

B-Physics Anomalies: Footprints in Cosmology?

Diego Guadagnoli^{*†}

*Laboratoire d'Annecy-le-Vieux de Physique Théorique UMR5108,
CNRS et Université de Savoie Mont-Blanc,
B.P. 110, Annecy-le-Vieux, F-74941 Annecy Cedex, France
E-mail: guadagno@lapth.cnrs.fr*

A whole body of B -meson decays display persistent deviations with respect to the Standard-Model (SM) predictions. These deviations concern coherent sets of data, *all of them with two leptons* in the final state. Deviations are in the fact that decays to different leptons appear to depart or not from the SM predictions depending on the considered lepton. This can be explained by some *new* interaction that distinguishes between the lepton species, i.e. one that violates Lepton Universality (LU). Here we explore the question whether models for such beyond-SM effect may leave footprints in the cosmos, e.g. in observables related to Dark Matter or to neutrino fluxes.

*2nd World Summit: Exploring the Dark Side of the Universe
25-29 June, 2018
University of Antilles, Pointe-à-Pitre, Guadeloupe, France*

^{*}Speaker.

[†]

1. Introduction

A whole range of $b \rightarrow s$ data involving a $\mu^+ \mu^-$ pair display a consistent pattern, with experimental data below the respective Standard-Model (SM) prediction, for di-lepton invariant masses below the charmonium threshold. This is true for the $B^0 \rightarrow K^0 \mu^+ \mu^-$, the $B^+ \rightarrow K^+ \mu^+ \mu^-$ and the $B^+ \rightarrow K^{*+} \mu^+ \mu^-$ decays [1], for the $B_s^0 \rightarrow \phi \mu^+ \mu^-$ decay [2] and, very recently, even in hyperon channels for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay [3, 4]. With these data alone, however, it is presently impossible to establish beyond-SM effects, as branching-ratio measurements suffer in general from sizable theoretical uncertainties due to hadronic form factors. On the other hand, such problem is basically absent if one considers suitable ratios of branching ratios to different lepton channels. Dedicated measurements exist on such ratios, and actually constitute the most alluring set of anomalies. In the $b \rightarrow s$ case these measurements are [5, 6]

$$\begin{aligned} R_K([1, 6] \text{GeV}^2) &\equiv \frac{\mathcal{B}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\mathcal{B}(B^+ \rightarrow K^+ e^+ e^-)} \Big|_{q^2 \in [1, 6] \text{GeV}^2} = 0.745_{-0.074}^{+0.090} (\text{stat}) \pm 0.036 (\text{syst}) , \\ R_{K^*0}([0.045, 1.1] \text{GeV}^2) &= 0.660_{-0.070}^{+0.110} \pm 0.024 , \\ R_{K^*0}([1.1, 6] \text{GeV}^2) &= 0.685_{-0.069}^{+0.113} \pm 0.047 , \end{aligned} \quad (1.1)$$

where we have omitted the definition of R_{K^*0} , analogous to the R_K one, and where q^2 denotes the invariant mass squared of the di-lepton pair. All of the above measurements are predicted to be unity (first and third of them) and respectively close to it (second one) within the SM, with a few-percent accuracy [7, 8, 9, 10]. Therefore, the R_K and R_{K^*0} measurements each imply a discrepancy between 2 and 2.6σ [5, 6], at face value signalling lepton-universality violation (LUV) beyond the SM. While the electron-channel measurement would be an obvious culprit for the discrepancies, because of bremsstrahlung and lower statistics, disagreement is rather in the muon channel, see [1, 11, 12]. The fact that muons are among the most reliable objects within LHCb would somewhat disfavour systematic effects as an explanation, although of course it cannot be excluded. Importantly, the emerging picture can be established from ratios alone, but it is supported by the other measurements mentioned above, whose theory error is more debated [13].

Equally interesting results come from measurements of the ratios $R(D^{(*)}) \equiv \mathcal{B}(B \rightarrow D^{(*)} \tau \nu) / \mathcal{B}(B \rightarrow D^{(*)} \ell \nu)$ [14, 15, 16, 17]. A simultaneous fit to all these $R(D)$ and $R(D^*)$ measurements yields a discrepancy with respect to the SM predictions with a significance of about 4σ [18], comparable to the global significance of $b \rightarrow s$ anomalies. Note also that $b \rightarrow c$ anomalies come jointly from several experiments: B factories and LHCb.

2. Theory considerations

A few basic considerations suggest to take both $b \rightarrow s$ and $b \rightarrow c$ measurements on an equal footing. First, both sets hint at dynamics that distinguishes between the different species of leptons, i.e. beyond-SM Lepton-Universality Violation (LUV). Besides, either of $b \rightarrow s$ and $b \rightarrow c$ data significances are around 4σ [13], as mentioned. Finally, these two sets of measurements correspond to two sets of observables ($b \rightarrow s$ and $b \rightarrow c$) related by the SM $SU(2)_L$ symmetry [19, 20].

$b \rightarrow s$ transitions: why interesting – There is a crucial difference between $b \rightarrow s$ and $b \rightarrow c$ transitions, that poses a major model-building obstacle, namely the fact that the $b \rightarrow s$ current is

expected to be way more sensitive to new effects than the $b \rightarrow c$ current, because the latter arises in the SM already at tree level. Conversely, the $b \rightarrow s$ current is a flavour-changing neutral current (FCNC), and by construction it has two built-in suppression mechanisms: a loop suppression, and what is known as ‘GIM’ suppression [21]. In the loop giving rise to the $b \rightarrow s$ amplitude, the contribution from each up-type quark q_u goes as $(V_{\text{CKM}}^\dagger)_{bq_u} \cdot (V_{\text{CKM}})_{q_us} \cdot f(m_{q_u})$, with V_{CKM} the quark-mixing, or Cabibbo-Kobayashi-Maskawa matrix [22, 23], and $f(m_{q_u})$ a function of the up-type quark masses m_{q_u} running in the loop. So, if these 3 masses were equal, the corresponding contributions would sum up to zero (GIM mechanism). In practice, in the $b \rightarrow s$ processes of interest to us the short-distance part is dominated by the top loop. The large top mass, $m_t^2/m_W^2 = O(1)$, implies a ‘hard’ (i.e. powerlike) GIM breaking. This comes with two consequences: first, one can shrink the loop dynamics to a point, and describe the decay as an effective interaction of the kind $\mathcal{H} = \sum_i C_i/\Lambda^2 (\bar{b}\Gamma_q^{(i)}s) (\bar{\ell}\Gamma_\ell^{(i)}\ell)$, with $\Gamma_{q,\ell}^{(i)}$ strings of Dirac matrices. Second, among the measurable FCNCs, $b \rightarrow s$ transitions are the closest to third-generation physics. If the proximity of m_t to the EW scale is not an accident, then the top is possibly a portal to new states. In this case, $b \rightarrow s$ decays would provide a convenient indirect probe of any such physics.

EFT understanding – An Effective-Field-Theory (EFT) interpretation of the flavour discrepancies is regarded as the ‘level-0’ understanding. In its absence, one would have to suppose that the new physics is very light. Let us consider the following Hamiltonian, which is part of the full $\bar{b} \rightarrow \bar{s}\ell\ell$ one

$$\mathcal{H}_{\text{SM+NP}}(\bar{b} \rightarrow \bar{s}\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \frac{\alpha_{em}(m_b)}{4\pi} \times \left[\bar{b}_L \gamma^\lambda s_L \bar{\ell} \left(C_9^{(\ell)} \gamma_\lambda + C_{10}^{(\ell)} \gamma_\lambda \gamma_5 \right) \ell \right] + \text{H.c.}, \quad (2.1)$$

where the index (ℓ) on the Wilson coefficients $C_{9,10}$ denotes that the corresponding new-physics shift distinguishes between lepton flavours, whereas the SM contribution does not. The SM contributions are such that $C_9 \simeq -C_{10}$ at the m_b scale, yielding (accidentally) an approximate $(V - A) \times (V - A)$ structure. Advocating likewise $C_{9,\text{NP}}^{(\mu)} = -C_{10,\text{NP}}^{(\mu)}$ for the new-physics shifts (note, in the μ -channel only) turns out to account at one stroke for all $b \rightarrow s$ discrepancies [24, 25]. Further global fits by different groups consistently show that the by far most favourite solutions are either a negative new-physics (NP) contribution to C_9 , with $C_{9,\text{NP}}^{(\mu)} \sim -30\% C_{9,\text{SM}}$, or NP in the mentioned $SU(2)_L$ -invariant direction $C_{9,\text{NP}}^{(\mu)} = -C_{10,\text{NP}}^{(\mu)} \simeq -12\% |C_{9,\text{SM}}|$ [13]. Note that such a solution is approximately RGE-stable.

The latter solution is especially interesting from a UV point of view, because it amounts to a $(V - A)_{\text{quark}} \times (V - A)_{\text{lepton}}$ operator, that can in turn be promoted to an $SU(2)_L$ -invariant, which is what one would expect of interactions arising above the EWSB scale [19, 20]. Let us then focus on this solution: $C_9^{(\ell)} \approx -C_{10}^{(\ell)}$ and $|C_{9,\text{NP}}^{(\mu)}| \gg |C_{9,\text{NP}}^{(e)}|$. Such a pattern, with effects much larger for muons than for electrons, can be generated from a purely third-generation interaction [26]. Such interaction is expected, e.g., in partial-compositeness frameworks [27].

Model-building – Up to now we have restricted ourselves to EFT considerations. Needless to say, a really satisfactory theory understanding requires that the previously identified EFT shifts be generated through some new UV dynamics. Models in this respect exist in profusion, and typically fall into two main categories: extensions invoking one or more new vector bosons, or else ‘leptoquarks’. It is impossible to represent all this work in this limited space. UV-complete models able to address both $b \rightarrow s$ and $b \rightarrow c$ anomalies are in Refs. [28, 29, 30, 31, 32, 33, 34, 35].

Models for the anomalies have to face various challenges. The first one is the mentioned fact that, in the SM, $B \rightarrow D^{(*)} \tau \nu$ and $B \rightarrow K^{(*)} \ell \ell$ decays arise respectively at tree and loop level, whereas the NP corrections hinted at by data are in either case of $O(15-25\%)$. This issue is relevant if we seek a common explanation of $b \rightarrow c \tau \nu$ and $b \rightarrow s \ell \ell$ discrepancies. A second obstacle is inherent in the fact that the needed NP is of the kind $J_q \times J_\ell$, i.e. the product of a quark and a lepton current. In most UV setups, such operators are typically accompanied by $J_q \times J_q$ and $J_\ell \times J_\ell$ structures, that are severely constrained by data, respectively from B_s -mixing observables, and from purely leptonic LFV or LUV decays. Finally, a third obstacle emerges from the observation that most model-building attempts advocate new charged, and possibly colored, states, with masses not larger than $O(\text{tens of TeV})$ and with significant couplings to 3rd-generation SM fermions. These conditions make constraints from direct searches, in particular of resonances decaying to $\tau\tau$ pairs, especially relevant, see [36, 37].

3. Footprints in the cosmos?

An interesting question, constituting the core of the present contribution, is whether flavour anomalies may possibly leave footprints in some cosmo/astro observables. This is a difficult question, that requires a disclaimer to start with. In order to establish robust connections between flavour and cosmo/astro observables, we arguably first need a proper theory understanding of the anomalies, i.e. a widely accepted model with beyond-SM d.o.f. that account for the flavour anomalies, and are possibly suited to well-defined connections with cosmo/astro data. We are not there yet, because the picture is established only at the Effective-Theory level, where additional d.o.f. are hidden by definition. In these circumstances, we may still ask ourselves: (a) whether general, *model-independent* relations can be established between the observed anomalies and any existing cosmological observation; (b) whether suitable cosmo/astro observables may say something about the different models for the anomalies.

I am not aware of any relation fulfilling item (a). As concerns item (b), we may argue that, depending on the microscopic d.o.f. (whether real or hypothetical) for which cosmo/astro probes exist, the question can be restated accordingly. Let us consider the concrete examples of particle, thermal DM, of neutrinos, and of axions. In the case of DM, item (b) becomes the question whether models for the anomalies do also provide DM candidates. In the case of neutrinos, whether models for the anomalies may modify neutrino physics in a way that is testable in the cosmos; finally, in the case of axions, whether models for the anomalies may also solve the strong-CP problem, possibly via an axion-like particle. We may already give a tentative negative answer to the latter question, considering that the (QCD) axion mass scale is way too low with respect to the scales hinted at by anomalies, that are in the range $\text{few} \times [1, 10] \text{ TeV}$.

As concerns models explaining at the same time $b \rightarrow s$ anomalies and WIMP DM, the motivation starts from the consideration that both phenomena would require new particles in roughly the same ballpark mass range. Due to the page boundary, it is impossible to even quote here all the existing literature on the subject. The reader is referred to the discussion in Ref. [38], and to the references therein. In Ref. [38] these models are usefully classified into two main categories: ‘portal’ models, whereby the d.o.f. responsible for the new contributions to $b \rightarrow s$ amplitudes also

mediates DM production in the early Universe, and ‘loop’ models, whose defining feature is that new contributions to $b \rightarrow s$ amplitudes occur in loops containing DM.

A frequent strategy towards marrying flavour anomalies to DM is to: (1) introduce a new gauge group, e.g. a $U(1)'$; (2) engineer (or just assume) this extension to generate large enough couplings to the structures $\bar{s}b$ and $\bar{\mu}\mu$, yet couplings small enough for all other fermion combinations so as to pass constraints; (3) the DM is provided by an additional d.o.f. made stable by a discrete-symmetry remnant of the broken $U(1)'$. A naturally arising question is then that of DM detection. One should first keep in mind that in such models DM is more elusive by construction, for indirect and direct detection alike. In fact, annihilation yields $\mu\mu$ and $\tau\tau$ way more often than ee by construction, and DM is more coupled to heavy than to light quarks, implying less interaction with nucleons.

Let us turn to the question whether an explanation for the anomalies may distort neutrino spectra. An example of a paper addressing this question is Ref. [39], whose line of argument is worth summarising here. One starts from leptoquarks coupled dominantly to heavy flavours ($b \rightarrow s$ anomalies naturally suggest such scenario [26], as mentioned). The main point in [39] is that this scenario induces non-negligible (high-energy) neutrino - gluon interactions. As a consequence, experiments such as IceCube may get larger-than-expected sensitivity to such leptoquarks, not to mention that IceCube-Gen2 will have much larger statistics. Unfortunately, most of the relevant coupling-vs.-mass parameter space is constrained by LHC searches and precision measurements at LEP, such as $Z \rightarrow \tau\tau$. The observation is interesting nonetheless, and prompts the question whether such induced neutrino ‘strong force’ may be probed elsewhere.

4. Conclusions

In $b \rightarrow s$ and $b \rightarrow c$ decays there are persistent discrepancies with respect to the SM. While experimental data look solid, a genuine theory understanding is still elusive. This contribution speculates on whether these anomalies may have footprints in astro/cosmo observables. We argue that, without an established theory of the anomalies, it may still be premature to try and establish general connections with phenomena in the cosmos. Yet, astro/cosmo quantities may soon prove precious to discern among models that are being proposed, because the latter may induce, e.g., distortions in neutrino fluxes, or DD/ID signatures of the DM candidate(s) of these models, if any, or yet other signatures that this workshop offered the opportunity to brainstorm about.

References

- [1] LHCb collaboration, R. Aaij et al., *Differential branching fractions and isospin asymmetries of $B \rightarrow K^{(*)}\mu^+\mu^-$ decays*, *JHEP* **06** (2014) 133, [1403.8044].
- [2] LHCb collaboration, R. Aaij et al., *Angular analysis and differential branching fraction of the decay $B_s^0 \rightarrow \phi\mu^+\mu^-$* , *JHEP* **09** (2015) 179, [1506.08777].
- [3] LHCb collaboration, R. Aaij et al., *Differential branching fraction and angular analysis of $\Lambda_b^0 \rightarrow \Lambda\mu^+\mu^-$ decays*, *JHEP* **06** (2015) 115, [1503.07138].
- [4] W. Detmold and S. Meinel, *$\Lambda_b \rightarrow \Lambda\ell^+\ell^-$ form factors, differential branching fraction, and angular observables from lattice QCD with relativistic b quarks*, *Phys. Rev.* **D93** (2016) 074501, [1602.01399].

- [5] LHCb collaboration, R. Aaij et al., *Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **113** (2014) 151601, [1406.6482].
- [6] LHCb collaboration, R. Aaij et al., *Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays*, *JHEP* **08** (2017) 055, [1705.05802].
- [7] M. Bordone, G. Isidori and A. Pattori, *On the Standard Model predictions for R_K and R_{K^*}* , *Eur. Phys. J.* **C76** (2016) 440, [1605.07633].
- [8] C. Bobeth, G. Hiller and G. Piranishvili, *Angular distributions of $\bar{B} \rightarrow \bar{K} \ell^+ \ell^-$ decays*, *JHEP* **12** (2007) 040, [0709.4174].
- [9] HPQCD collaboration, C. Bouchard, G. P. Lepage, C. Monahan, H. Na and J. Shigemitsu, *Standard Model Predictions for $B \rightarrow K \ell^+ \ell^-$ with Form Factors from Lattice QCD*, *Phys. Rev. Lett.* **111** (2013) 162002, [1306.0434].
- [10] G. Hiller and F. Kruger, *More model independent analysis of $b \rightarrow s$ processes*, *Phys. Rev.* **D69** (2004) 074020, [hep-ph/0310219].
- [11] LHCb collaboration, R. Aaij et al., *Differential branching fraction and angular analysis of the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay*, *JHEP* **02** (2013) 105, [1209.4284].
- [12] LHCb collaboration, R. Aaij et al., *Measurements of the S-wave fraction in $B^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$ decays and the $B^0 \rightarrow K^*(892)^0 \mu^+ \mu^-$ differential branching fraction*, *JHEP* **11** (2016) 047, [1606.04731].
- [13] B. Capdevila et al., *JHEP* **1801**, 093 (2018) [arXiv:1704.05340 [hep-ph]]; W. Altmannshofer, P. Stangl and D. M. Straub, *Phys. Rev. D* **96** (2017) no.5, 055008 [arXiv:1704.05435 [hep-ph]]; G. D'Amico et al., *JHEP* **1709** (2017) 010 [arXiv:1704.05438 [hep-ph]]; L. S. Geng et al., *Phys. Rev. D* **96** (2017) no.9, 093006 [arXiv:1704.05446 [hep-ph]]; M. Ciuchini et al., *Eur. Phys. J. C* **77** (2017) no.10, 688 [arXiv:1704.05447 [hep-ph]]; T. Hurth et al., *Phys. Rev. D* **96** (2017) no.9, 095034 [arXiv:1705.06274 [hep-ph]].
- [14] BABAR collaboration, J. P. Lees et al., *Evidence for an excess of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ decays*, *Phys. Rev. Lett.* **109** (2012) 101802, [1205.5442].
- [15] LHCb collaboration, R. Aaij et al., *Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau) / \mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \mu^- \bar{\nu}_\mu)$* , *Phys. Rev. Lett.* **115** (2015) 111803, [1506.08614].
- [16] BELLE collaboration, M. Huschle et al., *Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ relative to $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell$ decays with hadronic tagging at Belle*, *Phys. Rev.* **D92** (2015) 072014, [1507.03233].
- [17] P. Goldenzweig, talk at the “Rencontres de Moriond, EW interactions and unified theories”, 12-19 Mar, 2016.
- [18] HFLAV collaboration, Y. Amhis et al., *Averages of b-hadron, c-hadron, and τ -lepton properties as of summer 2016*, *Eur. Phys. J.* **C77** (2017) 895, [1612.07233].
- [19] R. Alonso, B. Grinstein and J. Martin Camalich, *$SU(2) \times U(1)$ gauge invariance and the shape of new physics in rare B decays*, *Phys. Rev. Lett.* **113** (2014) 241802, [1407.7044].
- [20] B. Bhattacharya, A. Datta, D. London and S. Shivashankara, *Simultaneous Explanation of the R_K and $R(D^{*})$ Puzzles*, *Phys. Lett.* **B742** (2015) 370–374, [1412.7164].
- [21] S. L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions with Lepton-Hadron Symmetry*, *Phys. Rev.* **D2** (1970) 1285–1292.

- [22] N. Cabibbo, *Unitary Symmetry and Leptonic Decays*, *Phys. Rev. Lett.* **10** (1963) 531–533.
- [23] M. Kobayashi and T. Maskawa, *CP Violation in the Renormalizable Theory of Weak Interaction*, *Prog. Theor. Phys.* **49** (1973) 652–657.
- [24] G. Hiller and M. Schmaltz, *R_K and future $b \rightarrow s\ell\ell$ physics beyond the standard model opportunities*, *Phys. Rev.* **D90** (2014) 054014, [1408.1627].
- [25] D. Ghosh, M. Nardecchia and S. A. Renner, *Hint of Lepton Flavour Non-Universality in B Meson Decays*, *JHEP* **12** (2014) 131, [1408.4097].
- [26] S. L. Glashow, D. Guadagnoli and K. Lane, *Lepton Flavor Violation in B Decays?*, *Phys. Rev. Lett.* **114** (2015) 091801, [1411.0565].
- [27] B. Gripaios, M. Nardecchia and S. A. Renner, *Composite leptoquarks and anomalies in B-meson decays*, *JHEP* **05** (2015) 006, [1412.1791].
- [28] N. Assad, B. Fornal and B. Grinstein, *Baryon Number and Lepton Universality Violation in Leptoquark and Diquark Models*, *Phys. Lett.* **B777** (2018) 324–331, [1708.06350].
- [29] L. Di Luzio, A. Greljo and M. Nardecchia, *Gauge leptoquark as the origin of B-physics anomalies*, *Phys. Rev.* **D96** (2017) 115011, [1708.08450].
- [30] L. Calibbi, A. Crivellin and T. Li, *A model of vector leptoquarks in view of the B-physics anomalies*, 1709.00692.
- [31] M. Bordone, C. Cornella, J. Fuentes-Martin and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, *Phys. Lett.* **B779** (2018) 317–323, [1712.01368].
- [32] R. Barbieri and A. Tesi, *B-decay anomalies in Pati-Salam SU(4)*, *Eur. Phys. J.* **C78** (2018) 193, [1712.06844].
- [33] D. Becirevic, I. Dorsner, S. Fajfer, D. A. Faroughy, N. Kosnik and O. Sumensari, *Scalar leptoquarks from GUT to accommodate the B-physics anomalies*, 1806.05689.
- [34] S. Trifinopoulos, *Revisiting R-parity violating interactions as an explanation of the B-physics anomalies*, 1807.01638.
- [35] M. Blanke and A. Crivellin, *B Meson Anomalies in a Pati-Salam Model within the Randall-Sundrum Background*, *Phys. Rev. Lett.* **121** (2018) 011801, [1801.07256].
- [36] A. Greljo, G. Isidori and D. Marzocca, *On the breaking of Lepton Flavor Universality in B decays*, *JHEP* **07** (2015) 142, [1506.01705].
- [37] D. A. Faroughy, A. Greljo and J. F. Kamenik, *Confronting lepton flavor universality violation in B decays with high- p_T tau lepton searches at LHC*, *Phys. Lett.* **B764** (2017) 126–134, [1609.07138].
- [38] A. Vicente, *Anomalies in $b \rightarrow s$ transitions and dark matter*, *Adv. High Energy Phys.* **2018** (2018) 3905848, [1803.04703].
- [39] D. Becirevic, B. Panes, O. Sumensari and R. Zukanovich Funchal, *Seeking leptoquarks in IceCube*, *JHEP* **06** (2018) 032, [1803.10112].