2018 Update on $\varepsilon_K$ with lattice QCD inputs

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SWME Collaboration

We present updated results for $\varepsilon_K$ determined directly from the standard model (SM) with lattice QCD inputs such as $\hat{B}_K$, $|V_{cb}|$, $|V_{us}|$, $\xi_0$, $\xi_2$, $\xi_{LD}$, $F_K$, and $m_c$. We find that the standard model with exclusive $|V_{cb}|$ and other lattice QCD inputs describes only 70% of the experimental value of $|\varepsilon_K|$ and does not explain its remaining 30%, which leads to a strong tension in $|\varepsilon_K|$ at the 4$\sigma$ level between the SM theory and experiment. We also find that this tension disappears when we use the inclusive value of $|V_{cb}|$ obtained using the heavy quark expansion based on QCD sum rules.

The 36th Annual International Symposium on Lattice Field Theory - LATTICE2018
22-28 July, 2018
Michigan State University, East Lansing, Michigan, USA.

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2. Input parameters: $|V_{cb}|$ and $\xi_0$

In Table 1, we present updated results for both exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$. Recently, HFLAV reported them in Ref. [5]. The results for exclusive $|V_{cb}|$ are obtained using lattice QCD results for the semileptonic form factors of Refs. [6, 7, 8]. Here, we use the combined results (ex-combined) for exclusive $|V_{cb}|$ and the results of the 1S scheme for inclusive $|V_{cb}|$ to evaluate $\varepsilon_k$. For more details on $|V_{cb}|$ and the related caveats, refer to Ref. [1].

The absorptive part of long distance effects on $\varepsilon_k$ is parametrized into $\xi_0$.

$$\xi_0 = \frac{\text{Im} A_0}{\text{Re} A_0}, \quad \xi_2 = \frac{\text{Im} A_2}{\text{Re} A_2}, \quad \text{Re} \left( \frac{e'}{e} \right) = \frac{\omega}{\sqrt{2} |\varepsilon_k|} (\xi_2 - \xi_0). \quad (2.1)$$

There are two independent methods to determine $\xi_0$ in lattice QCD: one is the indirect method and the other is the direct method. In the indirect method, one can determine $\xi_0$ using Eq. (2.1) with lattice QCD input $\xi_2$ and with experimental results for $e'/e$, $\varepsilon_k$, and $\omega$. In the direct method, one can determine $\xi_0$ directly using lattice QCD results for $\text{Im} A_0$ combined with experimental results for $\text{Re} A_0$. In Table 2 (a), we summarize results for $\xi_0$ calculated by RBC-UKQCD using the indirect and direct methods. Here, we use the results of the indirect method for $\xi_0$ to evaluate $\varepsilon_k$.

In Ref. [9], RBC-UKQCD also reported the S-wave scattering phase shift for the $I = 0$ channel: $\delta_0 = 23.8(49)(12)$, which is different from those of the dispersion relations [10, 11] by $\approx 3 \sigma$. In Ref. [12], they have accumulated higher statistics to obtain $\delta_0 = 19.1(25)(12)$, which is about $5 \sigma$ different from those of the dispersion analyses. They introduce a $\sigma$ operator and make all possible combinations with the $\sigma$ and $\pi - \pi$ operators. Then, RBC-UKQCD has obtained $\delta_0 = 32.8(12)(30)$.
\[ \xi_K \text{ with lattice QCD inputs} \]

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<table>
<thead>
<tr>
<th>Method</th>
<th>( \xi_0 )</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indirect</td>
<td>(-1.63(19) \times 10^{-4})</td>
<td>[13]</td>
</tr>
<tr>
<td>Direct</td>
<td>(-0.57(49) \times 10^{-4})</td>
<td>[9]</td>
</tr>
</tbody>
</table>

(a) \( \xi_0 \)

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>( \delta_0 )</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBC-UKQC-15</td>
<td>23.8(49)(12)°</td>
<td>[9]</td>
</tr>
<tr>
<td>RBC-UKQC-18</td>
<td>32.8(12)(30)°</td>
<td>[12]</td>
</tr>
<tr>
<td>CGL-2001</td>
<td>39.2(15)°</td>
<td>[10, 14]</td>
</tr>
</tbody>
</table>

(b) \( \delta_0 \)

Table 2: The absorptive long distance effect \( \xi_0 \) and S-wave \( I = 0 \) scattering phase shift \( \delta_0 \).

<table>
<thead>
<tr>
<th>WP</th>
<th>CKMfitter</th>
<th>UTfit</th>
<th>AOF</th>
<th>Input Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>0.22509(29)</td>
<td>0.22497(69)</td>
<td>0.2248(6)</td>
<td>( \eta_{\ell} )</td>
<td>1.72(27)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.1598(76)</td>
<td>0.153(13)</td>
<td>0.146(22)</td>
<td>( \eta_{tt} )</td>
<td>0.5765(65)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.3499(63)</td>
<td>0.343(11)</td>
<td>0.333(16)</td>
<td>( \eta_{ct} )</td>
<td>0.496(47)</td>
</tr>
</tbody>
</table>

(a) Wolfenstein parameters

Table 3: (a) Wolfenstein parameters and (b) QCD corrections: \( \eta_{ij} \) with \( i, j = c, t \).

which is consistent with those of the dispersion relations. These results are presented in Table 2 (b) and Figure 2 (c).

3. Input parameters: Wolfenstein parameters, \( \bar{B}_K, \xi_{LD}, \) and others

In Table 3 (a), we summarize the Wolfenstein parameters on the market. The CKMfitter and UTfit collaboration provide the Wolfenstein parameters determined by the global unitarity triangle (UT) fit. Unfortunately, \( \xi_K, \bar{B}_K, \) and \( |V_{cb}| \) are used as inputs to the global UT fit, which leads to unwanted correlation with \( \xi_K \). We want to avoid this correlation, and so take another input set from the angle-only fit (AOF) suggested in Ref. [15]. The AOF does not use \( \xi_K, \bar{B}_K, \) and \( |V_{cb}| \) as input to determine the UT apex \( (\rho, \eta) \). Here the \( \lambda \) parameter is determined from \( |V_{us}| \) which is obtained from the \( K_{2} \) and \( K_{3} \) decays using lattice QCD results for the form factors and decay constants. The \( A \) parameter is determined from \( |V_{cb}| \).

In the FLAG review [22], they present lattice QCD results for \( \bar{B}_K \) with \( N_f = 2, N_f = 2 + 1, \) and \( N_f = 2 + 1 + 1 \). Here, we use the results for \( \bar{B}_K \) with \( N_f = 2 + 1 \), which is obtained by taking a global average over the four data points from BMW 11 [23], Laiho 11 [24], RBC-UKQCD 14 [25], and SWME 15 [26]. In Table 4 (a), we present the FLAG 17 result for \( \bar{B}_K \) with \( N_f = 2 + 1 \), which is used to evaluate \( \xi_K \).

The dispersive long distance (LD) effect is defined as

\[
\xi_{LD} = \frac{m_{LD}'}{\sqrt{2} \Delta M_K}, \quad m_{LD}' = -\text{Im} \left[ \langle K^0 | H_{\omega} | C \rangle \langle C | H_{\omega} | K^0 \rangle \right] m_{K^0} - E_C
\] (3.1)
**ξ with lattice QCD inputs**

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<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>( \hat{B}_K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWME 15</td>
<td>[26]</td>
<td>0.735(5)(36)</td>
</tr>
<tr>
<td>RBC/UKQCD 14</td>
<td>[25]</td>
<td>0.7499(24)(150)</td>
</tr>
<tr>
<td>Laiho 11</td>
<td>[24]</td>
<td>0.7628(38)(205)</td>
</tr>
<tr>
<td>BMW 11</td>
<td>[23]</td>
<td>0.7727(81)(84)</td>
</tr>
<tr>
<td>FLAG 17</td>
<td>[22]</td>
<td>0.7625(97)</td>
</tr>
</tbody>
</table>

(a) \( \hat{B}_K \)

Table 4: (a) Results for \( \hat{B}_K \) and (b) other input parameters.

If the CPT invariance is well respected, the overall contribution of the \( \xi_{LD} \) to \( \epsilon_K \) is about ±2%.

Lattice QCD tools to calculate \( \xi_{LD} \) are well established in Refs. [28, 29, 30]. In addition, there have been a number of attempts to calculate \( \xi_{LD} \) on the lattice [31, 32]. In them, RBC-UKQCD used a pion mass of 329 MeV and a kaon mass of 591 MeV, and so the energy of the 2 pion and 3 pion states are heavier than the kaon mass. Hence, the sign of the denominator in Eq. 3.1 is opposite to that of the physical contribution. Therefore, this work belongs to the category of exploratory study rather than to that of precision measurement.

In Ref. [33], they use chiral perturbation theory to estimate the size of \( \xi_{LD} \) and claim that

\[
\xi_{LD} = -0.4(3) \times \frac{\xi_0}{\sqrt{2}} \tag{3.2}
\]

where we use the indirect results for \( \xi_0 \) and its error. Here, we call this method the BGI estimate for \( \xi_{LD} \). In Refs. [28, 34], RBC-UKQCD provides another estimate for \( \xi_{LD} \):

\[
\xi_{LD} = (0 \pm 1.6)\%	ag{3.3}
\]

Here, we call this method the RBC-UKQCD estimate for \( \xi_{LD} \).

In Table 3 (b), we present higher order QCD corrections: \( \eta_{ij} \) with \( i, j = t, c \). In Table 4 (b), we present other input parameters needed to evaluate \( \epsilon_K \). Since Lattice 2017, three parameters: \( m_t(m_t), m_K, F_K \) have been updated. The \( m_t(m_t) \) parameter is the scale-invariant (SI) top quark mass renormalized in the MS scheme. The pole mass of top quarks comes from Ref. [18]: \( M_t = 173.5 \pm 1.1 \) GeV. We convert the top quark pole mass into the SI top quark mass using the four-loop perturbation formula. For more details, refer to Ref. [1].

4. Results for \( \epsilon_K \)

In Fig. 1, we present results for \( |\epsilon_K| \) evaluated directly from the standard model (SM) with lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical evaluation of \( |\epsilon_K| \) using the FLAG-2017 \( \hat{B}_K \), AOF for Wolfenstein parameters, and exclusive \( |V_{cb}| \), and the RBC-UKQCD estimate for \( \xi_{LD} \). The red curve in Fig. 1 represents the experimental value of \( |\epsilon_K| \). In Fig. 1 (b), the blue curve represents the same as in Fig. 1 (a) except for using the inclusive \( |V_{cb}| \).
Our results for $|\epsilon_K|$ are summarized in Table 5. Here, the superscript $^\text{SM}$ means that it is obtained directly from the standard model, the subscript $\text{excl}$ ($\text{incl}$) means that it is obtained using exclusive (inclusive) $|V_{cb}|$, and the superscript $^\text{Exp}$ represents the experimental value. Results in Table 5 (a) are obtained using the RBC-UKQCD estimate for $\xi_{\text{LD}}$ and those in Table 5 (b) are obtained using the BGI estimate for $\xi_{\text{LD}}$. In Table 5 (a), we find that the theoretical evaluation of $|\epsilon_K|$ with lattice QCD inputs (with exclusive $|V_{cb}|$) $|\epsilon_K|^{\text{SM}}_{\text{excl}}$ has 4.2σ tension with the experimental result $|\epsilon_K|^{\text{Exp}}$, while there is no tension with inclusive $|V_{cb}|$ (heavy quark expansion with QCD sum rules).

<table>
<thead>
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<th>parameter</th>
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<th>value</th>
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<tr>
<td>$</td>
<td>\epsilon_K</td>
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<tr>
<td>$</td>
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<td>^{\text{SM}}_{\text{incl}}$</td>
</tr>
<tr>
<td>$</td>
<td>\epsilon_K</td>
<td>^{\text{Exp}}$</td>
</tr>
</tbody>
</table>

Table 5: $|\epsilon_K|$ in units of $1.0 \times 10^{-3}$.

In Fig. 2 (a), we plot the $\Delta \epsilon_K \equiv |\epsilon_K|^{\text{Exp}} - |\epsilon_K|^{\text{SM}}_{\text{excl}}$ in units of σ (the total error) as a function of time starting from 2012. In 2012, $\Delta \epsilon_K$ was 2.5σ, but now it is 4.2σ. In Fig. 2 (b), we plot the history of the average $\Delta \epsilon_K$ and the error $\sigma_{\Delta \epsilon_K}$. We find that the average has increased with some fluctuations by 27% during the period of 2012-2018, and its error has decreased monotonically by 25% in the same period.

In Table 6 (a), we present the error budget for $|\epsilon_K|_{\text{excl}}^{\text{SM}}$. Here, we find that the largest error comes from $|V_{cb}|$. Hence, it is essential to reduce the error in $|V_{cb}|$ significantly.

In Table 6 (b), we present how the values of $\Delta \epsilon_K$ have changed from 2015 [3] to 2018 [1]. Here, we find that the positive shift of $\Delta \epsilon_K$ is about the same for the inclusive and exclusive $|V_{cb}|$. This reflects the changes in other parameters since 2015.

Acknowledgments

We thank Shoji Hashimoto and Takashi Kaneko for helpful discussion on $|V_{cb}|$. The research...
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![Figure 2: Time history of (a) \( \Delta \varepsilon_K / \sigma \), and (b) \( \Delta \varepsilon_K \) and \( \sigma \Delta \varepsilon_K \).](image)

(a) Time evolution of \( \Delta \varepsilon_K / \sigma \)

(b) Time evolution of the average and error of \( \Delta \varepsilon_K \)

<table>
<thead>
<tr>
<th>source</th>
<th>error (%)</th>
<th>memo</th>
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<tbody>
<tr>
<td>(</td>
<td>V_{cb}</td>
<td>)</td>
</tr>
<tr>
<td>( \bar{\eta} )</td>
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<td>AOF</td>
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<td>( \eta_{ct} )</td>
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<td>( c-t ) Box</td>
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<td>( \eta_{cc} )</td>
<td>9.1</td>
<td>( c-c ) Box</td>
</tr>
<tr>
<td>...</td>
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</tr>
</tbody>
</table>

(a) Error budget for \( |\varepsilon_K|_{\text{SM excl}} \)

| year | Inclusive \( |V_{cb}| \) | Exclusive \( |V_{cb}| \) |
|------|-----------------|-----------------|
| 2015 | 0.33\( \sigma \) | 3.4\( \sigma \) |
| 2018 | 1.1\( \sigma \)  | 4.2\( \sigma \) |

(b) Results for \( \Delta \varepsilon_K \).

Table 6: Error budget for \( |\varepsilon_K|_{\text{SM excl}} \) and history of \( \Delta \varepsilon_K \)

of W. Lee is supported by the Creative Research Initiatives Program (No. 2017013332) of the NRF grant funded by the Korean government (MEST). J.A.B. is supported by the Basic Science Research Program of the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. 2015024974). W. Lee would like to acknowledge the support from the KISTI supercomputing center through the strategic support program for the supercomputing application research (No. KSC-2016-C3-0072). Computations were carried out on the DAVID GPU clusters at Seoul National University.

References

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