supersymmetric Yang-Mills theory

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In $\mathcal{N}=1$ supersymmetric Yang-Mills theory the superpartner of the gluon is the gluino, which is a spin 1/2 Majorana particle in the adjoint representation of the gauge group. Combining three gluinos, it is possible to form colour neutral bound states, analogous to baryons in QCD. The correlation functions of the corresponding baryonic operators contain a contribution represented by a “sunset diagram”, and in addition, unlike in QCD, another contribution represented by a “spectacle diagram”. We present first results from an implementation and calculation of these objects, obtained from numerical simulations of supersymmetric Yang-Mills theory.

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1. Introduction

Despite being an extremely successful effective theory, the Standard Model of particle physics needs some extensions. $\mathcal{N} = 1$ supersymmetric Yang-Mills theory (SYM) is the supersymmetric extension of the gluonic sector of the Standard Model. The superpartner of the gluon is the gluino. Gauge invariance together with supersymmetry requires gluinos to transform according to the adjoint representation of the gauge group $SU(N)$. SYM is similar to QCD in some aspects. It is asymptotically free at high energies, and due to confinement, the low-energy degrees of freedom are expected to be colour-neutral bound states of gluons and gluinos [1]. Their masses can be computed non-perturbatively using powerful computing machines.

Supersymmetry (SUSY) is the essential feature distinguishing SYM from QCD. If SUSY is unbroken in the continuum, as non-perturbative studies of the supersymmetric Ward identities indicate [2], the bound states are arranged in supermultiplets, see [3, 4] for numerical investigations for gauge groups $SU(2)$ and $SU(3)$. Effective actions for the low-energy degrees of freedom in SYM have been constructed in [5] and extended in [6, 7]. They predict supermultiplets consisting of mesons, glueballs, and gluino-gluon bound states. It is, however, also possible to construct colour singlet bound states of three gluinos, analogous to baryons in QCD. These baryonic states are not described by the low-energy effective actions [5, 6] and it is unknown how they fit into the supermultiplets. In this paper we investigate these interesting objects.

2. $\mathcal{N} = 1$ supersymmetric Yang-Mills theory

The continuum Lagrangian of SYM in Euclidean space-time is given by

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \frac{1}{2} \bar{\lambda}^a \gamma_\mu (\mathcal{D}_\mu \lambda)^a + \frac{m_g}{2} \bar{\lambda}^a \lambda^a, \quad (2.1)$$

where $F_{\mu\nu}^a$ is the non-Abelian field strength tensor built from the gauge field $A_\mu(x)$, which represents the gluon. The gluino field $\lambda(x)$, the fermion superpartner of the gluon, is minimally coupled to the gauge field. The gluino satisfies the Majorana condition

$$\bar{\lambda}(x) = \lambda^T(x) C, \quad (2.2)$$

where C is the charge conjugation matrix. The gluino transforms under the adjoint representation of the gauge group. The covariant derivative thus acts as

$$(\mathcal{D}_\mu \lambda)^a = \partial_\mu \lambda^a + g_0 f_{bc}^a A_\mu^b \lambda^c. \quad (2.3)$$

A non-zero mass m_g of the gluino in supersymmetric Yang-Mills theory causes a soft breaking of supersymmetry. For vanishing gluino mass it is expected that supersymmetry is unbroken in the continuum [8].

For non-perturbative studies of SYM the theory is regularised on a Euclidean space-time lattice. The introduction of a space-time lattice regulator necessarily breaks supersymmetry [9]. Models that preserve part of an extended superalgebra are discussed in Ref. [10]. In our simulations, we use the ‘‘Curci-Veneziano’’ lattice action [11]

$$S = S_g + S_f, \quad (2.4)$$

where the gauge part (S_g) of the full action is given by the standard Wilson action

$$S_g = \frac{\beta}{2} \sum_x \sum_{\mu \neq \nu} \left(1 - \frac{1}{N_c} \text{Re tr} U_{\mu\nu}(x) \right). \quad (2.5)$$

Here $\beta \equiv 2N_c/g_0^2$ is the bare lattice gauge coupling for the $SU(N_c)$ gauge field and $U_{\mu\nu}(x)$ is the standard plaquette variable. The fermion part of the action in equation (2.4) is

$$S_f \equiv \frac{1}{2} \sum_x \left\{ \bar{\lambda}^a(x) \lambda^a(x) - \kappa \sum_{\mu=1}^4 \left[\bar{\lambda}^a(x + \hat{\mu}) V_{\mu}^{ab}(x) (1 + \gamma_{\mu}) \lambda^b(x) + \bar{\lambda}^a(x) V_{\mu}^{abT}(x) (1 - \gamma_{\mu}) \lambda^b(x + \hat{\mu}) \right] + \frac{i}{4} g_0 \kappa c_{SW} \bar{\lambda}^a(x) \sigma_{\mu\nu} P_{\mu\nu;ab}^{(cl)}(x) \lambda^b(x) \right\}. \quad (2.6)$$

Here κ is the hopping parameter, which is related to the bare gluino mass by $\kappa = 1/(2m_g + 8)$, γ_{μ} denotes a Dirac matrix, and $\sigma_{\mu\nu} = -\frac{1}{2}[\gamma_{\mu}, \gamma_{\nu}]$. The links $V_{\mu}^{ab}(x)$ are the gauge field variables in the adjoint representation, obtained from links $U_{\mu}(x)$ in the fundamental representation by

$$V_{\mu}^{ab}(x) \equiv 2 \text{tr} \left(U_{\mu}^{\dagger}(x) T^a U_{\mu}(x) T^b \right), \quad (2.7)$$

where T^a are the group generators of $SU(N_c)$. Finally, the last term in the fermion action contains the clover-symmetrised lattice field strength tensor:

$$P_{\mu\nu}^{(cl)}(x) = \frac{1}{4a} \sum_{i=1}^4 \frac{1}{2ig_0a} \left(U_{\mu\nu}^{(i)}(x) - U_{\mu\nu}^{(i)\dagger}(x) \right). \quad (2.8)$$

The coefficient c_{SW} is tuned up to one-loop order in perturbation theory to improve the convergence of on-shell observables to the continuum limit [12].

3. Baryons in Supersymmetric Yang-Mills theory

By combining three gluino fields $\lambda(x)$, it is possible to construct a colourless object, provided all the colour indices are contracted with indices of invariant colour tensors t_{abc} for $a, b, c = 1, \dots, N_c^2 - 1$. The corresponding particles are analogous to baryons in Quantum Chromodynamics (QCD), and will therefore also be called ‘‘baryons’’. Due to the fact that gluinos are in the adjoint representation of the gauge group, these objects can be constructed for both $SU(2)$ and $SU(3)$.

Interpolating fields for baryons will generally be of the form

$$W(x) = t_{abc} \lambda_a(x) (\lambda_b^T(x) \Gamma \lambda_c(x)) \quad (3.1)$$

with an invariant colour tensor t_{abc} and a matrix Γ in Dirac space. In the case of $SU(2)$ one has $t_{abc} = \varepsilon_{abc}$ and Γ has to be symmetric. For $SU(3)$ the t_{abc} can be chosen as the completely antisymmetric structure constants f_{abc} or as the completely symmetric tensor d_{abc} . The spin matrix Γ then has to have the opposite symmetry type in order that $W \neq 0$. Depending on the choice of Γ , the baryon field $W(x)$ carries either spin 1/2 only or both spin 1/2 and spin 3/2. These parts can be singled out with suitable projection operators [13]. A possibility, which we considered, is the choice $t_{abc} = \varepsilon_{abc}$ and $\Gamma = C\gamma_4$ for gauge group $SU(2)$.

The conjugate field is

$$\bar{W}(x) = -(CW(x))^T. \quad (3.2)$$

The masses of baryons can be determined from the corresponding correlation functions

$$\begin{aligned}
D(x, y) &= \langle W(x) \bar{W}(y) \rangle \\
D^{\alpha\delta}(x, y) &= \langle W^\alpha(x) \bar{W}^\delta(y) \rangle \\
&= -\langle W^\alpha(x) C^{\delta\alpha'} W^{\alpha'}(y) \rangle \\
&= -t_{abc} t_{a'b'c'} \Gamma^{\beta\gamma} \Gamma^{\beta'\gamma'} C^{\delta\alpha'} \langle \lambda_a^\alpha(x) \lambda_b^\beta(x) \lambda_c^\gamma(x) \lambda_{a'}^{\alpha'}(y) \lambda_{b'}^{\beta'}(y) \lambda_{c'}^{\gamma'}(y) \rangle. \quad (3.3)
\end{aligned}$$

The fermionic expectation value can be decomposed into gluino propagators by means of Wick's theorem. In SYM, in contrast to QCD, every pair of fermion fields contributes due to the Majorana nature of the gluino. In order to obtain the Wick contractions, it is convenient to express the correlation function in terms of six λ , by converting factors $\bar{\lambda}$ into λ by means of the Majorana condition. The Wick contractions are

$$\langle \lambda_a^\alpha(x) \lambda_b^\beta(y) \rangle = K_{ab}^{\alpha\beta}(x, y) = -(\Delta(x, y) C)_{ab}^{\alpha\beta}, \quad (3.4)$$

where $\Delta = Q^{-1}$ is the fermion propagator. Taking into account the fermionic signs, we get 15 terms:

$$\begin{aligned}
D^{\alpha\alpha'}(x, y) &= t_{a'b'c'} t_{abc} \Gamma^{\beta\gamma} \Gamma^{\beta'\gamma'} \times \{ \\
&\quad + 2 \Delta_{aa'}^{\alpha\alpha'}(x, y) \Delta_{bb'}^{\beta\beta'}(x, y) \Delta_{cc'}^{\gamma\gamma'}(x, y) \\
&\quad + 4 \Delta_{ab'}^{\alpha\beta'}(x, y) \Delta_{bc'}^{\beta\gamma'}(x, y) \Delta_{ca'}^{\gamma\alpha'}(x, y) \\
&\quad + 2 \Delta_{ab}^{\alpha\beta}(x, x) \Delta_{ca'}^{\delta\alpha'}(x, y) \Delta_{c'b'}^{\delta'\beta'}(y, y) C^{\gamma\delta} C^{\gamma'\delta'} \\
&\quad + 4 \Delta_{ab}^{\alpha\beta}(x, x) \Delta_{b'c}^{\beta'\gamma}(y, x) \Delta_{c'a'}^{\gamma'\alpha'}(y, y) \\
&\quad + 1 \Delta_{aa'}^{\alpha\alpha'}(x, y) \Delta_{bc}^{\beta\delta}(x, x) \Delta_{c'b'}^{\delta'\beta'}(y, y) C^{\gamma\delta} C^{\delta'\gamma'} \\
&\quad + 2 \Delta_{ac}^{\alpha\delta'}(x, y) \Delta_{bc}^{\beta\delta}(x, x) \Delta_{b'a'}^{\beta'\alpha'}(y, y) C^{\gamma\delta} C^{\gamma'\delta'} \}. \quad (3.5)
\end{aligned}$$

Depending on the topology of the associated fermion line diagrams, the contributions can be divided into two types called ‘‘Sunset’’ and ‘‘Spectacle’’, which are represented in Fig.1.

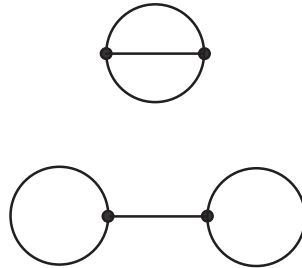


Figure 1: Contributions to the propagator of a three-gluino state. The second contribution, which we call ‘‘spectacle graph’’ comes from contractions of the form $\lambda(x)\lambda(x)$, which are allowed for Majorana fermions.

The corresponding algebraic expressions are given by

$$D_{\text{sunset}}(x, y) = t_{a'b'c'} t_{abc} \Gamma^{\beta\gamma} \Gamma^{\beta'\gamma'} P_{\pm}^{\alpha\alpha'} \times \{ \quad (3.6)$$

$$+ 2 \Delta_{aa'}^{\alpha\alpha'}(x, y) \Delta_{bb'}^{\beta\beta'}(x, y) \Delta_{cc'}^{\gamma\gamma'}(x, y) \quad (3.7)$$

$$+ 4 \Delta_{ab'}^{\alpha\beta'}(x, y) \Delta_{bc'}^{\beta\gamma'}(x, y) \Delta_{ca'}^{\gamma\alpha'}(x, y) \} \quad (3.8)$$

$$D_{\text{spectacle}}(x, y) = t_{a'b'c'} t_{abc} \Gamma^{\beta\gamma} \Gamma^{\beta'\gamma'} P_{\pm}^{\alpha\alpha'} \times \{$$

$$+ 2 \Delta_{ab}^{\alpha\beta}(x, x) \Delta_{ca'}^{\delta\alpha'}(x, y) \Delta_{c'b'}^{\delta'\beta'}(y, y) C^{\gamma\delta} C^{\gamma'\delta'}$$

$$+ 4 \Delta_{ab}^{\alpha\beta}(x, x) \Delta_{b'c}^{\beta'\gamma'}(y, x) \Delta_{c'a'}^{\gamma'\alpha'}(y, y)$$

$$+ 1 \Delta_{aa'}^{\alpha\alpha'}(x, y) \Delta_{bc}^{\beta\delta}(x, x) \Delta_{c'b'}^{\delta'\beta'}(y, y) C^{\gamma\delta} C^{\delta'\gamma'}$$

$$+ 2 \Delta_{ac}^{\alpha\delta'}(x, y) \Delta_{bc}^{\beta\delta}(x, x) \Delta_{b'a'}^{\beta'\alpha'}(y, y) C^{\gamma\delta} C^{\gamma'\delta'} \} \quad (3.9)$$

In contrast to mesonic or glueball correlation functions, the baryonic correlation functions are not symmetric or antisymmetric with respect to Euclidean time, if no parity projection is included, see e. g. [14]. Therefore we include the projection onto parity eigenstates by means of the parity projector P_{\pm} , which is defined as $P_{\pm} = \frac{1}{2}(1 \pm \gamma_4)$ for zero momentum.

We have implemented the contributions to the baryonic correlation function into the measurement code of our SYM collaboration. The numerically quite expensive part is the evaluation of the spectacle piece. We have calculated it by means of combining the exact contribution of the lowest eigenmodes of the Wilson-Dirac matrix with the stochastic estimator technique.

Presently some preliminary results have been obtained. Fig. 2 shows the two pieces of the baryonic correlation function and its sum for gauge group SU(2). In this case the sunset piece

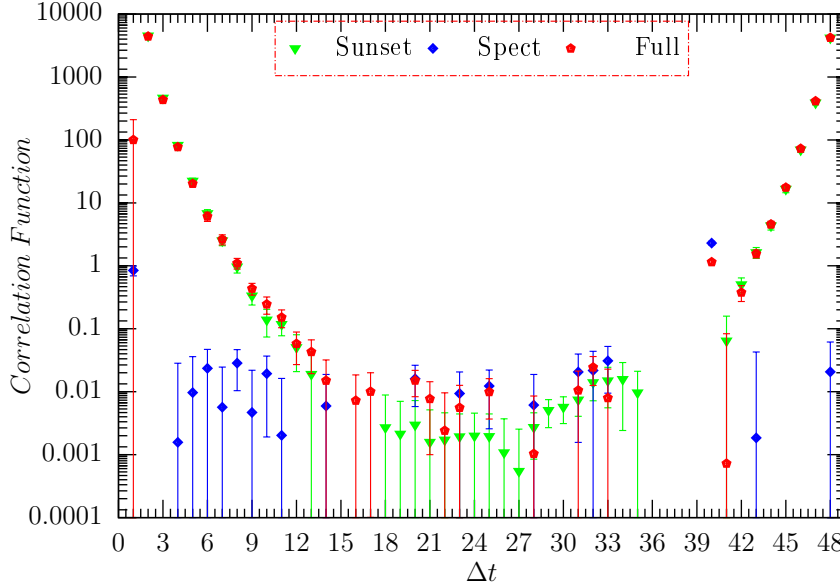


Figure 2: Numerical results for correlation functions at each time slice for gauge group SU(2), lattice volume, $V = 24^3 \cdot 48$, $\beta = 1.75$ and $\kappa = 0.14925$.

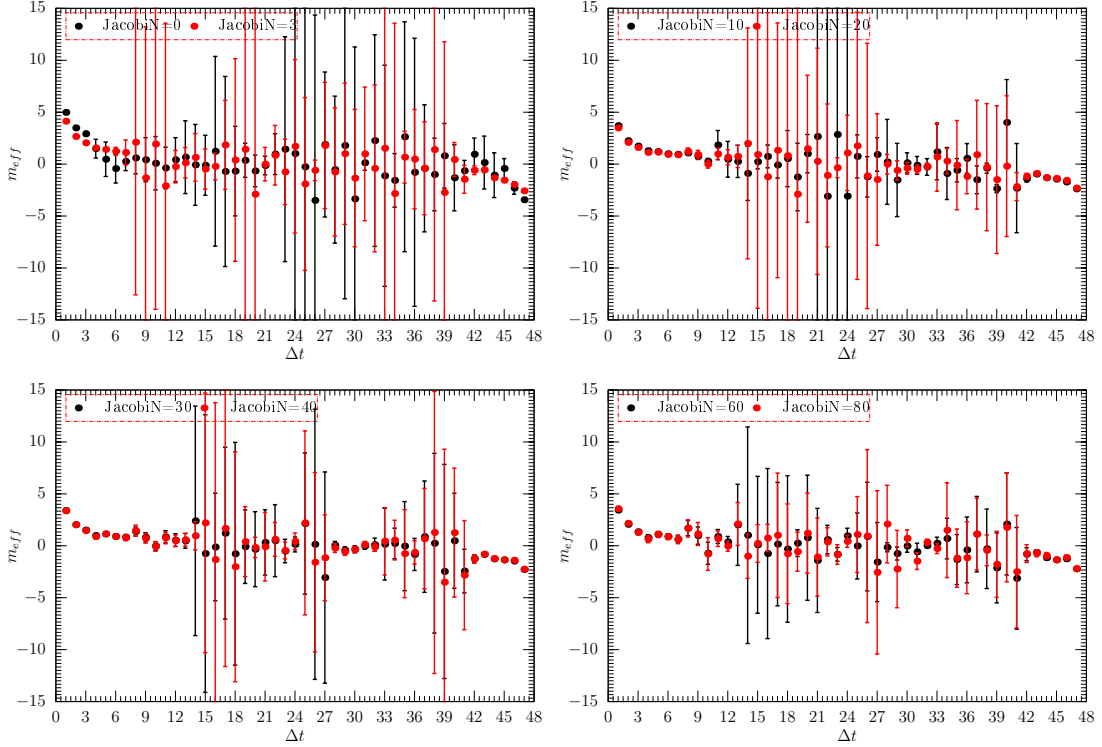


Figure 3: Effective mass plots for fixed APE pre-smearing with level 40 and different levels of Jacobi smearing of the Dirac fermion matrix.

appears to dominate the correlation function for not too large times. Fig. 3 shows the effective mass for different smearings.

4. Conclusion and outlook

In this paper we constructed the correlation function for a three-gluino state in supersymmetric Yang-Mills theory and presented preliminary estimates of its mass using Monte Carlo simulation at non-zero gluino mass and at finite inverse gauge coupling β . With better numerical results we expect to be able to perform chiral and continuum extrapolations in the future, and to find out how the baryonic states fit into the supermultiplet found in Ref. [3].

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