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Confinement, Instanton-dyons and Monopoles

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Confinement in QCD vacuum has been traditionally explained in terms of monopoles, and now quark-gluon plasma produced in heavy ion collisions is described as a dual plasma containing as quasiparticles not only quarks and gluons but also magnetic monopoles, dominating the ensemble near T_c. Chiral symmetry breaking was traditionally described in terms of instantons. At finite temperatures the nonzero Polyakov line VEV splits them into instanton-dyons. The semiclassical ensemble of those was shown to describe well both the deconfinement and chiral phase transitions. Furthermore, the semiclassical theory describes multiple phase transitions of Roberge-Weiss type, when periodicity phases of fermions cross the holonomy phases. The interrelation of monopoles and instanton-dyons is explained in terms of the so called Poisson duality between them: both describe the same nonperturbative phenomena and lead to the same partition function. However the QCD monopoles have action $S \sim log(1/g^2)$ and thus are not classical fields.

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1. Introduction

1.1 Monopoles in QGP

Discovery of 't Hooft-Polyakov monopoles in Georgi-Glashow model has led to significant progress in theories with extended supersymmetry, posessing adjoint scalars. But lattice studies revealed monopoles also in pure gauge theories. Among their applications is the famous "dual superconductor" model, explaining *confinement* by Bose-Einstein condensation of monopoles, at $T < T_c$. There are more recent applications to physics of quark-gluon plasma and heavy ion collisions, a *"dual plasma"* made of electrically charged quasiparticles, quarks and gluons, and magnetically charged monopoles.

Some of those were presented in the previous conferences and, due to space restriction, we will just enumerate few latest works. Extensive numerical study of monopole Bose-Einstein condensation in the Bose Coulomb gas have been made in [1]. In this paper the QCD thermodynamics and and correlation functions known from the lattice were very accurately reproduced. Chiral symmetry breaking in the monopole language has been demonstrated in [2]. Rather detailed and successful study of the monopole contribution to jet quenching in heavy ion collisions has been done in refs. [3, 4].

1.2 Semiclassical theory at finite T: the instanton-dyons

Historically, discussion of chiral symmetry breaking started from Nambu-Jona-Lasinio (NJL) model [5], in which hypothetical strong attraction between fermions have been introduced. Based on analogy to theory of superconductivity, it explained formation of effective quark masses and massless pions. Two decades later the instanton liquid model (ILM) [6] have identified the non-perturbative interaction with the instanton-induced 't Hooft Lagrangian. Unlike the 4-fermion interaction in the NJL model, it was not always an attractive thus explicitly violating the $U(1)_a$ symmetry. The ILM also proposed another view on the chiral symmetry breaking, related with *collectivization* of the topological fermionic zero modes into the so called zero mode zone (ZMZ). Multiple numerical simulations of the ensemble of interacting instantons were done, for a review see [7], which were able to reproduce point-to-point correlation functions corresponding to different mesons and baryons, known from phenomenology and lattice studies.

For the instanton-dyons the 't Hooft -Polyakov solution is used with the time component of the gauge field A_4 as an adjoint scalar. The semiclassical theory built on them obviously can only be used in the Euclidean time formulation: an analytic continuation of A_4 to Minkowski time include an imaginary field which makes no sense. So, the instanton-dyons cannot be used as quasiparticles. And yet, the presence of magnetic charge of the instanton-dyons does suggest, that they should *somehow* be related to particle-monopoles.

Relating these developments to confinement, it was shown in Ref. [8, 9] that the nonzero holonomy (or the average Polyakov loop which is the confinement order parameter) can split instantons into N_c constituents, known as the instanton-dyons or instanton-monopoles. Henceforth we refer to them as dyons for simplicity. Ensembles of dyons via mean-field methods were studied analytically [10, 11, 12, 13] as well as numerically [14, 15, 16].

These works could reproduce both the deconfinement and chiral phase transitions occurring at same temperature in QCD, and also explain extra phase transitions in beyond QCD theories with modified quark periodicity phases by “jumps" of the zero modes, from one type of dyon to another one. A very symmetric arrangement of the quark phases for $N_c = N_f$ QCD was proposed by [30], called $Z_{N_c}QCD$. It is a “democratic" distribution of those, so that each type of the dyons have a zero mode for one quark flavor. Such $Z_{N_c}QCD$ has been studied in the mean field framework [13], by statistical simulations [16] and also by lattice simulations [31]. In the dilute limit it also has been studied by [32]. As is shown in Fig.1, Z_2QCD has strong first order deconfinement transition, while chiral restoration never happens!

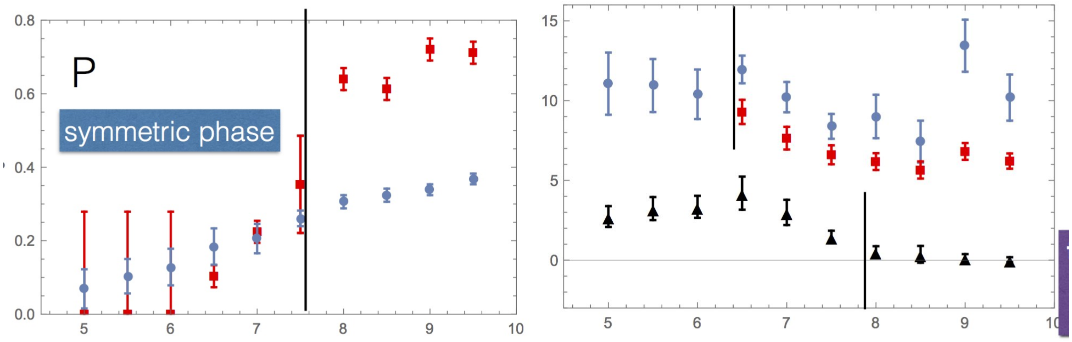

Figure 1: (left) The mean Polyakov line P versus the action parameter $S = 8\pi^2/g^2(T) \sim log(T)$. Red squares are for Z_2QCD while blue circles are for the usual QCD with quarks as fermions, both with $N_c = N_f = 2$. (right). The quark condensate versus the density parameter S. Black triangles correspond to the usual QCD: and they display chiral symmetry restoration. Blue and red poins are for two flavor condensates of the Z_2QCD: to the left of vertical line there is a “symmetric phase" in which both types of dyons and condensates are the same. Note that there is no tendency to chiral symmetry restoration even at high T.

1.3 Instanton-dyons in lattice QCD

Whether the resulting semiclassical theory can indeed provide accurate description of these phenomena, can be investigated only through first principles lattice gauge theory techniques. One of the early studies of the kind was done by Gattringer [17] who used QCD Dirac operator with two different temporal periodicity conditions on $SU(3)$ pure gauge configurations as a tool to locate dyons. These results reported the existence of dyon in gauge theories without fermions. Further lattice studies have been performed by Mueller-Preussker, Ilgenfritz and collaborators [18, 19], along similar lines. Clusters of local topological fluctuations were identified using a local definition of topological charge, from the eigenvectors of the valence Dirac operator with generalized periodicity conditions. Observed correlation between the topological clusters and local eigenvalues of the Polyakov loop provided further evidence for the presence of dyons in QCD vacuum.

This section is based on ongoing work by R.Larsen, S.Sharma and myself, which in turn uses gauge fields of 2+1 flavor QCD with domain wall fermions on lattices of size $32^3 \times 8$, at $T = 1.0, 1.08T_c$ generated by the RBC-LLNL collaboration [20]. We use the so called overlap Dirac operator [21, 22], a particular realization of fermions on the lattice that has exact chiral invariance, and therefore an exact index theorem [23] and configuration with unit topological charge $|Q| = 1$.

The main result of this study is direct demonstration that the topology responsible for quark condensate can indeed be very accurately described in terms of instanton-dyons. Unlike in earlier works, there are no more any “unidentified topological clusters". In Fig.. 2 we show some case of well separated and strongly overlapping dyons.

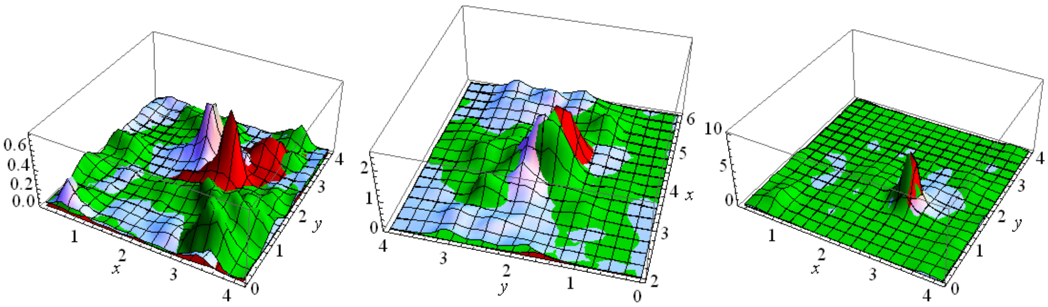

Figure 2: Wavefunction density $\rho(x,y)$ of the overlap fermion zero mode of three statistically independent QCD configurations at T_c (left panel) and at $T = 1.08T_c$ (middle and right panels). The three different colors represent the zero-modes at there different temporal periodicity phases $\phi = \pi$ (red), $\phi = \pi/3$ (blue), $\phi = -\pi/3$ (green) respectively. The peak height has been normalized such that the $\phi = \pi$ case has unit density. The x and y coordinates are in the units of $1/T$.

In the case of well separated dyons comparison with these analytic formulae is straightforward and successful. But even in strongly overlapping case, e.g. depicted in the right panel of Fig. 2, one can show that it is not a caloron with zero holonomy but 3 dyons, and accurately fix their locations. General expressions for the zero mode density provide remarkably accurate description of the shape of zero modes in all cases studied.

Another example I would like to show is Fig.??. In just single configuration, one finds that L-type dyons (possessing zero mode at fermionic phase $\phi = \pi$) have only localized $L\bar{L}$ molecules and no quark condensate, while for somewhat more numerous M dyons at the phase $\phi = \pi/3$ the zero mode is delocalized and shared between multiple dyons. It means that chiral phase transition happens at different T in those sectors.

2. Two theories, one answer: the Poisson duality

A gradual understanding of this statement began some time ago, but remained rather unnoticed

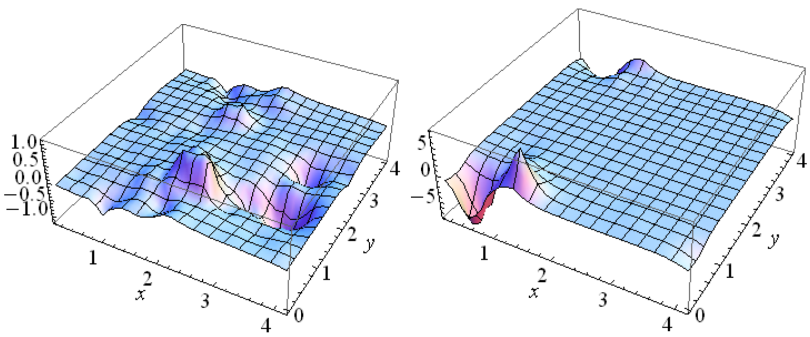

Figure 3: Chiral density $\rho_5(x,y)$ of the first near-zero mode for two different quark periodicity phases $\phi = \pi/3, \pi$, at the same QCD configuration at $T = 1.08\ T_c$ shown earlier in the mid-panel of Fig. 1..

by the larger community. One reason for that was the setting in which it was shown, which was based on extended supersymmetry. Only in these cases was one able to derive reliably *both* partition functions – in terms of monopoles and instanton-dyons – and show them to be equal [24, 25, 26]. Furthermore, they were not summed up to an analytic answer, but shown instead to be related by the so-called "Poisson duality."

2.1 Rotator

Another classic example, which display features important for physics to be discussed in this book, is a rotating object, which we will call the *rotator* or *the top*. What is special in this case is that the coordinates describing its location are *angles*, which are always defined with some natural periodicity conditions. Definition of the path integrals in such cases require important additional features[1].

The key questions and solutions can be explained following Schulman [27] using the simplest $SO(2)$ top, a particle moving on a circle. Its location is defined by the angle $\alpha \in [0, 2\pi]$ and its (initial) action contains only the kinetic term

$$S = \oint dt \frac{\Lambda}{2} \dot{\alpha}^2 \tag{2.1}$$

with $\Lambda = mR^2$ the corresponding moment of inertia for rotation.

All possible paths are naturally split into topological homotopy classes, defined by their *winding number*. The paths belonging to different classes cannot be continuously deformed to each other. Therefore a fundamental question arises: *How should one normalize those disjoint path integrals over classes of paths?* Clearly, there is no natural way to define their relative normalization, or rather their relative *phase*.

Following Aharonov and Bohm [28] one may provide a direct physical interpretation of this setting. Suppose our particle has an Abelian electric charge, and certain device (existing in extra dimensions invisible to the rotator) creates a nonzero magnetic field flux $\Phi \neq 0$ through the circle. Stokes theorem relates it to the circulation of the gauge field

$$\oint d\alpha A_\alpha = \int \vec{B} d\vec{S}$$

While $A_\mu(x)$ is gauge-dependent, its circulation (called holonomy) is gauge invariant, since it is related to the field flux[2].

The extra phase is thus physical. Furthermore, it propagates into the energy spectra and the partition function. One can write it in a Hamiltonian way, as the sum over states with the angular momentum m at temperature T

$$Z_1 = \sum_{m=-\infty}^{\infty} \exp\left(-\frac{m^2}{2\Lambda T} + im\omega\right), \tag{2.2}$$

[1] In the Feynman-Hibbs book does not have its discussion, and contains only a comment that the authors cannot describe, say, an electron with spin $1/2$, and that it was a "serious limitation" of the approach.

[2] For non-Abelian case there is no Stokes theorem, but gauge invariance of all closed paths is still true: it follows from direct calculation of gauge transformation of path-ordered-exponents .

where ω is the holonomy phase, which is so far arbitrary.

Although physical, the effect is invisible at the classical level. This can be seen from the inclusion of the additional term in the action $\sim (\omega/2\pi)\int d\tau\dot{\alpha}$ which would "explain" the holonomy phase. This term in Lagrangian however is a full derivative, $\dot{\alpha}$, so the action depends on the endpoints of the paths only, and is insensitive to its smooth deformations. It therefore generates no contribution to classical equations of motion, thus failing to "exert any force" on the particle in classical sense. In summary, an appearance of the holonomy phase is our first nontrivial quantum effect, not coming from the classical action.

Now one can also use Lagrangian approach, looking for paths periodic in Euclidean time on the Matsubara circle. Classes of paths which make a different number n of rotations around the original circle can be defined as "straight" classical periodic paths

$$\alpha_n(\tau) = 2\pi n \frac{\tau}{\beta}\,, \tag{2.3}$$

plus small fluctuations around them. Carrying out a Gaussian integral over them leads to the following partition function,

$$Z_2 = \sum_{n=-\infty}^{\infty} \sqrt{2\pi\Lambda T}\,\exp\left(-\frac{T\Lambda}{2}(2\pi n-\omega)^2\right). \tag{2.4}$$

The key point here is that these quantum numbers, m used for Z_1 and n for Z_2, are very different in nature. The dependence on the temperature is different. Also, for Z_1 each term of the sum is periodic in ω, while for Z_2, this property is also true, but recovered only after summation over n.

In spite of such differences, both expressions are in fact the same! In this toy model, it is possible to do the sums numerically and plot the results. Furthermore, one can also derive the analytic expressions, expressible in terms of the elliptic theta function of the third kind

$$Z_1 = Z_2 = \theta_3\left(-\frac{\omega}{2},\exp\left(-\frac{1}{2\Lambda T}\right)\right), \tag{2.5}$$

which is plotted in Fig. 4 for few values of the temperature T.

In order to prove that one may use the Jacobi identity

$$\theta_3(z,t) = (-it)^{-1/2}e^{z^2/i\pi t}\theta_3(z/t,-1/t)$$

As emphasized by our recent work [29], one can observe that two statistical sums are related by the Poisson summation formula, in a form

$$\sum_{n=-\infty}^{\infty} f(\omega+nP) = \sum_{l=-\infty}^{\infty}\frac{1}{P}\tilde{f}\left(\frac{l}{P}\right)e^{i2\pi l\omega/P}\,, \tag{2.6}$$

where $f(x)$ is some function, $\tilde{f}$ is its Fourier transform, and P is the period of both sums as a function of the "phase" ω. In this particular example the function is Gaussian, with Fourier transform being a periodic Gaussian: but we will later encounter examples of the Poisson duality with other functions as well.

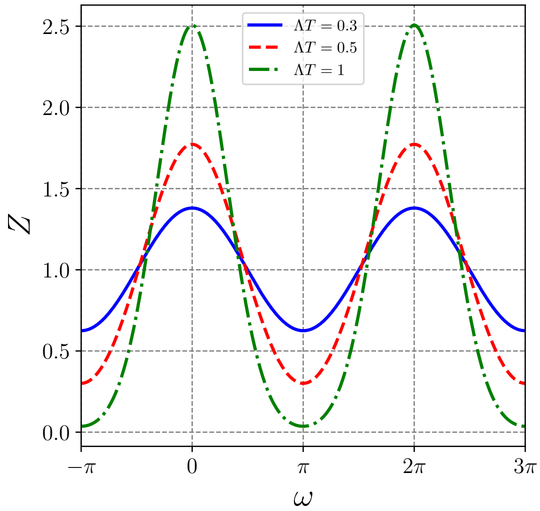

Figure 4: The partition function Z of the rotator as a function of the external Aharonov-Bohm phase ω (two periods are shown to emphasize its periodicity). The (blue) solid, (red) dashed and (green) dash-dotted curves are for $\Lambda T = 0.3, 0.5, 1$.

2.2 Poisson duality in $\mathcal{N}$=4 theory

The SU(2) monopole has four collective coordinates, three of which are related with translational symmetry and location in space, while the fourth is rotation around the τ^3 color direction,

$$\hat{\Omega} = \exp(i\alpha\hat{\tau}^3/2)\,. \tag{2.7}$$

Note that such rotation leaves unchanged the presumed VEVs of the Higgses and holonomies, as well as the Abelian $A^3_\mu \sim 1/r$ tails of the monopole solution. Nevertheless, these rotations are meaningful because they do rotate the monopole core – made up of non-Abelian A^1_μ, A^2_μ fields – nontrivially. It is this rotation in the angle α that makes the monopole problem similar to a quantum rotator. As was explained by Julia and Zee [33], the corresponding integer angular momentum is nothing but the electric charge of the rotating monopole, denoted by q.

Now that we understand the monopoles and their rotated states, one can define the partition function at certain temperature, which (anticipating the next sections) we will call $T \equiv 1/\beta$,

$$Z_{mono} = \sum_{k=1}^{\infty}\sum_{q=-\infty}^{\infty}\left(\frac{\beta}{g^2}\right)^8 \frac{k^{11/2}}{\beta^{3/2}M^{5/2}} \exp\left(ik\sigma - iq\omega - \beta kM - \frac{\beta\phi^2 q^2}{2kM}\right), \tag{2.8}$$

where k is the magnetic charge of the monopole. The derivation can be found in the original paper, and we only comment that the temperature in the exponent only appears twice, in the denominators of the mass and the rotation terms, as expected. The two other terms in the exponent, $\exp(ik\sigma - iq\omega)$, are the only places where holonomies appear, as the phases picked up by magnetic and electric charges over the circle.

Now we derive an alternative 4d version of the theory, in which we will look at gauge field configurations in all coordinates including the compactified “time coordinate" τ. These objects are versions of instantons, split by a nonzero holonomy into instanton constituents. Since these gauge field configurations need to be periodic on the circle, and this condition can be satisfied by paths adding arbitrary number n of rotations, their actions are

$$S^n_{mono} = \left(\frac{4\pi}{g^2}\right)\left(\beta^2|\phi|^2 + |\omega - 2\pi n|^2\right)^{\frac{1}{2}}, \tag{2.9}$$

including the contribution from the scalar VEV ϕ, the electric holonomy ω, and the winding number of the path n. In the absence of the holonomies, the first term would be M/T as one would expect.

The partition function then takes the form [34]

$$Z_{inst} = \sum_{k=1}^{\infty}\sum_{n=-\infty}^{\infty}\left(\frac{\beta}{g^2}\right)^9 \frac{k^6}{(\beta M)^3} \times \exp\left(ik\sigma - \beta k M - \frac{kM}{2\phi^2\beta}(\omega - 2\pi n)^2\right), \tag{2.10}$$

where $M = (4\pi\phi/g^2)$, the BPS monopole mass without holonomies; thus the second term in the exponent is interpreted as just the Boltzmann factor. The “temperature" appears in the unusual place in the last term (like for the rotator toy model). The actions of the instantons are large at high-T (small circumference β); the semiclassical instanton theory works best at high-T.

The Poisson duality relation between these two partition functions, Eqs. (2.8) and (2.10), was originally pointed out by Dorey and collaborators [34]. In this book, following [29], it was explained earlier using the toy model of a *quantum rotator*. In fact the Poisson duality relation between two sums is in this case exactly the same.

2.3 Poisson duality in QCD

The authors of [29] went further, performing the Poisson duality transformation over the semiclassical sum over twisted instanton-dyons. While we will derive it later, in chapter **??**, and here just present the resulting expression for the semiclassical partition function as

$$Z_{inst} = \sum_n e^{-\left(\frac{4\pi}{g_0^2}\right)|2\pi n - \omega|} \tag{2.11}$$

It is periodic in the holonomy, as it should be. Note that, unlike in Eq. (2.10), it has a modulus rather than a square of the corresponding expression in the exponent. This is due to the fact that the sizes of L_n and their masses are all defined by the same combination $|2\pi n - \omega|T$ and therefore the moment of inertia $\Lambda \sim 1/|2\pi n\beta - v|$.

Using the general Poisson relation, Eq. (2.6), the Fourier transform of the corresponding function appearing in the sum in Eq. (2.11) reads

$$F\left(e^{-A|x|}\right) \equiv \int_{-\infty}^{\infty} dx\, e^{i2\pi\nu x - A|x|} = \frac{2A}{A^2 + (2\pi\nu)^2}, \tag{2.12}$$

and therefore the monopole partition function is

$$Z_{mono} \sim \sum_{q=-\infty}^{\infty} e^{iq\omega - S(q)}, \tag{2.13}$$

where

$$S(q) = \log\left(\left(\frac{4\pi}{g_0^2}\right)^2 + q^2\right) \approx 2\log\left(\frac{4\pi}{g_0^2}\right) + q^2\left(\frac{g_0^2}{4\pi}\right)^2 + \ldots, \tag{2.14}$$

where the last equality is for $q \ll 4\pi/g_0^2$. The resulting partition function can be interpreted as being generated by *moving and rotating monopoles*. The results are a bit surprising. First, the action of a monopole, although still formally large in weak coupling, is only a logarithm of the semiclassical parameter; these monopoles are therefore quite light. Second is the issue of monopole rotation. The very presence of an object that admits rotational states implies that the monopole core is not spherically symmetric. The Poisson-rewritten partition function has demonstrated that the rotating monopoles are *not* the rigid rotators, because their action, Eq. (2.14), depends on the angular momentum q and is quadratic only for small values of q. The slow (logarithmic) increase of the action with q implies that the dyons are in fact shrinking with increased rotation. In the moment of inertia, this shrinkage is more important than the growth in the mass, as the size appears quadratically. As strange as it sounds, it reflects on the corresponding behavior of the instanton-dyons L_n with the increasing n.

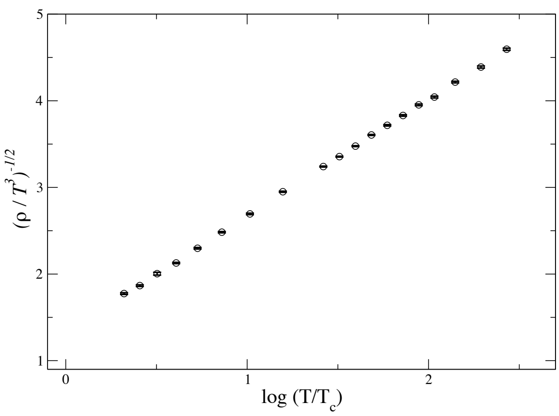

Figure 5: The normalized monopole density in SU(2) gauge theory in power -1/2, $(\rho/T^3)^{-1/2}$ versus $log(T/T_c)$ shows an apparent linear dependence. (From [35].)

3. Summary

Let me start with the main conclusions of this talk. Two nonperturbative theories describing finite-T QCD, based on monopoles and instanton-dyons, do describe *the same physics*, and produce the same partition function. They are just so to say Hamiltonian and Lagrangian (or Minkowskian and Euclidean) approaches.

The second important consequence of these studies is the realization of the fact that *In QCD-like theories without adjoint scalars, monopoles are not classical objects.* While it came with some surprise, the evidences for that where in front of us for a long time. In particular, it has been demonstrated rather clearly by [35] that their density is *not* power of T, but only a power of its log. The monopole action is

$$S_{mono} \sim log(1/g^2) \sim log(log(T))$$

.

These two approaches should be used, one or the other, depending on the problem. Monopoles are quasiparticles, and thus can be used outside of Euclidean formulation in out-of-equilibrium settings. Doing path integral Monte Carlo with monopoles is hard, but possible. The theory based on instanton dyons in purely Euclidean construction, but it is much simpler technically. Its semiclassical nature allows for systematic improvement of its accuracy.

One more important conclusion is that improved chiral fermions on the lattice allow to see that the instanton-dyons constitute complete and rather accurate description of the lowest Dirac eigenstates, responsible for chiral symmetry breaking.

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