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BRST invariant $d = 2$ condensates in Gribov-Zwanziger theory

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In this proceeding, $SU(N)$ Yang-Mills theory is quantized in the linear covariant gauges, while taking into account the issue of Gribov copies, we construct the one-loop effective potential for a set of mass dimension 2 condensates, including the Gribov parameter, that refines the infrared region of the Gribov-Zwanziger theory, whilst respecting the renormalization group invariance and BRST symmetry.

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1. Introduction

The analytical explanation of quark and gluon confinement has been a big challenge in recent decades. At low temperatures (or in the infrared regime, IR), where confinement happens, the coupling constant g^2 is large, hence the perturbative formalism can not be used in this regime. This research main focus has been on the gluon propagator and also ghost propagator in the IR. The gluon propagator is suppressed to a nonvanishing value at zero momentum violating the positivity and the ghost propagator is not enhanced at at large volume according to the lattice data [1]. A possible analytical explanation for this behavior is gotten through the adding of dimension 2 condensates to the Gribov-Zwanziger (GZ) formalism, yielding to so-called Refined Gribov-Zwanziger (RGZ) framework [2, 3] that fits with the lattice data quite well [1].

In this present study, the analysis of a non-trivial minimum of the effective action, which leads us to a dynamical transformation of the GZ action into the RGZ action, has been done in the presence of the $\langle A^h A^h \rangle$ and $\langle \bar{\varphi}\varphi \rangle$ condensates at one-loop following earlier steps of [4], suitably generalized to respect BRST invariance following recent developments by some of us in the field.

In this proceeding, we highlight a few steps, a comprehensive paper will be presented elsewhere.

2. The Gribov-Zwanziger action in the linear covariant gauge

In the IR region, the Gribov copies appear. Since the coupling constant g^2 is large, these copies can not be eliminated [5]. A way to work around this problem is to restrict the functional integral to a specific region Ω in field space, a solution proposed by Gribov using the Landau gauge [5]. Moreover, this solution given by Gribov can be generalized to linear covariant gauge [6]:

$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = i\alpha b^a, \qquad \mathcal{M}^{ab}(A^h) = -\partial_\mu D^{ab}_\mu(A^h) > 0\}. \tag{2.1}$$

whereby the Hermitian Faddeev-Popov-related operator, $\mathcal{M}^{ab}(A^h) = -\delta^{ab}\partial^2 + gf^{abc}(A^h)^c_\mu \partial_\mu$, is positive. In (2.1), A^h_μ is a non-local power series in the gauge field, gotten from the minimization of the functional $f_A[u]$ along the gauge orbit of A_μ [7, 8, 9],

$$\begin{aligned} f_A[u] &\equiv \min_{\{u\}} \mathrm{Tr}\int d^4x\, A^u_\mu A^u_\mu, \\ A^u_\mu &= u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \end{aligned} \tag{2.2}$$

A local minimum is found and given by

$$\begin{aligned} A^h_\mu &= \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\phi_\nu\,, \qquad \partial_\mu A^h_\mu = 0\,, \\ \phi_\nu &= A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + O(A^3). \end{aligned} \tag{2.3}$$

Here we highlight that A^h_μ is gauge invariant order by order [6]. The field A^h_μ can be localized by introducing an auxiliary Stueckelberg field ξ^a [6, 10],

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \tag{2.4}$$

while

$$h = e^{ig\,\xi^a T^a}, \tag{2.5}$$

Now, the local gauge invariance of A^h_μ under a gauge transformation $u \in SU(N)$ can be obtained from

$$h \to u^\dagger h, \qquad h^\dagger \to h^\dagger u, \qquad A_\mu \to u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \tag{2.6}$$

Now, considering the BRST invariance, the Gribov-Zwanziger action in the linear covariant gauges, the total action is given by

$$S = S_{YM} + S_{GF} + S_{GZ} + S_\varepsilon, \tag{2.7}$$

whereby S_{YM} is the Yang-Mills action,

$$S_{YM} = \frac{1}{4}\int d^4x F^a_{\mu\nu} F^a_{\mu\nu}, \tag{2.8}$$

S_{GF} is the Faddeev-Popov gauge-fixing in linear covariant gauges,

$$S_{GF} = \int d^4x \left(\frac{\alpha}{2} b^a b^a + i b^a\, \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu(A) c^b \right), \tag{2.9}$$

α being the gauge parameter and $\alpha = 0$ in Landau gauge; S_{GZ} is the Gribov-Zwanziger action in its local form,

$$\begin{aligned} S_{GZ} = & \int d^4x \left[\bar{\varphi}^{ac}_\mu \partial_\nu D^{ab}_\nu(A^h) \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \partial_\nu (D^{ab}_\nu(A^h) \omega^{bc}_\mu) \right] \\ & - \gamma^2 g \int d^4x \left[f^{abc} (A^h)^a_\mu \varphi^{bc}_\mu + f^{abc} (A^h)^a_\mu \bar{\varphi}^{bc}_\mu + \frac{d}{g}(N_c^2 - 1)\gamma^2 \right], \end{aligned} \tag{2.10}$$

with $(\bar{\varphi}^{ac}_\mu, \varphi^{ac}_\mu)$ a pair of complex-conjugate bosonic fields, $(\bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$ a pair of anti-commuting complex-conjugate fields; and γ the Gribov parameter which is dynamically fixed by a gap equation that gives us the horizon function [11, 12],

$$\langle f^{abc} (A^h)^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) \rangle = 2d(N^2 - 1)\frac{\gamma^2}{g^2}, \tag{2.11}$$

which can also be rewritten as [4]

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0, \tag{2.12}$$

whereby Γ is the quantum action defined by

$$e^{-\Gamma} = \int [d\Phi] e^{-S}. \tag{2.13}$$

The last term from (2.7),

$$S_\varepsilon = \int d^4x\, \varepsilon^a\, \partial_\mu (A^h)^a_\mu \tag{2.14}$$

ensures, through the Lagrange multiplier ε, the transversality of the composite operator $(A^h)^a_\mu$, $\partial_\mu (A^h)^a_\mu = 0$.

The action S, (2.7), enjoys an exact BRST invariance, $sS = 0$ and $s^2 = 0$ [6]

$$\begin{aligned}
&sA^a_\mu = -D^{ab}_\mu c^b\,, && sc^a = \frac{g}{2} f^{abc} c^b c^c\,,\\
&s\bar{c}^a = ib^a\,, && sb^a = 0\,,\\
&s\varphi^{ab}_\mu = 0\,, && s\omega^{ab}_\mu = 0\,,\\
&s\bar{\omega}^{ab}_\mu = 0\,, && s\bar{\varphi}^{ab}_\mu = 0\,,\\
&s\varepsilon^a = 0\,, && s(A^h)^a_\mu = 0\,,\\
&sh^{ij} = -igc^a (T^a)^{ik} h^{kj}\,. &&
\end{aligned} \tag{2.15}$$

3. Refined Gribov-Zwanziger Action

The BRST invariant $d = 2$ condensates, $\langle A^a_{h,\mu} A^a_{h,\mu} \rangle$ and $\langle \bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu \rangle$, cause non-perturbative dynamical instabilities disturbing the Gribov-Zwanziger formalism [2, 3, 4]. $\langle \bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu \rangle$ guarantees that the gluon propagator is non-vanishing at zero momentum, and $\langle A^a_\mu A^a_\mu \rangle$ assures to fit the result with the lattice data [4]. The refined Gribov-Zwanziger action (RGZ) is obtained adding these condensates to the GZ action via the *local composite operator (LCO) formalism*, see [4]. The operators $A^h A^h$ and $\bar{\varphi}\varphi$ will be added to the action via two BRST invariant bosonic sources τ and Q,

$$s\tau = 0 \quad \text{and} \quad sQ = 0. \tag{3.1}$$

From here, we opted for the Landau gauge $\partial A = 0$ for convenience, so that we can work with $A^h = A$, as formally proven in [6]. Then, the action with these operators is written as

$$\Sigma = S + S_{A^2} + S_{\varphi\bar{\varphi}} + S_{\text{vac}}, \tag{3.2}$$

whereby S is given by (2.7) and we also have

$$\begin{aligned}
S_{A^2} &= \int d^{\mathcal{D}}x \frac{\tau}{2} A^a_\mu A^a_\mu,\\
S_{\bar{\varphi}\varphi} &= \int d^{\mathcal{D}}x Q \bar{\varphi}^{ac}_\mu \varphi^{ac}_\mu,\\
S_{\text{vac}} &= -\int d^{\mathcal{D}}x \left(\frac{\zeta}{2}\tau^2 + \alpha QQ + \chi Q\tau \right).
\end{aligned} \tag{3.3}$$

The parameters α, χ and ζ are the LCO parameters which guarantee that the divergences of the kind $\langle A^2(x) A^2(y) \rangle_{x\to y}$, etc. can be properly dealt with, see [4].

4. The Effective Action Calculus

In order to get the effective action, we have written the energy functional as

$$e^{-W(J)} = \int [d\Phi] e^{-\Sigma}, \tag{4.1}$$

where Σ is given by (3.2).

The action Σ, (3.2), has three terms quadratic in the sources and they should be removed to facilitate calculations and interpret our results in terms of the vacuum energy. The easiest way to remove these terms is by introducing two auxiliary fields σ_1 and σ_2 via two identities

$$
\begin{aligned}
1 &= \int [\mathcal{D}\sigma_1]\, e^{-\frac{1}{2}\int d^dx \left(\sigma_1 + \frac{\bar{a}}{2}A^2 + \bar{b}Q + \bar{c}\tau\right)^2}, \\
1 &= \int [\mathcal{D}\sigma_2]\, e^{+\frac{1}{2}\int d^dx \left(\sigma_2 + \bar{d}\bar{\varphi}\varphi + \bar{e}Q + \frac{\bar{f}}{2}A^2\right)^2},
\end{aligned} \tag{4.2}
$$

multiplying the integral in (4.1). If we choose

$$
\begin{aligned}
\bar{a} &= -\frac{Z_A}{\sqrt{Z_{\zeta\zeta}\zeta}}\mu^{\epsilon/2} \\
\bar{b} &= \frac{Z_{QQ}Z_{\chi\chi}\chi}{\sqrt{Z_{\zeta\zeta}\zeta}}\mu^{-\epsilon/2}, \\
\bar{c} &= Z_{\tau\tau}\sqrt{Z_{\zeta\zeta}\zeta}\mu^{-\epsilon/2}, \\
\bar{d} &= \frac{Z_\varphi}{\sqrt{\frac{Z_{\chi\chi}^2\chi^2}{Z_{\zeta\zeta}\zeta} - 2Z_{\alpha\alpha}\alpha}}\mu^{\epsilon/2}, \\
\bar{e} &= Z_{QQ}\sqrt{\frac{Z_{\chi\chi}^2\chi^2}{Z_{\zeta\zeta}\zeta} - 2Z_{\alpha\alpha}\alpha}\; \mu^{-\epsilon/2}, \\
\bar{f} &= \frac{Z_A}{\sqrt{Z_{\zeta\zeta}\zeta}}\left(\frac{Z_{\tau Q}Z_{\zeta\zeta}\zeta - Z_{QQ}Z_{\chi\chi}\chi}{Z_{QQ}\sqrt{Z_{\chi\chi}^2\chi^2 - 2Z_{\alpha\alpha}Z_{\zeta\zeta}\zeta\alpha}}\right)\mu^{\epsilon/2},
\end{aligned} \tag{4.3}
$$

we can remove the quadratic terms in sources. In the $\overline{\text{MS}}$ scheme and at one loop, the Z factors are given by [4]

$$
\begin{aligned}
Z_A &= 1 + \frac{13}{3}\frac{Ng^2}{16\pi^2\epsilon}, \qquad \tilde{Z}_\zeta = Z_{\zeta\zeta}Z_{\tau\tau}^2 = 1 - \frac{13}{3}\frac{Ng^2}{16\pi^2\epsilon}, \qquad Z_{\zeta\zeta} = 1 + \frac{22}{3}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_g &= 1 - \frac{11}{3}\frac{Ng^2}{16\pi^2\epsilon}, \qquad Z_{\tau\tau} = 1 - \frac{35}{6}\frac{Ng^2}{16\pi^2\epsilon}, \qquad Z_{QQ} = Z_g Z_A^{1/2} = 1 - \frac{3}{2}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_{\chi\chi} &= 1, \qquad Z_{\tau Q} = 0, \qquad \tilde{Z}_\alpha = Z_{\alpha\alpha}Z_{QQ}^2 = 1 + \frac{35}{6}\frac{Ng^2}{16\pi^2\epsilon}, \\
Z_{\alpha\alpha} &= 1 + \frac{53}{6}\frac{Ng^2}{16\pi^2\epsilon}, \qquad Z_\varphi = Z_{\bar{\varphi}} = Z_g^{-1}Z_A^{-1/2} = 1 + \frac{3}{2}\frac{Ng^2}{16\pi^2\epsilon}.
\end{aligned} \tag{4.4}
$$

Therefore, (4.1) becomes

$$
\begin{aligned}
e^{-W(Q,\tau)} = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_{1,3}] \exp \Bigg[&-S_{GZ} - \frac{1}{2}\int d^dx \Bigg(2\bar{c}\sigma_1\tau + 2\sigma_3 Q\left(1 - \frac{\bar{b}^2}{\bar{e}^2}\right)\sigma_1^2 \\
&- \frac{1}{\bar{e}^2}(\sigma_3^2 - 2\bar{b}\sigma_1\sigma_3) + \left(\left(\bar{a} - \frac{\bar{f}\bar{b}}{\bar{e}}\right)\langle\sigma_1\rangle + \frac{\bar{f}}{\bar{e}}\langle\sigma_3\rangle\right)A^2 \\
&- 2\frac{\bar{d}}{\bar{e}}(\bar{b}\langle\sigma_1\rangle - \langle\sigma_3\rangle)\bar{\varphi}\varphi \Bigg)\Bigg],
\end{aligned} \tag{4.5}
$$

where

$$\sigma_3 = \sigma_1 \bar{b} - \sigma_2 \bar{e}. \tag{4.6}$$

So far, all LCO parameters, sources and fields have been renormalized, except the auxiliary fields σ's. Analyzing the term $\bar{c}\sigma_1\tau = Z_{\tau\tau}\sqrt{Z_{\zeta\zeta}\zeta}\mu^{-\epsilon/2}\sigma_1\tau$ that appears in Eq.(4.5), it is easy to see that this field is indeed infinite, thus it also must be renormalized. As the Z-factors are infinite, if σ_1 would be finite, then the quantity multiplying τ would be infinite. This would not make sense as a physical (and thus finite) local operator $\mathcal{O}$. The original σ_1 field *should* then be infinite in order to get a finite quantity multiplying the finite source τ. It is then natural to define a renormalized finite field σ_1' by $\sigma_1' \equiv Z_{\tau\tau}\sqrt{Z_{\zeta\zeta}}\sigma_1 \equiv \sqrt{\widetilde{Z}_\zeta}\sigma_1$. Regarding σ_3, the term $\sigma_3 Q$ in Eq.(4.5) teach us, knowing that Q is finite, that σ_3 is already finite and thus should not be renormalized. In terms of the *finite* fields σ_1' and σ_3, the energy functional now reads :

$$\begin{aligned} e^{-W(Q,\tau)}S = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_{1,3}]\ \exp\ \Bigg[-S_{GZ} - \frac{1}{2}\int d^dx \Bigg(& -2\sqrt{\zeta}\sigma_1'\tau + 2\sigma_3 Q\left(1-\frac{\bar{b}^2}{\bar{e}^2}\right)\frac{{\sigma_1'}^2}{\widetilde{Z}_\zeta} \\ & -\frac{1}{\bar{e}^2}\left(\sigma_3^2 - 2\bar{b}\frac{\sigma_1'}{\sqrt{\widetilde{Z}_\zeta}}\sigma_3\left(\bar{a}-\frac{\bar{f}\bar{b}}{\bar{e}}\right)\frac{\langle\sigma_1'\rangle}{\sqrt{\widetilde{Z}_\zeta}} + \frac{\bar{f}}{\bar{e}}\langle\sigma_3\rangle\right)A^2 \\ & -2\frac{\bar{d}}{\bar{e}}\left(\bar{b}\frac{\langle\sigma_1'\rangle}{\sqrt{\widetilde{Z}_\zeta}} - \langle\sigma_3\rangle\right)\overline{\varphi}\varphi\Bigg)\Bigg]. \end{aligned} \tag{4.7}$$

In this expression, all LCO parameters, sources and fields are now finite, and infinities are only present in the renormalization factors Z's, explicitly written or present in the coefficients $\bar{a},...,\bar{f}$. At one loop, $\chi = 0$, $Z_{\tau Q} = 0$ [4] which implies that $\bar{b} = \bar{f} = 0$, then $\sigma_3 = -\bar{e}\sigma_2$. Now, by analysis of the term $\sigma_3 Q = -\bar{e}\sigma_2 Q$ in (4.7), a same reasoning as above for σ_1 shows that σ_2 is infinite and should be renormalized defining a new finite field σ_2' through $\sigma_2' \equiv Z_{QQ}\sqrt{Z_{\alpha\alpha}}\sigma_2 \equiv \sqrt{\widetilde{Z}_\alpha}\sigma_2$. Hence, the energy functional in terms of the finite fields σ_1' and σ_2' and with one-loop coefficients reads :

$$\begin{aligned} e^{-W(Q,\tau)} = \int [\mathcal{D}\Phi][\mathcal{D}\sigma_{1,2}]\exp\ \Bigg[-S_{GZ} - \frac{1}{2}\int d^dx \Bigg(& \frac{{\sigma_1'}^2}{\widetilde{Z}_\zeta} - \frac{{\sigma_2'}^2}{\widetilde{Z}_\alpha} + \bar{a}\frac{\langle\sigma_1'\rangle}{\sqrt{\widetilde{Z}_\zeta}}A^2 - 2\bar{d}\frac{\langle\sigma_2'\rangle}{\sqrt{\widetilde{Z}_\alpha}}\overline{\varphi}\varphi \\ & -2\sqrt{\zeta}\sigma_1'\tau + 2\sqrt{-2\alpha}\sigma_2' Q\Bigg)\Bigg] \end{aligned} \tag{4.8}$$

In this expression, infinities are now localized in four different places, only in the renormalization factors $\tilde{Z}_\zeta$, $\tilde{Z}_\alpha$, and in those hidden in $\bar{a}$ and $\bar{d}$.

In order to have an expression of the form $\frac{m^2}{2}A^2 - M^2\bar{\varphi}\varphi$, we defined the effective masses, m^2 and M^2, respectively linked to $\langle AA\rangle$ and $\langle\bar{\varphi}\varphi\rangle$ by:

$$m^2 \equiv \frac{\bar{a}}{\sqrt{\widetilde{Z}_\zeta}}\langle\sigma_1'\rangle = \left(1+\frac{17}{6}\frac{Ng^2}{16\pi^2\epsilon}\right)\sqrt{\frac{13Ng^2}{9(N^2-1)}}\langle\sigma_1'\rangle + \mathcal{O}(g^4), \tag{4.9}$$

$$M^2 \equiv \frac{\bar{d}}{\sqrt{\widetilde{Z_\alpha}}} \langle \sigma_2' \rangle = -\left(1 - \frac{35}{6}\frac{Ng^2}{16\pi^2\epsilon}\right)\sqrt{\frac{35Ng^2}{48(N^2-1)^2}}\langle \sigma_2' \rangle + \mathcal{O}(g^4). \tag{4.10}$$

where the last equalities follow from considering the first order term of the Z-factors in $\overline{\text{MS}}$ scheme and $\alpha = \frac{\alpha_0}{g^2} = -\frac{24(N^2-1)^2}{35Ng^2}$ and $\zeta = \frac{\zeta_0}{g^2} = \frac{9(N^2-1)}{13Ng^2}$ [4].

The ghost fields c, $\bar{c}$, ω, $\bar{\omega}$ give us just an overall factor. Now, to integrate over the φ and $\bar{\varphi}$ fields, we proceed as follows

$$\bar{\varphi}^{ab}_\mu = U^{ab}_\mu + iV^{ab}_\mu, \ \ \varphi^{ab}_\mu = U^{ab}_\mu - iV^{ab}_\mu. \tag{4.11}$$

Then

$$\begin{aligned}\int [\mathcal{D}U,V] \quad & e^{-\int d^dx [V^{ab}_\mu P^{ac,bd}_{\mu\nu} V^{cd}_\nu + U^{ab}_\mu P^{ac,bd}_{\mu\nu} U^{cd}_\nu - 2g\gamma^2 f^{abc} A^a_\mu U^{bc}_\mu]} = \\ & = \frac{1}{\det P^{ac,bd}_{\mu\nu}} e^{\int d^dx N g^2 \gamma^4 A^a_\mu P^{-1}_{\mu\nu} \delta^{ab} A^b_\nu},\end{aligned} \tag{4.12}$$

with

$$P^{ac,bd}_{\mu\nu} \equiv (\partial^2 - M^2)\delta^{ac}\delta^{bd}\delta_{\mu\nu}. \tag{4.13}$$

Therefore, the first contribution Γ_a to the effective potential is obtained by :

$$\Omega\Gamma_a = \ln\det P^{ac,bd}_{\mu\nu} = \mathrm{Tr}\ln P^{ac,bd}_{\mu\nu}, \tag{4.14}$$

resulting in

$$\begin{aligned}\Gamma_a &= (N^2-1)^2\left[-\frac{1}{\epsilon}\frac{M^4}{4\pi^2} + \frac{M^4}{8\pi^2}\ln\frac{M^2}{\bar{\mu}^2} - \frac{M^4}{8\pi^2}\right] \\ &= \frac{35Ng^2}{48}\frac{\langle\sigma_2'\rangle^2}{4\pi^2}\left(-\frac{1}{\epsilon} - \frac{1}{2} + \frac{1}{2}\ln\left(-\sqrt{\frac{35Ng^2}{48(N^2-1)^2}}\frac{\langle\sigma_2'\rangle}{\bar{\mu}^2}\right)\right) + \mathcal{O}(g^4),\end{aligned} \tag{4.15}$$

whereby $\bar{\mu}$ is the energy scale. The second contribution Γ_b to the effective potential comes from the gluon field A_μ. The quadratic part of the action containing A_μ is

$$e^{-\frac{1}{2}\int d^dx A^a_\mu R^{ab}_{\mu\nu} A^b_\nu} \tag{4.16}$$

where

$$R^{ab}_{\mu\nu} \equiv \delta^{ab}\left[\left(-\partial^2 + m^2 - \frac{2N\gamma^4 g^2}{\partial^2 - M^2}\right)\delta_{\mu\nu} - \partial_\mu\partial_\nu\left(\frac{1}{\alpha} - 1\right)\right]. \tag{4.17}$$

Therefore,

$$\Omega\Gamma_b = \frac{1}{2}\ln\det R^{ac,bd}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\ln R^{ac,bd}_{\mu\nu}, \tag{4.18}$$

resulting in

$$\begin{aligned}\Gamma_b = &-\frac{(N^2-1)}{2(4\pi)^2}\left(\frac{3}{\epsilon} + \frac{5}{4}\right)(m^4 - 4\gamma^4 g^2 N) + \frac{3(N^2-1)}{4(4\pi)^2}\left[x_1^2\ln\left(\frac{-x_1}{\bar{\mu}^2}\right) + x_2^2\ln\left(\frac{-x_2}{\bar{\mu}^2}\right)\right. \\ &\left. - M^4\ln\left(\frac{M^2}{\bar{\mu}^2}\right)\right],\end{aligned} \tag{4.19}$$

where x_1 and x_2 are the solutions of the equation $x^2+(M^2+m^2)x+M^2m^2+\lambda^4=0$,

$$x_1 = -\frac{1}{2}\left(m^2+M^2+\sqrt{(m^2-M^2)^2-4\lambda^4}\right),$$
$$x_2 = -\frac{1}{2}\left(m^2+M^2-\sqrt{(m^2-M^2)^2+-4\lambda^4}\right). \quad (4.20)$$

The third part of effective potential Γ_c is from the Gribov-Zwanziger action,

$$\Gamma_c = -d\gamma_0^4(N^2-1). \quad (4.21)$$

Knowing that $Z_{\gamma^2} = Z_g^{-1/2} Z_A^{-1/4}$, we get

$$\gamma_0^4 = Z_{\gamma^2}^2\gamma^4, \quad \text{with} \quad Z_{\gamma^2}^2 = 1+\frac{3}{2}\frac{g^2N}{16\pi^2\epsilon}, \quad (4.22)$$

hence

$$\Gamma_c = (N^2-1)\gamma^4\left(-4+\frac{3Ng^2}{32\pi^2}-\frac{3Ng^2}{8\pi^2\epsilon}\right)+\mathcal{O}(g^4). \quad (4.23)$$

And the last contribution comes from $\langle\sigma_1'\rangle^2$ and $\langle\sigma_2'\rangle^2$:

$$\Gamma_d = \frac{1}{2}\left(\frac{1}{\tilde{Z}_\zeta}\langle\sigma_1'\rangle^2-\frac{1}{\tilde{Z}_\alpha}\langle\sigma_2'\rangle^2\right)$$
$$= \frac{\langle\sigma_1'\rangle^2}{2}-\frac{\langle\sigma_2'\rangle^2}{2}+\frac{13}{6}\frac{Ng^2}{16\pi^2\epsilon}\langle\sigma_1'\rangle^2+\frac{35}{12}\frac{Ng^2}{16\pi^2\epsilon}\langle\sigma_2'\rangle^2+\mathcal{O}(g^4). \quad (4.24)$$

The full effective potential given by $\Gamma = \Gamma_a+\Gamma_b+\Gamma_c+\Gamma_d$ is finite when $\epsilon\to 0$ at first order in g^2. Therefore, it can be written as

$$\Gamma(m^2,M^2,\lambda^4) = \frac{9(N^2-1)}{13Ng^2}\frac{m^4}{2}-\frac{48(N^2-1)^2}{35Ng^2}\frac{M^4}{2}-\frac{2\lambda^4(N^2-1)}{Ng^2}$$
$$-\frac{N^2-1}{16\pi^2}\left\{-2\lambda^4+\frac{5}{8}m^4+2(N^2-1)M^4-\left(2(N^2-1)-\frac{3}{4}\right)\ln\left(\frac{M^2}{\bar{\mu}^2}\right)M^4\right\}$$
$$+\frac{3}{8}\frac{N^2-1}{16\pi^2}\left\{m^4+M^4-2\lambda^4+(m^2+M^2)\sqrt{(m^2-M^2)^2-4\lambda^4}\right\}$$
$$\times\ln\left[\frac{1}{2\bar{\mu}^2}\left(m^2+M^2+\sqrt{(m^2-M^2)^2-4\lambda^4}\right)\right]$$
$$+\frac{3}{8}\frac{N^2-1}{16\pi^2}\left\{m^4+M^4-2\lambda^4-(m^2+M^2)\sqrt{(m^2-M^2)^2-4\lambda^4}\right\}$$
$$\times\ln\left[\frac{1}{2\bar{\mu}^2}\left(m^2+M^2-\sqrt{(m^2-M^2)^2-4\lambda^4}\right)\right]. \quad (4.25)$$

with

$$m^2 = \sqrt{\frac{13Ng^2}{9(N^2-1)}}\langle\sigma_1'\rangle, \quad (4.26)$$

$$M^2 = -\sqrt{\frac{35Ng^2}{48(N^2-1)^2}}\langle\sigma_2'\rangle. \quad (4.27)$$

and $\lambda^4 \equiv 2Ng^2\gamma^4$. The next step is to analyze the gap equations given by

$$\frac{\partial\Gamma}{\partial M^2} = 0, \qquad \frac{\partial\Gamma}{\partial m^2} = 0, \qquad \frac{\partial\Gamma}{\partial\lambda^4} = 0. \tag{4.28}$$

Unfortunately, an acceptable result was not obtained in this particular scheme. The resolution was to not fix a scheme, we rather rewrote the effective potential in a general scheme as has been done in e.g. [13]. Details of this procedure will be published elsewhere. The effective potential (4.25), in general scheme, becomes

$$\begin{aligned}\Gamma_{gen}(m^2,M^2,\lambda^4,b_0) = &\frac{9(N^2-1)}{26Ng^2}m^4 - \frac{24(N^2-1)^2}{35Ng^2}M^4 - \frac{2(N^2-1)^2M^4}{16\pi^2}\left(1-\ln\left(\frac{M^2}{\bar{\mu}^2}\right)\right)\\ &-2\lambda^4\frac{N^2-1}{Ng^2} - 2\lambda^4\frac{N^2-1}{16\pi^2}(b_0-1)\\ &+\frac{3}{4}\frac{N^2-1}{16\pi^2}\left\{\frac{5}{4}(m^4+M^4-2\lambda^4) - \frac{m^2+M^{-}2\lambda^4}{2}\ln\left[\frac{m^2M^2+\lambda^4}{\bar{\mu}^4}\right]\right.\\ &\left.+(m^2+M^2)\sqrt{4\lambda^4-(m^2-M^2)^2}\arctan\left[\frac{\sqrt{4\lambda^4-(m^2-M^2)^2}}{m^2+M^2}\right] + \ln\left[\frac{M^2}{\bar{\mu}^2}\right]M^4\right\},\end{aligned} \tag{4.29}$$

b_0 being a parameter related to the chosen scheme for the coupling. It was fixed, at the end, by the matching our values for the complex conjugate poles masses of the transverse gluon propagator to those estimated from lattice data [1] when the gap equations are solved for. The effective masses m^2 and M^2 and the Gribov parameter γ^2 were gotten in function of the parameter b_0 and $\bar{\mu}$ in units $\Lambda = 1$, $N = 3$ and also with $N = 2$. Notice that these poles masses are gauge and scheme independent [6], so we benefitted from this to fix the parameters b_0 and $\bar{\mu}$ by using a minimal external lattice input to determine the "optimum scheme". We got $b_0 = -3.643$ and $\bar{\mu} = 1.429$. With this procedure, we obtained a reasonable value for the coupling constant, namely 0.382. Therefore, in this case, the perturbative result is relatively trustworthy. The Gribov parameter γ^2 is 0.637 and the vacuum energy is −26.955. The Hessian determinant is positive and also the second derivatives,

$$\left.\frac{\partial^2\Gamma_{gen}}{\partial M^{2^2}}\right|_{solved} = 1.668 \qquad \left.\frac{\partial^2\Gamma_{gen}}{\partial m^{2^2}}\right|_{solved} = 0.216 \qquad \left.\frac{\partial^2\Gamma_{gen}}{\partial M^2\partial m^2}\right|_{solved} = 0.011. \tag{4.30}$$

Then, the solution does correspond to a minimum.

The future step will be to extend this research to finite temperatures and to study if the deconfinement transitions reflects itself in a change in the propagator behavior and to check if, with the Polyakov loop added to the game, we observe the transition also in that order parameter.

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