Semi-leptonic form factors for $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$

Jonathan M. Flynn

Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK

Ryan C. Hill

Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK
DISCnet Centre for Doctoral Training, University of Southampton, Southampton SO17 1BJ, UK

Andreas Jüttner

Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, UK

Amarjit Soni

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Justus Tobias Tsang

Higgs Centre for Theoretical Physics, The University of Edinburgh, EH9 3FD, UK

Oliver Witzel[*]

Department of Physics, University of Colorado Boulder, Boulder, CO, USA
E-mail: Oliver.Witzel@colorado.edu

Semi-leptonic $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ decays provide an alternative *b*-decay channel to determine the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ or to obtain *R*-ratios to investigate lepton flavor universality violations. In addition, these decays may shed further light on the discrepancies seen in the analysis of inclusive vs. exclusive decays. Using the nonperturbative methods of lattice QCD, theoretical results are obtained with good precision and full control over systematic uncertainties. This talk will highlight ongoing efforts of the *B*-physics program by the RBC-UKQCD collaboration.

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[*]Speaker.

1. Introduction

Since the discovery of the Higgs boson in 2012, no other new elementary particles have been discovered nor have direct signs of new physics been detected at the Large Hadron Collider (LHC). Hence testing the Standard Model (SM) at high precision is increasingly important, with flavor physics (electro-weak interactions changing quark flavor) of special relevance. At tree-level, the SM allows up-type quarks to decay to down-type quarks (and vice versa) by emitting a charged $W^\pm$ boson, but flavor changing neutral currents (FCNC) are suppressed and occur only at loop level. High precision calculations of processes allowed in the SM are needed to test for differences to experimental observations. Deviations could signal new physics, arising for example from virtual particles in loops.

New physics is expected to occur at higher energy scales and observing its effects is more likely if the decaying particle can release large amounts of energy. Decays of mesons containing a heavy *b*-quark provide many opportunities because the b-quark lives long enough for experimental investigation but also delivers more than 4 GeV energy. The large *b*-mass also allows a plethora of decay channels and correspondingly many tests of the SM. $B_{(s)}$-meson decays could allow indirect observation of new physics from virtual particles which would otherwise show up at currently inaccessible energy scales. It is important to perform tests for both tree-level as well as loop-level processes. For both cases tantalizing deviations between SM predictions and experimental measurements have been reported see e.g. [1, 2] and references within. Particularly striking are ratios investigating the universality of lepton flavors in semi-leptonic *B* decays, e.g., for *B* mesons decaying to a $D^{(*)}$ meson with either $\tau\nu_\tau$ or $\mu\nu_\mu$ leptons in the final state,

$$R_{D^{(*)}}^{\tau/\mu} \equiv \frac{BF(B \to D^{(*)}\tau\nu_\tau)}{BF(B \to D^{(*)}\mu\nu_\mu)}. \tag{1.1}$$

Currently the combined analysis for pseudoscalar and vector hadronic final states yields a tension of more than 3σ between SM prediction and the experimental values obtained from measurements by BaBar, Belle, and LHCb [3–8].

Our project for nonperturbative calculations of semileptonic $B_{(s)}$ decays includes operators for tree- and loop-level processes with one pseudoscalar or one vector hadronic final state [9], but here we focus on tree-level B_s-meson decays with a kaon or D_s meson in the final state. Experimentally many B_s decays are observed by LHCb and we intend to support their program by pursuing these calculations. The diagram corresponding to $B_s \to K\ell\nu$ decays is sketched in Fig. 1; for $B_s \to D_s\ell\nu$ decays kaons and D_s-mesons are simply exchanged and the $\bar{u}$ daughter quark becomes $\bar{c}$. Conventionally, the branching fraction measured experimentally is parameterized in the B_s-meson rest frame by

$$\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_K^2-M_K^2}}{q^4 M_{B_s}^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)M_{B_s}^2(E_K^2-M_K^2)|f_+(q^2)|^2\right.$$
$$\left. + \frac{3m_\ell^2}{8q^2}(M_{B_s}^2-M_K^2)^2|f_0(q^2)|^2\right]. \tag{1.2}$$

$$q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K$$
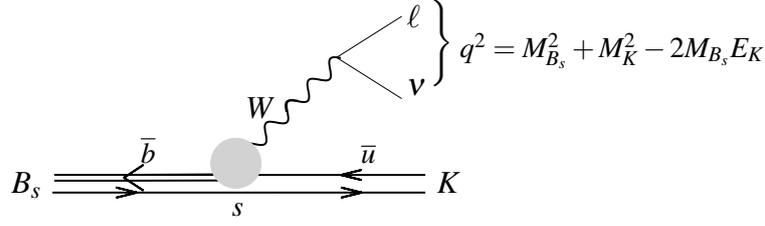
Figure 1: Sketch of tree-level weak semi-leptonic B_s decays mediated by a charged $W^{\pm}$ boson and a set-up with the B_s meson at rest.

	L	a^{-1}(GeV)	am_l	am_s	M_π(MeV)	# configs.	#sources
C1	24	1.784	0.005	0.040	338	1636	1
C2	24	1.784	0.010	0.040	434	1419	1
M1	32	2.383	0.004	0.030	301	628	2
M2	32	2.383	0.006	0.030	362	889	2
M3	32	2.383	0.008	0.030	411	544	2
F1	48	2.774	0.002144	0.02144	234	98	24

Table 1: Dynamical 2+1 flavor domain-wall fermion ensembles [10–13] used in this calculation. The lattice spacing is determined in combined analysis [12, 13] and the quoted values correspond to $a \sim 0.11$ fm, ~ 0.08 fm, ~ 0.07 fm.

The nonperturbative contributions are given by the form factors f_+ and f_0 which are related to the matrix element

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_{B_s}^2 - M_K^2}{q^2}q^\mu. \tag{1.3}$$

The weak decay is dominated by short distance contributions and hence we can consider the weak operator as a point-like object and implement the calculation using conventional lattice QCD techniques. In the following we report updates on our efforts to determine the form factors for $B_s \rightarrow K\ell\nu$ and $B_s \rightarrow D_s\ell\nu$ decays. Our calculations are based on a subset of RBC-UKQCD's 2+1 flavor domain wall fermion and Iwasaki gauge field ensembles [10–13] which we summarize in Tab. 1. Light and strange quarks are simulated using domain wall fermions [14–17], charm quarks are simulated by applying the Möbius domain wall action to heavy quarks [18], and bottom quarks are simulated using the relativistic heavy quark (RHQ) action [19, 20], a variant of the Fermilab action [21] with nonperturbatively tuned parameters [22]. Further details of the set-up and our project to compute bottom and charm physics can be found in Refs. [13, 23–26]. Here we focus on updates of our form factor calculations reporting in Section 2 on $B_s \rightarrow K\ell\nu$ decays and in Section 3 on $B_s \rightarrow D_s\ell\nu$, before summarizing in Section 4.

2. Form factors for semi-leptonic $B_s \rightarrow K\ell\nu$ decays

In order to extend our original work [24] and include for example the new ensemble F_1 at a third, finer lattice spacing, we first repeated our non-perturbative tuning of the RHQ parameters
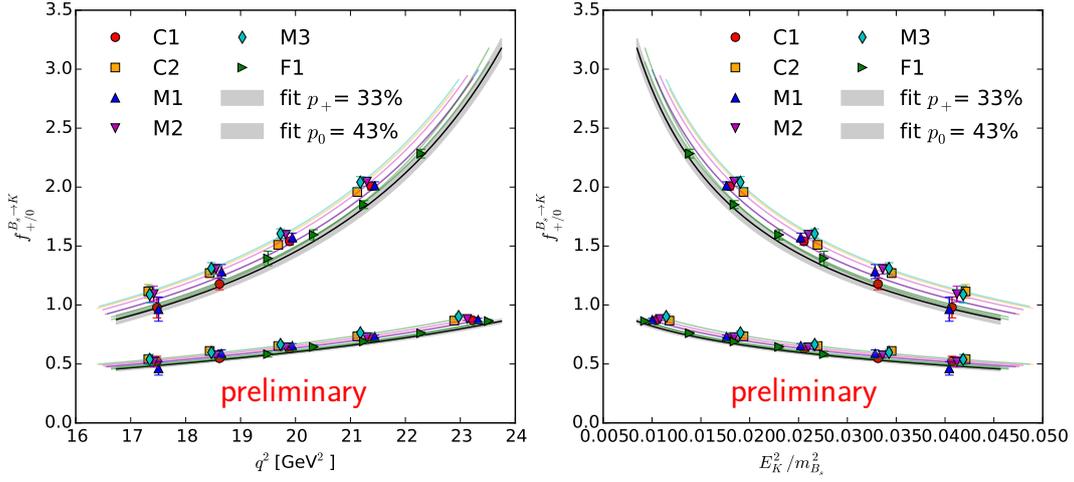
Figure 2: Chiral-continuum extrapolation of semi-leptonic form factors for $B_s \to K\ell\nu$ decays using HMχPT. Only statistical errors are shown. The plot on the left presents the form factors in units of q^2, while the plot on the right uses the kaon energy squared normalized by the B_s-meson mass.

[22] to reflect updated values of the lattice spacing and the physical mass of the strange quark [12]. Using the newly tuned RHQ parameters, we simulate physical b-quarks and use close-to physical values for the strange quark, whereas the mass of light quark is set to the unitary light quark mass on each ensemble. We calculate 3-point and 2-point functions to extract the form factors on each ensemble using discrete spatial lattice momenta up to $p^2 = 4(2\pi/L)^2$. This results in the set of colored data points for f_+ and f_0 shown in Fig. 2. Using an ansatz based on heavy meson chiral perturbation theory (HMχPT) [27, 28], we obtain a functional form to describe our data

$$f_{\text{pole}}(M_K, E_K, a^2) = \frac{1}{E_K + \Delta} c^{(1)} \cdot \left[1 + \frac{\delta f}{(4\pi f)^2} + c^{(2)} \frac{M_K^2}{\Lambda^2} + c^{(3)} \frac{E_K}{\Lambda} + c^{(4)} \frac{E_K^2}{\Lambda^2} + c^{(5)} \frac{a^2}{\Lambda^2 a_{32}^4} \right], \quad (2.1)$$

where δf are non-analytic logarithms of the kaon mass and the hard-kaon limit is taken by $M_K/E_K \to 0$. Next we perform a global fit to all data points for f_+ (f_0) to obtain form factors in the chiral-continuum limit. These fits have excellent p-values of 33% for f_+ (43% for f_0) and the outcome is shown by the black central line with gray error band in Fig. 2.

We aim for a continuum description of the form factors with a full statistical and systematic error budget. The latter is still work in progress. Thus uncertainties presented in the following are neither final nor complete. Using however our continuum limit result with a preliminary accounting of systematic effects, we can carry out an extrapolation over the entire range of allowed q^2 values. We do so by implementing a so called z expansion i.e. we map q^2 to the variable z using

$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}} \quad (2.2)$$

with $t_{\pm} = (M_{B_s} \pm M_K)^2$ and $t_0 \equiv t_{\text{opt}} = (M_{B_s} + M_K)(\sqrt{M_{B_s}} - \sqrt{M_K})^2$. The outcome of this kinematical extrapolation is presented in Fig. 3 where we use the parametrization by Bourrely, Caprini,
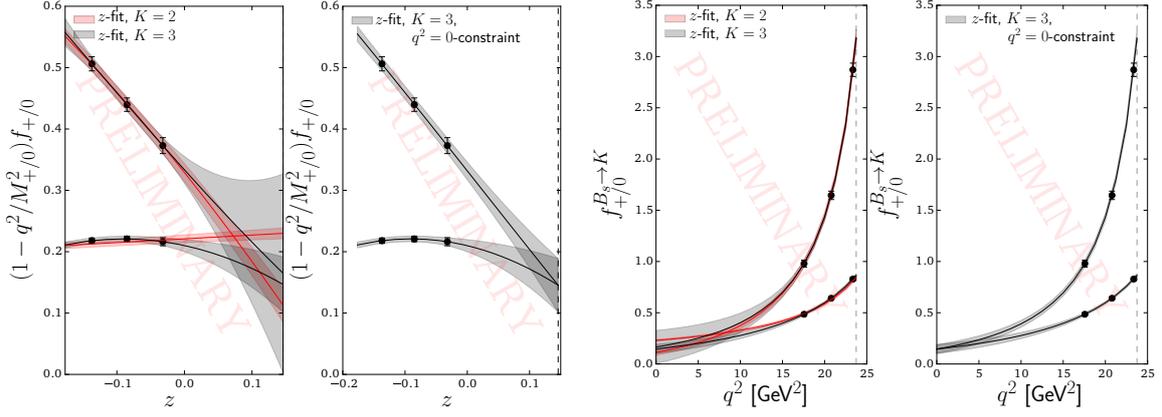
Figure 3: Kinematical extrapolation of the $B_s \to K\ell\nu$ form factors using the BCL z expansion over the full q^2 range for $K = 2$ or 3. On the left, the result are shown in units of the z extrapolation, on the right in physical GeV^2.

and Lellouch (BCL) [29]

$$
f_+(q^2) = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{k=0}^{K-1} b_+^{(k)} \left[z^k - \frac{k}{K}(-1)^{k-K} z^K \right]; \qquad f_0(q^2) \;\; = \frac{1}{1 - q^2/M_{B^*(0^+)}^2} \sum_{k=0}^{K-1} b_0^{(k)} z^k.
\tag{2.3}
$$

We show extrapolations of our results using $K = 2$ and 3 and in addition also implement the constraint $f_0(0) = f_+(0)$.

3. Form factors for $B_s \to D_s\ell\nu$ decays

Our determination of $B_s \to D_s\ell\nu$ form factors follows steps analogous to those for $B_s \to K\ell\nu$ replacing the light daughter quark with a charm quark. We also choose to maintain the same parametrization in terms of f_+ and f_0. However, the light quark mass only contributes to the sea-sector resulting in a mild (or flat) chiral extrapolation but, in addition, we need to perform an extra- or interpolation in the charm quark mass to obtain form factors for physical D_s mesons [13]. This step is necessary because on the coarse ensembles we cannot directly simulate a physical charm quark mass with our choice of heavy domain wall action and on the medium and fine ensembles we choose to bracket the physical value of the charm quark mass. In Fig. 4 we show the form factors f_+ and f_0 obtained from our simulated charm quark masses i.e. three charm-like masses are used to guide an extrapolation on the coarse ensembles (red symbols), while two charm-like masses enable an interpolation on the medium (blue symbols) and the fine (green symbols) ensembles. Again we obtain the result at physical quark masses performing a global fit, which for $B_s \to D_s\ell\nu$ is based on the ansatz

$$
f(q^2, a, M_\pi, M_{D_s}) = \frac{\alpha_0 + \alpha_1 M_{D_s} + \alpha_2 a^2 + \alpha_3 M_\pi^2}{1 + \alpha_4 q^2/M_{B_s}^2},
\tag{3.1}
$$

to account for a dependence on the charm-quark mass, the lattice spacing, and the (sea) pion mass. The obtained continuum limit is shown by the gray error band. As for $B_s \to K\ell\nu$, we are currently
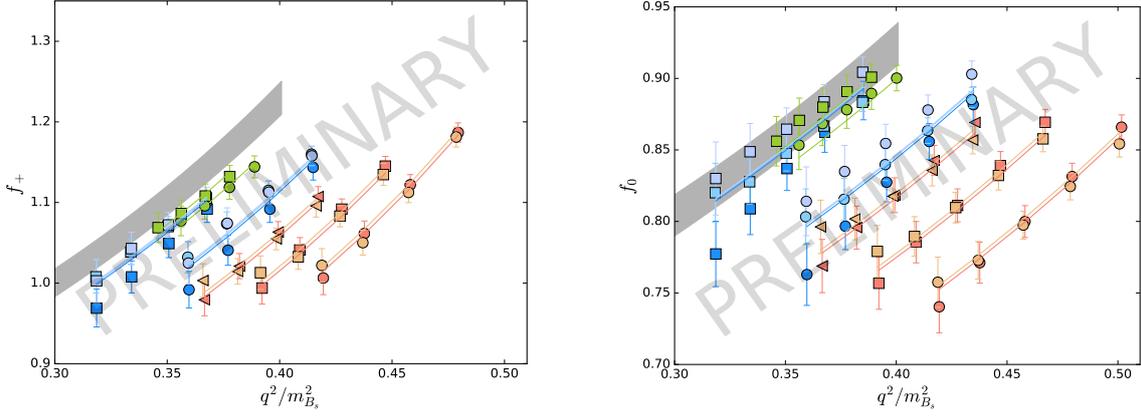
Figure 4: Semi-leptonic form factors $B_s \to D_s\ell\nu$ decays. The colored data points show our simulated data using charm-like masses to extra-/interpolate to the physical charm quark mass. Carrying out a global fit based on Eq. (3.1) we obtain the continuum limit (gray band) at physical quark masses.
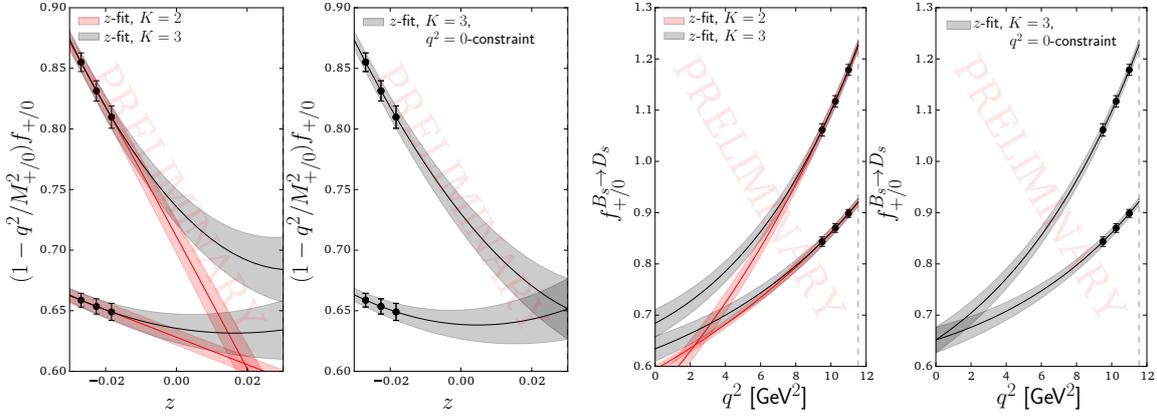
Figure 5: Kinematical extrapolation of the $B_s \to D_s\ell\nu$ form factors using the BCL z expansion over the full q^2 range. On the left, the result are shown in units of the z extrapolation, on the right in physical GeV^2.

in the process of accounting for all systematic uncertainties and hence do not have a final and complete error budget, yet. Nevertheless we can proceed and perform a kinematical z expansion using the BCL parametrization and show the current status using $K = 2$ or 3 in Fig. 5.

4. Summary

We reported updates on our calculation of semi-leptonic form factors for $B_s \to K\ell\nu$ and $B_s \to D_s\ell\nu$ decays. We are currently finalizing our error budgets. Our results will provide entirely independent determinations of the form factors so far also calculated by Atoui et al., HPQCD, and Fermilab/MILC [30–35]. In addition our results will allow to extract R ratios for B_s decays which may serve as proxy for corresponding B decays and in addition allow for the determination of the ratio of CKM matrix elements $|V_{cb}/V_{ub}|$ in combination with an experimental measurement. Such an independent determination may also help to resolve present discrepancies betweeen inclusive and exclusive determinations.

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