

Quark mass determinations with the RI-SMOM scheme and HISQ action

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Lattice QCD provides several avenues for the high precision determination of quark masses. Using the RI-SMOM scheme applied to lattice calculations with the HISQ action, we obtain mass renormalisation factors that we use to provide strange and charm quark masses with 1% precision. The calculation involves the study of various sources of systematic uncertainty, including an analysis of possible nonperturbative (condensate) contributions. These results allow a comparison of different mass determination methods of comparable precision. In particular we (HPQCD) find good agreement between RI-SMOM and current-current correlator determinations based on the same lattice QCD bare masses, providing a strong test of our understanding of systematic uncertainties.

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1. Motivation

It is well demonstrated that lattice QCD is a powerful framework for the extraction of Standard Model (SM) parameters to high precision. Such work is important as input for various SM calculations used in high energy experiments such as the ongoing work at the LHC [1]. Presented here is a summary of a calculation of the strange and charm quark masses by tuning of lattice computations to experimentally measured meson masses with renormalisation of the resulting bare quark masses in the RI-SMOM scheme, with a final perturbative matching to $\overline{\text{MS}}$, detailed in [2]. This method is able to produce percent level precision, comparable to that achieved by the different methodologies of [3] and [4]. In particular the work of [3] uses much of the same input data (bare quark masses and configurations) and this separate determination therefore provides a strong check on the understanding of the (different) systematics in both calculations.

2. The RI-SMOM scheme

The RI-SMOM scheme [5] defines renormalisation factors in terms of (Landau) gauge fixed inverse propagators and vertex functions in momentum space, on which renormalisation conditions are imposed. For example, the wavefunction renormalisation Z_q is fixed to be 1 in the free theory. All vertex renormalisation factors are calculated as the ratio of Z_q and the relevant amputated vertex function $\Lambda_{\mathcal{O}}$, with \mathcal{O} denoting the operator at the vertex. These vertex functions have a symmetric kinematic setup with $q^2 = p_1^2 = p_2^2 \equiv \mu^2$ where p_1 and p_2 are the ingoing and outgoing quark momenta and $q = p_1 - p_2$ is the momentum insertion at the vertex.

This scheme can be implemented nonperturbatively on the lattice as long as care is taken in the consideration of condensate contributions which are not present in the perturbative matching calculations to other schemes, performed in the continuum. For the purposes presented here this is done (as is typical) at vanishing valence quark mass which is achieved through an extrapolation to this point using multiple valence masses (see Section 4 and Section IV B of [2]).

Here the mass renormalisation factors Z_m are calculated on 2+1+1 HISQ [6] configurations generated by the MILC collaboration [7, 8] from the scalar vertex function and propagator, using lattice spacings in the range $\sim 0.06 - 0.12$ fm. Z_m is simply the inverse of Z_S . Calculations were done at multiple values of μ ranging from 2 to 5 GeV.

The RI-SMOM scheme is detailed in [5] and the framework for the implementation of such schemes with staggered quarks was developed in [9].

3. Systematic checks

In order to assess the level at which finite volume and mistuned or unphysical sea quark masses affect the results, Z_m was calculated on lattice ensembles with multiple volumes and multiple sea quark masses. No significant variation was seen between ensembles, an example being given by Figure 1. This is as expected from ultraviolet quantities. However, a significant (although small) and momentum dependent effect was observed as the tolerance of Landau gauge fixing was varied. The various data used in this analysis used a gauge fixing tolerance of either 10^{-7} or 10^{-14} with a μ dependent systematic error being added to the former to account for the effect seen in Figure 2 which shows the spread of bootstrap data samples as the tolerance is tightened.

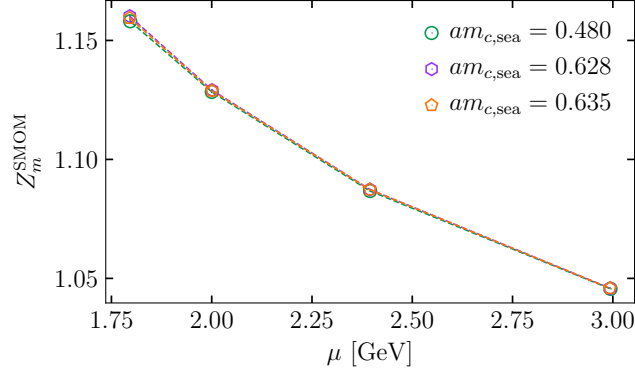


Figure 1: Dependence of Z_m^{SMOM} on the charm sea mass as a function of μ . No significant variation can be seen.

As the SMOM to $\overline{\text{MS}}$ matching calculations have been done at zero mass they neglect the effect of sea quark masses which are present in lattice calculations. We use matching factors that account for the massive charm in the sea for which details are provided in Appendix A of [2].

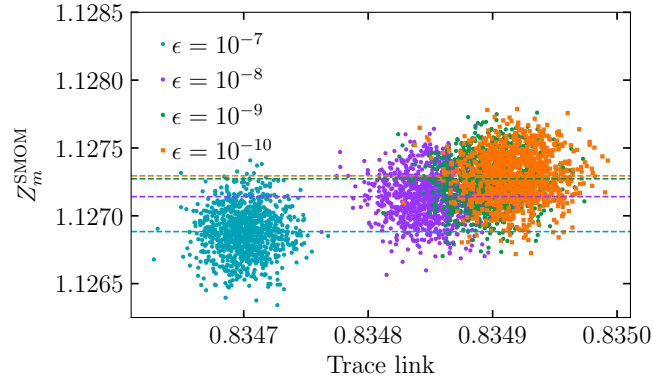


Figure 2: Scatter plot of bootstrap samples of Z_m^{SMOM} against the average trace link variable for different Landau gauge fixing tolerances. The mean values are shown by the dashed horizontal lines and can be seen to move substantially from tolerance 10^{-7} to 10^{-10} .

4. Mass extrapolation

In order to extrapolate to the zero valence mass point the calculation of Z_m was performed at three different valence masses and then extrapolated using a polynomial in am_{val} up to third order. This was found to give good χ^2 for all fits and can be demonstrated to provide an accurate representation of the mass dependence even up to the strange quark mass, as displayed in Figure 3 where the highest mass point was not included in the fit shown. All data used in final fits has been extrapolated to zero valence quark mass. We have been careful to propagate the effect of correlations between different μ values on a given ensemble through the calculation.

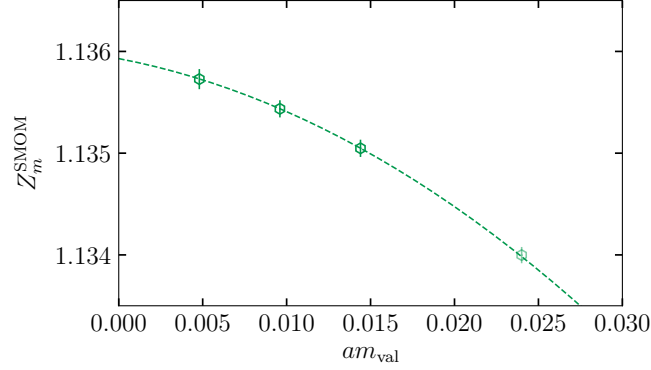


Figure 3: Valence mass extrapolation at $\mu = 3$ GeV on a lattice with a spacing of approximately 0.06 fm. The highest mass point is not included in the fit.

5. Condensate contributions

The extrapolation of the valence mass removes the contributions of chiral condensates that appear in the operator product expansion proportional to the mass. However, there will still be contributions from gluon condensates. The leading gauge invariant such condensate $\langle G_{\mu\nu}G^{\mu\nu} \rangle$ is expected to be small and will be suppressed by μ^4 but the gauge noninvariant quantities that we use here get contributions from gauge noninvariant condensates, most prominently $\langle A^2 \rangle$ which will only be suppressed by μ^2 [10]. Such a condensate is therefore allowed for in the final continuum extrapolation fit. This has been neglected in previous calculations of Z_m using this method.

6. Continuum extrapolation and extraction of quark masses

The data used in the final fit is constructed by multiplying the bare quark mass on each lattice by the relevant Z_m^{SMOM} , then multiplying by the $\overline{\text{MS}}$ matching factor and running to a reference scale of 3 GeV in the $\overline{\text{MS}}$ scheme. Having all data points perturbatively run to the same reference scale allows the effects of condensates to become visible.

The continuum extrapolation employed here accounts for sources of discretisation errors in both the bare masses and Z_m as well as condensate contributions, residual sea quark mass dependence and neglected α_s^3 terms in the matching to $\overline{\text{MS}}$. These extrapolations (done separately for the strange and charm masses) result in determinations with 1% precision whose values are in good agreement with the determinations of [3] and [4].

The greatest source of uncertainty in the final values (particularly for the strange quark) are the correlated and uncorrelated uncertainties on the bare quark masses. The uncorrelated uncertainties arise from the independent tuning done on each ensemble while the correlated uncertainties come from fitting and the w_0 value [11] used in each lattice spacing determination. After that the next largest sources are from the continuum extrapolation and from the condensate terms. Both could in principle be reduced by using finer lattices which allow a move to higher values of μ , which will lead to further suppression of the condensate contributions.

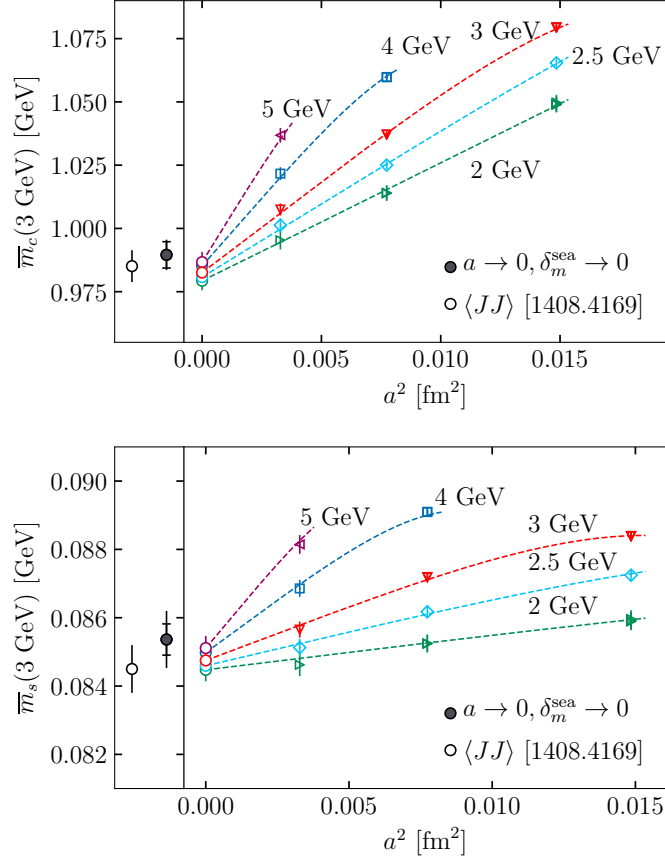


Figure 4: Continuum extrapolations of \bar{m}_c and \bar{m}_s with a fit accounting for condensates and sea mass mistunings. The circles at the end of the fit lines have the sea mass mistunings removed but still contain condensate contributions, while the filled circle on the left is the final answer, with condensate contributions removed. The empty circle on the far left is the result from [3].

7. Results

The final results in the $\overline{\text{MS}}$ scheme at a reference value of 3 GeV are

$$\begin{aligned}\bar{m}_c(3 \text{ GeV}, n_f = 4) &= 0.9896(61) \text{ GeV} \\ \bar{m}_s(3 \text{ GeV}, n_f = 4) &= 0.08536(85) \text{ GeV}.\end{aligned}\tag{7.1}$$

These can be run to the conventionally quoted scales of \bar{m}_c and 2 GeV respectively, giving

$$\begin{aligned}\bar{m}_c(\bar{m}_c, n_f = 4) &= 1.2757(84) \text{ GeV} \\ \bar{m}_s(2 \text{ GeV}, n_f = 4) &= 0.09449(96) \text{ GeV}.\end{aligned}\tag{7.2}$$

These results can be combined as described in [2] with other available $n_f = 4$ lattice determinations to give world averages of

$$\bar{m}_c(\bar{m}_c, n_f = 4)_{2+1+1 \text{ av.}} = 1.2753(65) \text{ GeV}\tag{7.3}$$

$$\bar{m}_s(2 \text{ GeV}, n_f = 4)_{2+1+1 \text{ av.}} = 0.09291(78) \text{ GeV}.$$

In summary, good 1% level precision agreement for the strange and charm quark masses has been achieved for different methodologies with different sources of systematic uncertainty, indicating good control over these uncertainties for lattice QCD quark mass determinations.

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