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Calculation of $K \to \pi l \nu$ form factor in $N_f = 2+1$ QCD at physical point on (10 fm)^3

J. Kakazu[*1], **K.-I. Ishikawa**[2,3], **N. Ishizuka**[1,4], **Y. Kuramashi**[1,4], **Y. Nakamura**[2], **Y. Namekawa**[1], **Y. Taniguchi**[1,4], **N. Ukita**[4], **T. Yamazaki**[1,4], **and T. Yoshie**[1,4]
(PACS Collaboration)

[1] *Graduate School of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8571, Japan*
[2] *RIKEN Center for Computational Science, Kobe, Hyogo 650-0047, Japan*
[3] *Graduate School of Science, Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8526, Japan*
[4] *Center for Computational Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8577, Japan*
E-mail: kakazu@het.ph.tsukuba.ac.jp

We present our preliminary result of the form factor of $K \to \pi l \nu$ semileptonic decays on the large volume configuration, $L \approx 10$ fm, with the physical m_π and m_K using the stout-smearing clover quark and Iwasaki gauge actions at $a^{-1} = 2.333$ GeV. From an interpolation using the data in small momentum transfers, we determine the semileptonic decay form factors at zero momentum transfer. The result is compared with the previous lattice calculations. We also estimate the value of $|V_{us}|$ by combining our result with the experimental value of the kaon semileptonic decay.

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1. Introduction

The weak semileptonic and leptonic decays of kaon have been investigated for a long time. The decays play an important role to determine V_{us}, which is one of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements [1] to describe the mixing of the mass eigenstates between up and strange quarks. From the unitary condition of the up quark part in the CKM matrix, $\Delta_u \equiv |V_{ub}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$ must be vanished in the standard model. Therefore, we can examine the existence of physics beyond the standard model by checking whether the $\Delta_u = 0$ or not.

We can determine the value of $|V_{us}|$ in the two different ways. One uses the form factor of the kaon semileptonic (K_{l3}) decay at zero momentum transfer. The other is the ratio of the meson decay constants, f_K/f_π, which is related to the leptonic kaon (K_{l2}) decay [2]. It is not possible to determine $|V_{us}|$ only from the experiments, because in experimental results, for instance a branching ratio, $|V_{us}|$ is multiplied to the form factor or the decay constants. Therefore, some theoretical evaluations for the form factor and decay constants are necessary. The lattice QCD calculation is the most precise way to determine these quantities.

The recent results of $|V_{us}|$ in PDG [3] are given by

$$|V_{us}| = \begin{cases} 0.2231(8) \ \text{(form factor)} \\ 0.2253(7) \ \text{(decay constant)} \\ 0.2256(8) \ \text{(unitarity condition)} \end{cases} .$$

The result from the form factor is estimated by combining the experimental value $|V_{us}|f_+(q^2 = 0) = 0.2165(4)$ in Ref. [4], and $f_+(0)$ in the FLAG's value [5]. The result from the decay constant ratio is estimated by using the experimental value of the K_{l2} decay [6], and f_K/f_π in the FLAG's value [5]. Another one is estimated by the unitarity condition $\Delta_u = 0$ using the most precise result of $|V_{ud}|$ [7] and ignoring $|V_{ub}|$ due to the small effect ($|V_{ub}| \approx O(10^{-3})$) in this estimation.

In the $|V_{us}|$ estimations, there is difference between the value from the unitarity condition and that from the form factor by about 2σ. It is still premature to conclude, however, that it is significant signal of new physics or not.

In order to check whether the difference is significant or not, we need more precise determination of the form factor by reducing uncertainties of the lattice QCD calculation. The uncertainties of the form factor in lattice calculations come from the chiral extrapolation, interpolation (or extrapolation) to zero momentum transfer due to the accessible momentum in lattice calculation, and finite size effect and the continuum extrapolation. In order to suppress these uncertainties as small as possible, we calculate the semileptonic form factors at physical point on the large volume of (10.8 fm)^3. Therefore, there is no systematic error from the chiral extrapolation, and the finite size effect is expected to be negligible in this calculation.

2. Calculation of form factors

The K_{l3} form factors $f_+(q^2)$ and $f_-(q^2)$ are defined by the matrix element of the weak vector current as,

$$\langle \pi(\vec{p}_\pi) | V_\mu | K(\vec{p}_K) \rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2), \tag{2.1}$$

where V_μ is vector current and $q = p_K - p_\pi$ is the momentum transfer. The scalar form factor $f_0(q^2)$ is defined by using $f_+(q^2)$ and $f_-(q^2)$ as,

$$f_0(q^2) = f_+(q^2) + \frac{-q^2}{(m_K^2 - m_\pi^2)} f_-(q^2) = f_+(q^2)\left(1 + \frac{-q^2}{(m_K^2 - m_\pi^2)}\xi(q^2)\right), \tag{2.2}$$

where $\xi(q^2) = f_-(q^2)/f_+(q^2)$. At $q^2 = 0$, the two form factors, $f_+(q^2)$ and $f_0(q^2)$ give the same value, $f_0(0) = f_+(0)$.

In order to obtain the form factors, we calculate the meson 3-point functions $C_\mu^{XY}(\vec{p}', \vec{p}, t)$ with the weak vector current given by

$$C_\mu^{XY}(\vec{p}', \vec{p}, t) = \langle 0|O_Y(\vec{p}', t_f) V_\mu(\vec{q}, t) O_X^\dagger(\vec{p}, t_i)|0\rangle \tag{2.3}$$

$$= \frac{Z_Y(\vec{P}') Z_X(\vec{P})}{4E_Y(\vec{P}')E_X(\vec{p})} \frac{1}{Z_V} \langle Y(\vec{p}')\left|V_\mu\right|X(\vec{p})\rangle e^{-E_Y(\vec{p}')(t_f - t)} e^{-E_X(\vec{p})(t - t_i)} + \cdots, \tag{2.4}$$

where Z_V is the renormalization factor of the vector current, $X, Y = \pi, K$, and $t_i < t < t_f$. Since we fix $\vec{p}' = \vec{0}$ in our calculation, we represent the 3-point function as $C_\mu^{XY}(\vec{p}, t)$ in the followings. $E_X(\vec{p})$ and $Z_X(\vec{p})$ are evaluated from the meson 2-point functions given by

$$C^X(\vec{p}, t) = \langle 0|O_X(\vec{p}, t) O_X^\dagger(\vec{p}, t_i)|0\rangle = \frac{|Z_X(\vec{p})|^2}{2E_X(\vec{p})}\left(e^{-E_X(\vec{p})|t - t_i|} + e^{-E_X(\vec{p})(T - |t - t_i|)}\right) + \cdots, \tag{2.5}$$

with the periodic boundary condition in the temporal direction. The terms of the dots $(\cdots)$ are contributions of excited states. The meson masses, m_π and m_K, are obtained from a single exponential fit to each 2-point function. Their energies are determined by the equation $E_X(\vec{p}) = \sqrt{m_X^2 + \vec{p}^2}$ using the fit result of m_X.

In order to calculate the form factors from $C_\mu^{XY}(\vec{p}, t)$ and $C^X(\vec{p}, t)$, we define the following three quantities [8] as

$$d_1(q, t) = \frac{C_4^{\pi K}(\vec{0}, t) C_4^{K\pi}(\vec{0}, t)}{C_4^{KK}(\vec{0}, t) C_4^{\pi\pi}(\vec{0}, t)} \to \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}(f_0(q^2_{max}))^2, \tag{2.6}$$

$$d_2(q, t) = \frac{C_4^{\pi K}(\vec{p}, t) C^{\pi}(\vec{0}, t)}{C_4^{\pi K}(\vec{0}, t) C^{\pi}(\vec{p}, t)} \to \left(\frac{E_\pi(\vec{p}) + m_K}{m_\pi + m_K} + \frac{E_\pi(\vec{p}) - m_K}{m_\pi + m_K}\xi(q^2)\right)\frac{f_+(q^2)}{f_0(q^2_{max})}, \tag{2.7}$$

$$d_3(q, t) = \frac{C_i^{\pi K}(\vec{p}, t) C_4^{KK}(\vec{p}, t)}{C_i^{KK}(\vec{p}, t) C_4^{\pi K}(\vec{p}, t)} \to \frac{(E_K(\vec{p}) + m_K)(1 - \xi(q^2))}{E_\pi(p) + m_K + (m_K - E_\pi(p))\xi(q^2)}. \tag{2.8}$$

The right arrow expresses the asymptotic limit of $t_i \ll t \ll t_f$, where $C_\mu^{XY}(\vec{p}, t)$ and $C^X(\vec{p}, t)$ are dominated by the π and K states. In the region, the quantities become independent of the time slice of the vector current t, so that each ratio in the right hand sides of Eqs. (2.6)–(2.8) is obtained by constant fits of the quantities. From $d_1(q, t)$, $f_0(q^2_{max})$ with $q^2_{max} = -(m_K - m_\pi)^2$ is obtained. By solving the simultaneous equations in Eqs. (2.7) and (2.8), we determine $f_+(q^2)$ and $\xi(q^2)$ at each q^2, and then $f_0(q^2)$ is evaluated from Eq. (2.2).

3. Simulation setup

We use the configuration generated at the physical point, $m_\pi = 0.135$ GeV, on the large volume corresponding to $La = Ta = 10.8$ fm ($L = T = 128$), which is a part of the PACS10 configuration [9]. The configurations were generated by using $N_f = 2+1$ non-perturbative Wilson clover quark action with 6 stout smearing link [10] (smearing parameter $\rho = 0.1$) and the improvement coefficient $c_{SW} = 1.11$, and the Iwasaki gauge action [11] at $\beta = 1.82$. The hopping parameters of degenerated light quarks and strange quark are $(\kappa_{ud}, \kappa_s) = (0.126117, 0.124902)$, respectively.

In our form factor calculation, we use 20 configurations in total. We adopt 8 positions of the source time $t_i = 0, 16, 32, 48, 64, 72, 96, 112$ per configuration, and 4 choices of the temporal axis thanks to the hypercube lattice. In the calculation of 2- and 3-point functions, we use $\mathbb{Z}(2) \otimes \mathbb{Z}(2)$ random wall source spread in the spatial sites, and also color and spin spaces [13]. The number of the random source is one in each t_i. The 3-point function is calculated using the sequential source technique at the sink time slice t_f, where the meson momentum is fixed to zero. We choose the temporal separation between the source and sink as $|t_f - t_i| = 36$ (= 3.0 fm). For the constant fits of the quantities $d_{1,2,3}(q,t)$ explained in the above, we use the fit range of $t = 15-21$ when $t_i = 0$. We calculate $C^{XY}_{\mu}(\vec{p},t)$ and $C^{X}(\vec{p},t)$ with the momentum $\vec{p} = (2\pi/L)\vec{n}$ of $|\vec{n}|^2 \leq 6$ without the twisted boundary condition, where $\vec{n}$ is an integer vector. The one elimination jackknife method is employed to estimate the statistical errors.

We suppress the wrapping around effect of the 3-point function, discussed in Ref. [12], by averaging the 3-point functions with the periodic and anti-periodic boundary conditions in the temporal direction.

4. Result

The left panel of Fig. 1 shows the momentum transfer dependence of $f_+(q^2)$ and $f_0(q^2)$. We obtain clear signals for both form factors. Since we have the data at very close to $q^2 = 0$, which are obtained from $d_{2,3}(q,t)$ with the momentum of $|\vec{n}|^2 = 4$, it is possible to carry out stable interpolations to obtain the form factors at $q^2 = 0$.

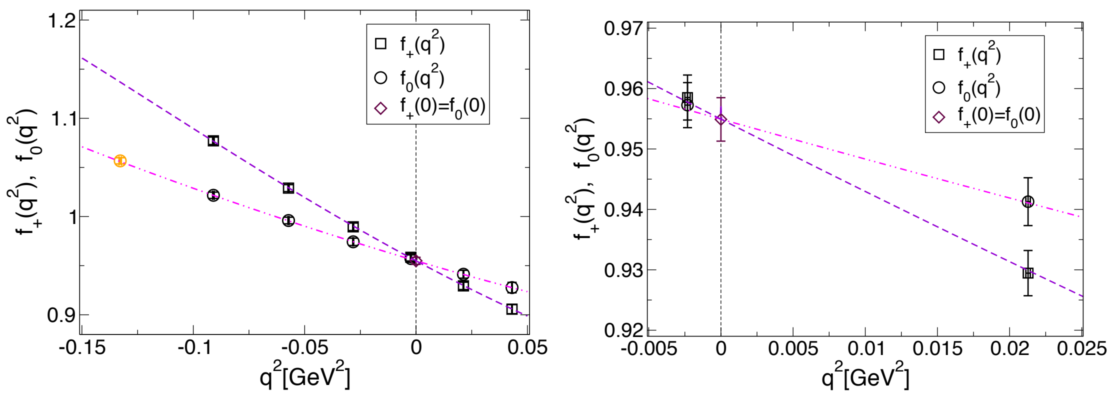

Figure 1: (left) q^2 dependence of K_{l3} form factors. The square and circle symbols denote $f_+(q^2)$ and $f_0(q^2)$, respectively. The dashed and dot-dashed curves represent the simultaneous fit result of $f_+(q^2)$ and $f_0(q^2)$, respectively, with the ChPT forms in Eqs.(4.1) and (4.2). The orange circle in the far left represents $f_0(q^2_{max})$, which is not included in the fit. The diamond symbol denotes the fit result of $f_+(0) = f_0(0)$. (right) The same figure as (left), but near $q^2 = 0$ region.

For a q^2 interpolation of the form factors, the NLO SU(2) ChPT formula [14] is employed, which is given by

$$f_+(q^2) = F_+\left(1+(C_0^+ + C_1^+ s)x + \frac{m_K^2}{(4\pi f_0)^2}\left(-\frac{3}{4}x\log x - xT_1^+(s) - T_2^+(s)\right)\right), \tag{4.1}$$

$$f_0(q^2) = F_0\left(1+(C_0^0 + C_1^0 s)x + \frac{m_K^2}{(4\pi f_0)^2}\left(-\frac{3}{4}x\log x + xT_1^0(s) - T_2^0(s)\right)\right), \tag{4.2}$$

where $s = -q^2/m_K^2$, $x = m_\pi^2/m_K^2$, $f_0 = 0.10508$GeV, and

$$T_1^+(s) = [(1-s)\log(1-s) + s(1-s/2)]3(1+s)/4s^2, \tag{4.3}$$

$$T_2^+(s) = [(1-s)\log(1-s) + s(1-s/2)](1-s)^2/4s^2, \tag{4.4}$$

$$T_1^0(s) = [\log(1-s) + s(1+s/2)](9+7s^2)/4s^2, \tag{4.5}$$

$$T_2^0(s) = [(1-s)\log(1-s) + s(1-s/2)](1-s)(3+5s)/4s^2. \tag{4.6}$$

In the interpolation, we use the five fit parameters, F_+, C_0^+, C_1^+, C_0^0, and C_1^0, while F_0 is fixed by use the constraint $f_+(0) = f_0(0)$. We also employ the monopole ansatz as another interpolation form given by

$$f_+(q^2) = \frac{F}{1+q^2/M_V^2} \quad \text{and} \quad f_0(q^2) = \frac{F}{1+q^2/M_S^2}, \tag{4.7}$$

where the fit parameters are F, M_V, and M_S, and $F = f_+(0) = f_0(0)$. Using the fit forms we perform the uncorrelated simultaneous fit with the data for $f_+(q^2)$ and $f_0(q^2)$.

The left panel of Fig. 1 shows the simultaneous fit result of $f_+(q^2)$ and $f_0(q^2)$ using the ChPT forms in Eqs. (4.1) and (4.2). The data of $f_0(q^2_{max})$ denoted by the orange circle symbol in the figure is not included in the fit. The ChPT forms well describe our data, and the value of χ^2/d.o.f. ≈ 0.03 in the uncorrelated fit. The right panel of Fig. 1 shows the fit result of the form factor at $q^2 = 0$. The monopole ansatz gives a similar result, whose uncorrelated χ^2/d.o.f. ≈ 0.03. From the two interpolations we obtain our preliminary results of the K_{l3} from factor at $q^2 = 0$ given by

$$f_+(0) = f_0(0) = \begin{cases} 0.9549(36) \ \text{(ChPT)} \\ 0.9552(36) \ \text{(monopole)} \end{cases}. \tag{4.8}$$

The systematic error of the fit form dependence is much smaller than the statistical error as expected.

We estimate the absolute value of the CKM matrix element $|V_{us}|$ from our preliminary result using the ChPT form by combining the experimental result $|V_{us}|f_+(0) = 0.2165(4)$ [4],

$$|V_{us}| = 0.22671(84)(41), \tag{4.9}$$

where the first error is statistical error, and the second comes from the experiment. The monopole fit result agrees with this value. Figure 2 shows the comparison of our results from the ChPT and monopole fits with the PDG's estimations [3], other lattice results [8, 13–17], and the one from the unitarity condition $\Delta_u = 0$. Our results are relatively higher than the PDG's value [3] estimated from the K_{l3} form factor and the other lattice calculations, while they are consistent with the ones determined from the decay constants [3, 9] and also the one of $\Delta_u = 0$.

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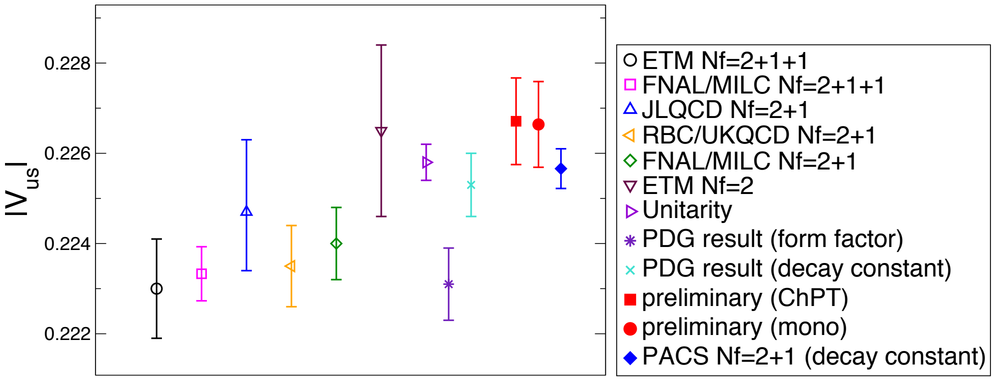

Figure 2: Comparison of $|V_{us}|$. The filled square and circle symbols are our results from the K_{l3} form factor with the ChPT and monopole fit, respectively. The filled diamond symbol is estimated by the decay constant ratio $f_\pi/f_K = 1.1914(16)$ calculated with the same configuration [9]. The star and cross symbols express the PDG's values [3] from the K_{l3} form factor and the ratio of the decay constants. Estimation from our f_π/f_K [9], The open right triangle symbol is estimated from the unitarity condition $\Delta_u = 0$. The other open symbols represent previous lattice QCD results from the K_{l3} form factor [8, 13–17].

In order to test consistency of our result with the experiment, we estimate the slopes of the form factors $\lambda_{+,0}$ defined by

$$\lambda_+ = \frac{m^2_{\pi^\pm\mathrm{phys}}}{f_+(0)} \frac{f_+(t)}{dt}\bigg|_{t=-q^2=0} \quad \text{and} \quad \lambda_0 = \frac{m^2_{\pi^\pm\mathrm{phys}}}{f_0(0)} \frac{f_0(t)}{dt}\bigg|_{t=-q^2=0}. \tag{4.10}$$

Our preliminary results obtained from the ChPT fit are

$$\lambda_+ = 2.48(8) \times 10^{-2},\ \lambda_0 = 1.36(9) \times 10^{-2}. \tag{4.11}$$

We find that the above results are consistent with the ones from the monopole fit as in $|V_{us}|$. They are in good agreement with the experimental results$\lambda_+ = 2.58(7) \times 10^{-2}$ and $\lambda_0 = 1.36(7) \times 10^{-2}$ [18].

5. Summary

We present preliminary results of the K_{l3} form factors $f_+(q^2)$ and $f_0(q^2)$ in $N_f = 2+1$ lattice QCD at the physical point ($m_\pi = 0.135$ GeV) on a large volume, whose spatial extent is 10.8 fm. Since we calculate the form factors in close to zero momentum transfer, we carry out stable interpolations by using the NLO SU(3) ChPT and monopole forms. From the interpolations we evaluate the slope of the form factors, and find that our results are consistent with the experimental values. Using our preliminary result of the form factors at the zero momentum transfer, $|V_{us}|$ is estimated by combining with the experimental value. Our value is relatively higher than the PDG's value and other lattice calculations, while it agrees with the values estimated from the decay constants and the one from the unitarity condition $\Delta_u = 0$. One of important future works in this calculation is to estimate systematic errors in the form factors, such as excited state contamination.

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