PROCEEDINGS OF SCIENCE

Constraints on Neutrino Masses from Cosmological Observations

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Fitting the predictions of the ΛCDM model to the Sachs-Wolfe effect, σ_8, the galaxy power spectrum $P_{\mathrm{gal}}(k)$, fluctuations of galaxy counts in spheres of radii ranging from $16/h$ to $128/h$ Mpc, Baryon Acoustic Oscillation (BAO) measurements, and $h = 0.678 \pm 0.009$, in various combinations, *with free spectral index* n_s, and free galaxy bias *and galaxy bias slope*, we obtain consistent measurements of the sum of neutrino masses $\sum m_\nu$. The results depend on h, so we have presented confidence contours in the $(\sum m_\nu, h)$ plane.

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1. Introduction

The power spectrum of linear density perturbations $P(k)$ becomes suppressed at large wavenumber k due to free-streaming of massive neutrinos that can not cluster on these small scales, and, more importantly, due to the slower growth of structure with massive neutrinos [1]. This suppression of $P(k)$ at large k affects σ_8 that is sensitive to $\log_{10}(k/(h \text{ Mpc}^{-1}))$ in the range -1.3 to -0.6, while the Sachs-Wolfe effect, that is sensitive in the range -3.1 to -2.7, is unsuppressed. It is therefore possible to measure neutrino masses by fitting the predictions of the ΛCDM model [2] to measurements of the Sachs-Wolfe effect and σ_8. Measurements of the galaxy power spectrum $P_{\text{gal}}(k)$ can also contribute to constrain neutrino masses if the bias b, defined by $P_{\text{gal}}(k) = b^2 P(k)$, is understood. The Sachs-Wolfe effect of fluctuations of the Cosmic Microwave Background (CMB) provides a direct measurement of density fluctuations [2, 3]. The relative mass fluctuation σ_8 in randomly placed spheres of radius $r_s = 8/h$ Mpc is measured with gravitational lensing and studies of rich galaxy clusters [4].

To be specific, we consider three active neutrino eigenstates with nearly the same mass, so $\sum m_\nu \approx 3m_\nu$. The suppression factor of $P(k)$ for large k is $f(k, \sum m_\nu) = 1 - 8 f_\nu$, where $f_\nu = \Omega_\nu/\Omega_m$ [1]. Ω_m is the total (dark plus baryonic plus neutrino) matter density today relative to the critical density, and includes the contribution $\Omega_\nu = h^{-2} \sum m_\nu / 93.04$ eV of neutrinos that are non-relativistic today.

In this note we outline the results of measurements of $\sum m_\nu$. For details we refer the reader to the talk at the Guadeloupe 2018 Conference [5], and to [6] and references therein.

2. Measurement of neutrino masses with the Sachs-Wolfe effect and σ_8

The ΛCDM model prediction for $P(k)$ [2] has three free parameters: the amplitude N^2, the spectral index n_s, and $\sum m_\nu$. We keep n_s fixed. We vary the two parameters N^2 and $\sum m_\nu$ to minimize a χ^2 with two terms corresponding to two observables: the Sachs-Wolfe effect that constrains N^2, and σ_8. We therefore have zero degrees of freedom. The result is a function of h, Ω_m, and n_s, so we define $\delta h \equiv (h - 0.678)/0.009$, $\delta\Omega_m \equiv (\Omega_m - 0.281)/0.003$, and $\delta n \equiv (n_s - 1)/0.038$, and obtain

$$\sum m_\nu = 0.595 + 0.047 \cdot \delta h + 0.226 \cdot \delta n + 0.022 \cdot \delta\Omega_m \pm 0.225 \text{ (stat)}^{+0.484}_{-0.152} \text{ (syst) eV}. \quad (2.1)$$

3. Test of scale invariance of the galaxy bias b

We count galaxies in an array of $N_s = N_x \times N_y$ spheres of radii r_s, and obtain their mean $\bar{N}$, and their root-mean-square (rms). All spheres have their center at redshift $z = 0.5$ to ensure the homogeneity of the galaxy selections. We compare $\sigma/\bar{N}$ obtained from galaxy counts, with the predicted relative mass fluctuation in the linear approximation corresponding to $P(k)$. The ratio of these two quantities is the bias b. This bias b depends on $\sum m_\nu$, h, and n_s. We find that the galaxy bias b is scale invariant, within the statistical uncertainties, if

$$\sum m_\nu = 0.939 + 0.035 \cdot \delta h + 0.089 \cdot \delta n \pm 0.008 \text{ eV}, \quad (3.1)$$

else scale invariance is broken. Since scale invariance depends on $\sum m_\nu$ we allow b to depend on k in the following fits.

4. Measurement of neutrino masses with the Sachs-Wolfe effect, σ_8, and $P_{\mathbf{gal}}(k)$

We fit the Sachs-Wolfe effect, σ_8, $h = 0.678 \pm 0.009$ [4], and the measurement of $P_{\rm gal}(k)$ with galaxies in the Sloan Digital Sky Survey SDSS-III by the BOSS Collaboration [7, 8]. We allow the galaxy bias b to depend on scale: $b \equiv b_0 + b_1 \log_{10}(k/h\ \mathrm{Mpc}^{-1})$. Minimizing the χ^2 with respect to $\sum m_\nu$, N^2, n_s, $h = 0.678 \pm 0.009$, b_0, and b_1, we obtain

$$\begin{aligned} \sum m_\nu &= 0.80 \pm 0.23 \text{ eV}, \\ N^2 &= (1.88 \pm 0.39) \times 10^{-10}, \\ n_s &= 1.064 \pm 0.068, \\ h &= 0.676 \pm 0.011, \\ b_0 &= 2.35 \pm 0.36, \\ b_1 &= 0.229 \pm 0.094, \end{aligned} \tag{4.1}$$

with $\chi^2 = 27.8$ for 18 degrees of freedom. The uncertainties have been multiplied by $\sqrt{(27.8/18)}$. Confidence contours are presented in Fig. 1. Fixing $b_1 = 0$ obtains $\chi^2 = 36.3$, so including the scale dependence of b is necessary. This measurement of $\sum m_\nu$ is interesting, but we do not use it in our final combination because of the high χ^2 per degree of freedom (and the need for a better understanding of the galaxy bias).

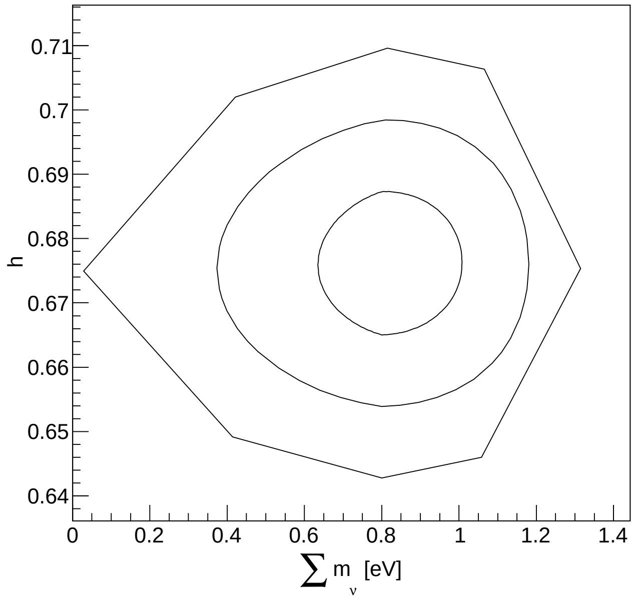

Figure 1: Contours corresponding to 1, 2, and 3 standard deviations in the $(\sum m_\nu, h)$ plane, from Sachs-Wolfe, σ_8, $h = 0.678 \pm 0.009$, and $P_{\rm gal}(k)$ measurements. Points on the contours have $\chi^2 - \chi^2_{\rm min} = 1, 4,$ and 9, respectively, where χ^2 has been minimized with respect to N^2, n_s, b_0, and b_1.

5. Measurement of neutrino masses with the Sachs-Wolfe effect, σ_8, and galaxy fluctuations

We repeat the measurements of Section 2 but add 4 more experimental constraints: $\sigma/\bar{N}$ of SDSS DR14 [6, 8, 9] galaxy counts in spheres of radius $r_s = 16/h, 32/h, 64/h$, and $128/h$ Mpc. We add two more parameters to be fit: b_0 and b_s which define the bias $b = b_0 - i_s b_s$, with $i_s = 0, 1, 2, 3$ for $r_s = 16/h, 32/h, 64/h$, and $128/h$ Mpc, respectively. From the Sachs-Wolfe effect, σ_8, and the 4 $\sigma/\bar{N}$ measurements we obtain

$$\sum m_\nu \;=\; 0.618 + 0.042 \cdot \delta h + 0.206 \cdot \delta n + 0.019 \cdot \delta\Omega_m \pm 0.209 \text{ (stat)}^{+0.420}_{-0.139} \text{ (syst) eV}, \quad (5.1)$$

with $\chi^2 = 1.1$ for 2 degrees of freedom. The variables that minimize the χ^2 are $\sum m_\nu$, N^2, b_0, and b_s. This result may be compared with (2.1).

6. Combination with BAO

In the companion talk and note in this Guadeloupe 2018 Conference [5] we obtained

$$\sum m_\nu = 0.711 - 0.335 \cdot \delta h + 0.050 \cdot \delta b \pm 0.063 \text{ eV}, \quad (6.1)$$

where $\delta b \equiv (\Omega_b h^2 - 0.02226)/0.00023$, from a study of Baryon Acoustic Oscillations (BAO) with SDSS DR13 galaxies and $\theta_{\rm MC}$ [8, 9, 10, 11]. We allow $\Omega_b h^2$ to vary by one standard deviation, i.e. $\delta b = 0 \pm 1$ [4]. Combining with (5.1) we obtain

$$\sum m_\nu \;=\; 0.697 - 0.276 \cdot \delta h + 0.032 \cdot \delta n + 0.003 \cdot \delta\Omega_m \pm 0.075 \text{ (stat)}^{+0.055}_{-0.028} \text{ (syst) eV}, \quad (6.2)$$

with $\chi^2 = 1.3$ for 3 degrees of freedom. Freeing n_s, and minimizing the χ^2 with respect to $\sum m_\nu$, N^2, n_s, $h = 0.678 \pm 0.009$, b_0, and b_s, we obtain

$$\begin{aligned}
\sum m_\nu &= 0.719 \pm 0.312 \text{ (stat)}^{+0.055}_{-0.028} \text{ (syst) eV}, \\
N^2 &= (2.09 \pm 0.33) \times 10^{-10}, \\
n_s &= 1.021 \pm 0.075, \\
h &= 0.678 \pm 0.008, \\
b_0 &= 1.751 \pm 0.060, \\
b_s &= -0.053 \pm 0.041, \qquad (6.3)
\end{aligned}$$

with $\chi^2 = 1.1$ for 2 degrees of freedom. The uncertainty of $\sum m_\nu$ is dominated by the uncertainty of h, so we present confidence contours in the $(\sum m_\nu, h)$ plane in Figure 2.

7. Tensions

Let us comment on Equations (6.1) and (5.1). Equation (6.1) is mainly determined by the precise measurement of the sound horizon angle $\theta_{\rm MC}$ by the Planck experiment, and by the assumption that the BAO wave stalls at redshift $z = z_* = 1089.9 \pm 0.4$. Equation (6.1) tells us that $(\sum m_\nu, h)$ lies on the diagonal shown in Figure 2 (with some uncertainty from $\Omega_b h^2$). Equation

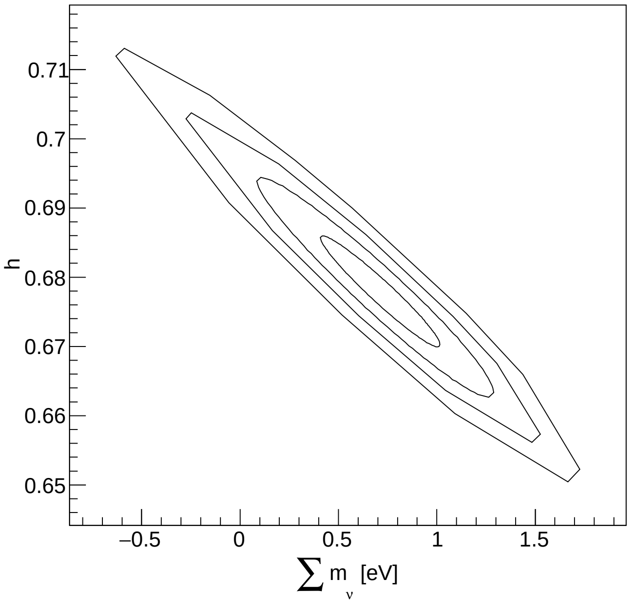

Figure 2: Contours corresponding to 1, 2, 3, and 4 standard deviations in the $(\sum m_\nu, h)$ plane, from Sachs-Wolfe, σ_8, 4 $\sigma/\overline{N}$, BAO, and $h = 0.678 \pm 0.009$ measurements. Points on the contours have $\chi^2 - \chi^2_{\text{min}} = 1, 4, 9$, and 16, respectively, where χ^2 has been minimized with respect to N^2, n_s, b_0, and b_s. The total uncertainty of $\sum m_\nu$ is dominated by the uncertainty of h. In this figure the systematic uncertainties presented in Equation (6.3) are not included.

(5.1) is a constraint mainly between $\sum m_\nu$ and n_s with large uncertainties. To determine $\sum m_\nu$ we need as input a value for h (or a value for n_s). In this article we have taken $h = 0.678 \pm 0.009$ from [4]. If $h = 0.678 \pm 0.009$ we obtain $\sum m_\nu = 0.719 \pm 0.312$ eV, and $n_s = 1.021 \pm 0.075$. If however $h = 0.688 \pm 0.009$ we obtain $\sum m_\nu = 0.412 \pm 0.328$ eV, and $n_s = 0.960 \pm 0.073$. And if $h \approx 0.697$, we obtain $\sum m_\nu \approx 0$ eV. Alternatively, if we fix $n_s = 1.0$, then $h = 0.681 \pm 0.005$ and $\sum m_\nu = 0.619 \pm 0.182$ eV. Or if we fix $n_s = 0.96$ as estimated from the spectrum of CMB fluctuations, then $h = 0.685 \pm 0.006$ and $\sum m_\nu = 0.440 \pm 0.189$ eV, see Figure 3. At the Guadeloupe 2018 Conference, Adam Riess, representing the SH$_0$ES Team, presented the latest direct measurement of the expansion parameter: $h = 0.7353 \pm 0.0162$, which corresponds to negative $\sum m_\nu$! Discussions on these tensions made the Guadeoulpe meeting extremely interesting. And the solution may come from an unexpected direction: gravitational waves from merging black holes are a “standard siren". The single black hole merger GW170817 already obtains $h = 0.70^{+0.12}_{-0.08}$, see the talk by Archil Kobakhidze!

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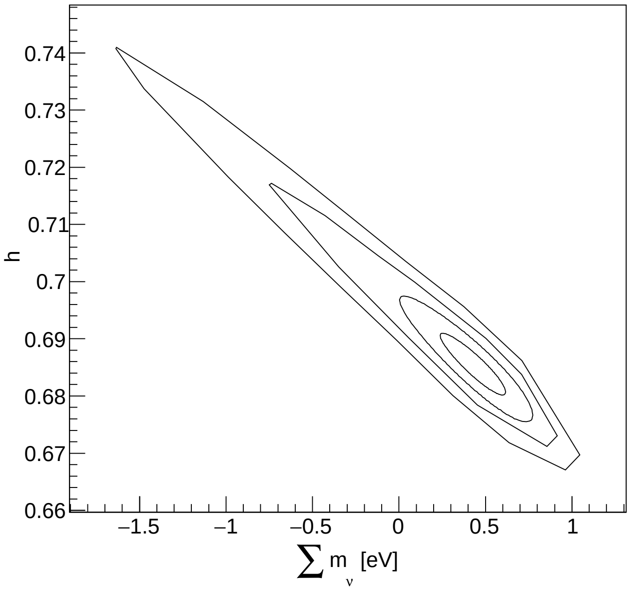

Figure 3: Contours corresponding to 1, 2, 3, and 4 standard deviations in the $(\sum m_\nu, h)$ plane, from Sachs-Wolfe, σ_8, 4 $\sigma/\bar{N}$, and BAO measurements. $n_s = 0.96$ is fixed (as estimated from the CMB fluctuation spectrum). Points on the contours have $\chi^2 - \chi^2_{\text{min}} = 1, 4, 9$, and 16, respectively, where χ^2 has been minimized with respect to N^2, b_0, and b_s. The total uncertainty of $\sum m_\nu$ is dominated by the uncertainty of h. In this figure the systematic uncertainties presented in Equation (6.3) are not included.

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