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2018 Update on ε_K with lattice QCD inputs

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We present updated results for ε_K determined directly from the standard model (SM) with lattice QCD inputs such as $\hat{B}_K$, $|V_{cb}|$, $|V_{us}|$, ξ_0, ξ_2, ξ_{LD}, F_K, and m_c. We find that the standard model with exclusive $|V_{cb}|$ and other lattice QCD inputs describes only 70% of the experimental value of $|\varepsilon_K|$ and does not explain its remaining 30%, which leads to a strong tension in $|\varepsilon_K|$ at the 4σ level between the SM theory and experiment. We also find that this tension disappears when we use the inclusive value of $|V_{cb}|$ obtained using the heavy quark expansion based on QCD sum rules.

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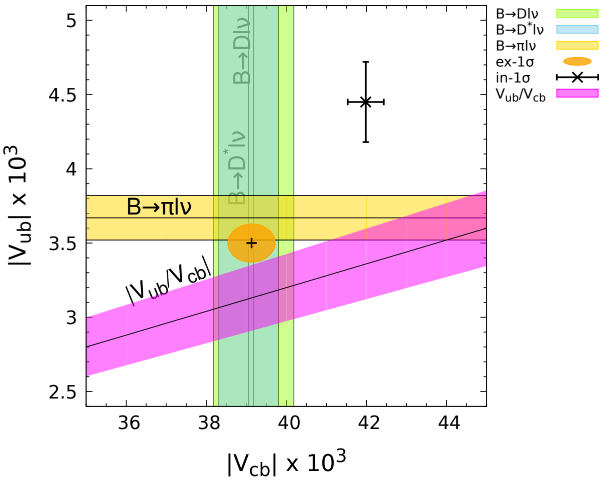
channel	value	Ref.
$B \to D^* \ell \bar{\nu}$	39.05(47)(58)	[5, 6]
$B \to D \ell \bar{\nu}$	39.18(94)(36)	[5, 7]
$\lvert V_{ub}\rvert / \lvert V_{cb}\rvert$	0.080(4)(4)	[5, 8]
ex-combined	39.13(59)	[5]

(a) Results for exclusive $|V_{cb}|$.

channel	value	Ref.
kinetic scheme	42.19(78)	[5]
1S scheme	41.98(45)	[5]
in-combined	42.03(39)	[1]

(b) Results for inclusive $|V_{cb}|$.



(c) $|V_{cb}|$ versus $|V_{ub}|$.

Table 1: $|V_{cb}|$: (a) exclusive $|V_{cb}|$, (b) inclusive $|V_{cb}|$, and (c) $|V_{cb}|$ versus $|V_{ub}|$.

1. Introduction

This paper is a brief summary of our previous paper [1]. This paper is also an update of our previous papers [2, 3, 4].

2. Input parameters: $|V_{cb}|$ and ξ_0

In Table 1, we present updated results for both exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$. Recently, HFLAV reported them in Ref. [5]. The results for exclusive $|V_{cb}|$ are obtained using lattice QCD results for the semileptonic form factors of Refs. [6, 7, 8]. Here, we use the combined results (ex-combined) for exclusive $|V_{cb}|$ and the results of the 1S scheme for inclusive $|V_{cb}|$ to evaluate ε_K. For more details on $|V_{cb}|$ and the related caveats, refer to Ref. [1].

The absorptive part of long distance effects on ε_K is parametrized into ξ_0.

$$\xi_0 = \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}, \qquad \xi_2 = \frac{\mathrm{Im} A_2}{\mathrm{Re} A_2}, \qquad \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|}(\xi_2 - \xi_0)\,. \tag{2.1}$$

There are two independent methods to determine ξ_0 in lattice QCD: one is the indirect method and the other is the direct method. In the indirect method, one can determine ξ_0 using Eq. (2.1) with lattice QCD input ξ_2 and with experimental results for ε'/ε, ε_K, and ω. In the direct method, one can determine ξ_0 directly using lattice QCD results for ImA_0 combined with experimental results for ReA_0. In Table 2 (a), we summarize results for ξ_0 calculated by RBC-UKQCD using the indirect and direct methods. Here, we use the results of the indirect method for ξ_0 to evaluate ε_K.

In Ref. [9], RBC-UKQCD also reported the S-wave scattering phase shift for the $I = 0$ channel: $\delta_0 = 23.8(49)(12)$, which is different from those of the dispersion relations [10, 11] by $\approx 3\sigma$. In Ref. [12], they have accumulated higher statistics to obtain $\delta_0 = 19.1(25)(12)$, which is about 5σ different from those of the dispersion analyses. They introduce a σ operator and make all possible combinations with the σ and $\pi - \pi$ operators. Then, RBC-UKQCD has obtained $\delta_0 = 32.8(12)(30)$

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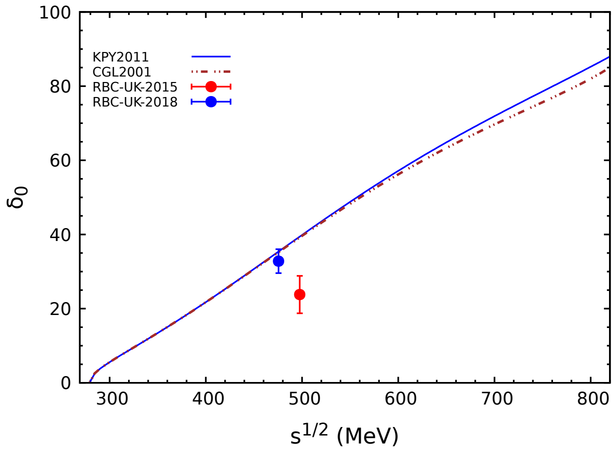
method	ξ_0	Ref.
indirect	$-1.63(19)\times 10^{-4}$	[13]
direct	$-0.57(49)\times 10^{-4}$	[9]

(a) ξ_0

Collaboration	δ_0	Ref.
RBC-UKQCD-15	$23.8(49)(12)^\circ$	[9]
RBC-UKQCD-18	$32.8(12)(30)^\circ$	[12]
KPY-2011	$39.1(6)^\circ$	[11]
CGL-2001	$39.2(15)^\circ$	[10, 14]

(b) Results for δ_0



(c) δ_0

Table 2: The absorptive long distance effect ξ_0 and S-wave $I=0$ scattering phase shift δ_0.

WP	CKMfitter		UTfit		AOF	
λ	0.22509(29)	[16]	0.22497(69)	[17]	0.2248(6)	[18]
$\bar{\rho}$	0.1598(76)	[16]	0.153(13)	[17]	0.146(22)	[19]
$\bar{\eta}$	0.3499(63)	[16]	0.343(11)	[17]	0.333(16)	[19]

(a) Wolfenstein parameters

Input	Value	Ref.
η_{cc}	1.72(27)	[3]
η_{tt}	0.5765(65)	[20]
η_{ct}	0.496(47)	[21]

(b) η_{ij}

Table 3: (a) Wolfenstein parameters and (b) QCD corrections: η_{ij} with $i,j=c,t$.

which is consistent with those of the dispersion relations. These results are presented in Table 2 (b) and Figure 2 (c).

3. Input parameters: Wolfenstein parameters, $\hat{B}_K$, $\xi_{\rm LD}$, and others

In Table 3 (a), we summarize the Wolfenstein parameters on the market. The CKMfitter and UTfit collaboration provide the Wolfenstein parameters determined by the global unitarity triangle (UT) fit. Unfortunately, ε_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to the global UT fit, which leads to unwanted correlation with ε_K. We want to avoid this correlation, and so take another input set from the angle-only fit (AOF) suggested in Ref. [15]. The AOF does not use ε_K, $\hat{B}_K$, and $|V_{cb}|$ as input to determine the UT apex $(\bar{\rho},\bar{\eta})$. Here the λ parameter is determined from $|V_{us}|$ which is obtained from the $K_{\ell 2}$ and $K_{\ell 3}$ decays using lattice QCD results for the form factors and decay constants. The A parameter is determined from $|V_{cb}|$.

In the FLAG review [22], they present lattice QCD results for $\hat{B}_K$ with $N_f=2$, $N_f=2+1$, and $N_f=2+1+1$. Here, we use the results for $\hat{B}_K$ with $N_f=2+1$, which is obtained by taking a global average over the four data points from BMW 11 [23], Laiho 11 [24], RBC-UKQCD 14 [25], and SWME 15 [26]. In Table 4 (a), we present the FLAG 17 result for $\hat{B}_K$ with $N_f=2+1$, which is used to evaluate ε_K.

The dispersive long distance (LD) effect is defined as

$$\xi_{\rm LD} = \frac{m'_{\rm LD}}{\sqrt{2}\Delta M_K}, \qquad m'_{\rm LD} = -{\rm Im}\left[\mathscr{P}\sum_C \frac{\langle \overline{K}^0|H_{\rm w}|C\rangle\langle C|H_{\rm w}|K^0\rangle}{m_{K^0}-E_C}\right] \tag{3.1}$$

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Collaboration	Ref.	$\hat{B}_K$
SWME 15	[26]	0.735(5)(36)
RBC/UKQCD 14	[25]	0.7499(24)(150)
Laiho 11	[24]	0.7628(38)(205)
BMW 11	[23]	0.7727(81)(84)
FLAG 17	[22]	0.7625(97)

(a) $\hat{B}_K$

Input	Value	Ref.
G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[18]
M_W	80.385(15) GeV	[18]
$m_c(m_c)$	1.2733(76) GeV	[27]
$m_t(m_t)$	163.65(105)(17) GeV	[1]
θ	$43.52(5)^\circ$	[18]
m_{K^0}	497.611(13) MeV	[18]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[18]
F_K	155.6(4) MeV	[18]

(b) Other parameters

Table 4: (a) Results for $\hat{B}_K$ and (b) other input parameters.

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If the CPT invariance is well respected, the overall contribution of the $\xi_{\rm LD}$ to ε_K is about $\pm 2\%$.

Lattice QCD tools to calculate $\xi_{\rm LD}$ are well established in Refs. [28, 29, 30]. In addition, there have been a number of attempts to calculate $\xi_{\rm LD}$ on the lattice [31, 32]. In them, RBC-UKQCD used a pion mass of 329 MeV and a kaon mass of 591 MeV, and so the energy of the 2 pion and 3 pion states are heavier than the kaon mass. Hence, the sign of the denominator in Eq. 3.1 is opposite to that of the physical contribution. Therefore, this work belongs to the category of exploratory study rather than to that of precision measurement.

In Ref. [33], they use chiral perturbation theory to estimate the size of $\xi_{\rm LD}$ and claim that

$$\xi_{\rm LD} = -0.4(3) \times \frac{\xi_0}{\sqrt{2}} \tag{3.2}$$

where we use the indirect results for ξ_0 and its error. Here, we call this method the BGI estimate for $\xi_{\rm LD}$. In Refs. [28, 34], RBC-UKQCD provides another estimate for $\xi_{\rm LD}$:

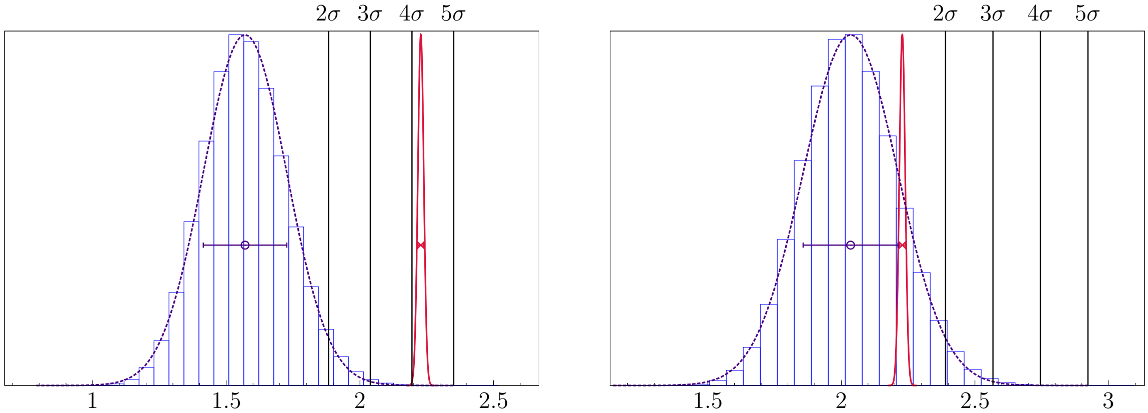
$$\xi_{\rm LD} = (0 \pm 1.6)\%. \tag{3.3}$$

Here, we call this method the RBC-UKQCD estimate for $\xi_{\rm LD}$.

In Table 3 (b), we present higher order QCD corrections: η_{ij} with $i, j = t, c$. In Table 4 (b), we present other input parameters needed to evaluate ε_K. Since Lattice 2017, three parameters: $m_t(m_t)$, m_{K^0}, F_K have been updated. The $m_t(m_t)$ parameter is the scale-invariant (SI) top quark mass renormalized in the $\overline{\rm MS}$ scheme. The pole mass of top quarks comes from Ref. [18]: $M_t = 173.5 \pm 1.1$ GeV. We convert the top quark pole mass into the SI top quark mass using the four-loop perturbation formula. For more details, refer to Ref. [1].

4. Results for ε_K

In Fig. 1, we present results for $|\varepsilon_K|$ evaluated directly from the standard model (SM) with lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical evaluation of $|\varepsilon_K|$ using the FLAG-2017 $\hat{B}_K$, AOF for Wolfenstein parameters, and exclusive $|V_{cb}|$, and the RBC-UKQCD estimate for $\xi_{\rm LD}$. The red curve in Fig. 1 represents the experimental value of $|\varepsilon_K|$. In Fig. 1 (b), the blue curve represents the same as in Fig. 1 (a) except for using the inclusive $|V_{cb}|$.



(a) Exclusive $|V_{cb}|$ (b) Inclusive $|V_{cb}|$

Figure 1: $|\varepsilon_K|$ with (a) exclusive $|V_{cb}|$ (left) and (b) inclusive $|V_{cb}|$ (right) in units of 1.0×10^{-3}.

Our results for $|\varepsilon_K|$ are summarized in Table 5. Here, the superscript $^{\text{SM}}$ means that it is obtained directly from the standard model, the subscript $_{\text{excl}}$ ($_{\text{incl}}$) means that it is obtained using exclusive (inclusive) $|V_{cb}|$, and the superscript $^{\text{Exp}}$ represents the experimental value. Results in Table 5 (a) are obtained using the RBC-UKQCD estimate for ξ_{LD} and those in Table 5 (b) are obtained using the BGI estimate for ξ_{LD}. In Table 5 (a), we find that the theoretical evaluation of $|\varepsilon_K|$ with lattice QCD inputs (with exclusive $|V_{cb}|$) $|\varepsilon_K|^{\text{SM}}_{\text{excl}}$ has 4.2σ tension with the experimental result $|\varepsilon_K|^{\text{Exp}}$, while there is no tension with inclusive $|V_{cb}|$ (heavy quark expansion with QCD sum rules).

parameter	method	value
$\lvert\varepsilon_K\rvert^{\text{SM}}_{\text{excl}}$	exclusive $\lvert V_{cb}\rvert$	1.570 ± 0.156
$\lvert\varepsilon_K\rvert^{\text{SM}}_{\text{incl}}$	inclusive $\lvert V_{cb}\rvert$	2.035 ± 0.178
$\lvert\varepsilon_K\rvert^{\text{Exp}}$	experiment	2.228 ± 0.011

(a) RBC-UKQCD estimate for ξ_{LD}

parameter	method	value
$\lvert\varepsilon_K\rvert^{\text{SM}}_{\text{excl}}$	exclusive $\lvert V_{cb}\rvert$	1.615 ± 0.158
$\lvert\varepsilon_K\rvert^{\text{SM}}_{\text{incl}}$	inclusive $\lvert V_{cb}\rvert$	2.079 ± 0.178
$\lvert\varepsilon_K\rvert^{\text{Exp}}$	experiment	2.228 ± 0.011

(b) BGI estimate for ξ_{LD}

Table 5: $|\varepsilon_K|$ in units of 1.0×10^{-3}.

In Fig. 2 (a), we plot the $\Delta\varepsilon_K \equiv |\varepsilon_K|^{\text{Exp}} - |\varepsilon_K|^{\text{SM}}_{\text{excl}}$ in units of σ (the total error) as a function of time starting from 2012. In 2012, $\Delta\varepsilon_K$ was 2.5σ, but now it is 4.2σ. In Fig. 2 (b), we plot the history of the average $\Delta\varepsilon_K$ and the error $\sigma_{\Delta\varepsilon_K}$. We find that the average has increased with some fluctuations by 27% during the period of 2012-2018, and its error has decreased monotonically by 25% in the same period.

In Table 6 (a), we present the error budget for $|\varepsilon_K|^{\text{SM}}_{\text{excl}}$. Here, we find that the largest error comes from $|V_{cb}|$. Hence, it is essential to reduce the error in $|V_{cb}|$ significantly.

In Table 6 (b), we present how the values of $\Delta\varepsilon_K$ have changed from 2015 [3] to 2018 [1]. Here, we find that the positive shift of $\Delta\varepsilon_K$ is about the same for the inclusive and exclusive $|V_{cb}|$. This reflects the changes in other parameters since 2015.

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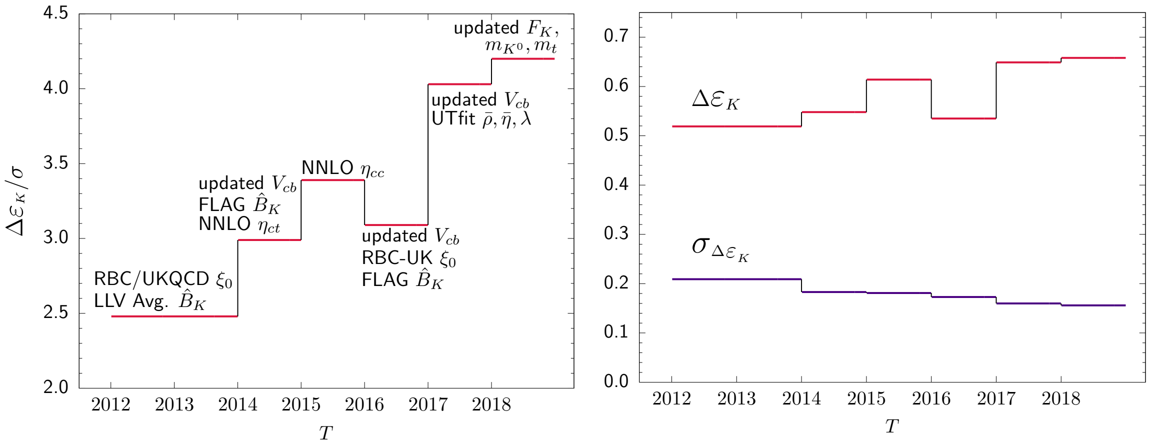

(a) Time evolution of $\Delta\varepsilon_K/\sigma$ (b) Time evolution of the average and error of $\Delta\varepsilon_K$

Figure 2: Time history of (a) $\Delta\varepsilon_K/\sigma$, and (b) $\Delta\varepsilon_K$ and $\sigma_{\Delta\varepsilon_K}$.

source	error (%)	memo
$\lvert V_{cb}\rvert$	31.4	ex-combined
$\bar\eta$	26.8	AOF
η_{ct}	21.5	$c-t$ Box
η_{cc}	9.1	$c-c$ Box
⋮	⋮	⋮

(a) Error budget for $|\varepsilon_K|^{\text{SM}}_{\text{excl}}$

year	Inclusive $\lvert V_{cb}\rvert$	Exclusive $\lvert V_{cb}\rvert$
2015	0.33σ	3.4σ
2018	1.1σ	4.2σ

(b) Results for $\Delta\varepsilon_K$.

Table 6: Error budget for $|\varepsilon_K|^{\text{SM}}_{\text{excl}}$ and history of $\Delta\varepsilon_K$

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