PROCEEDINGS OF SCIENCE

Compact Flavour-Exotic Tetraquarks in Large-N_c QCD — To Be or Not to Be?

Wolfgang Lucha
Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria
E-mail: `Wolfgang.Lucha@oeaw.ac.at`

Dmitri Melikhov[*]
Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria, and
D. V. Skobeltsyn Institute of Nuclear Physics, M. V. Lomonosov Moscow State University, 119991, Moscow, Russia, and
Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria
E-mail: `dmitri_melikhov@gmx.de`

Hagop Sazdjian
Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, Université Paris-Saclay, 91405 Orsay Cedex, France
E-mail: `sazdjian@ipno.in2p3.fr`

We analyse exotic tetraquark mesons — bound states formed by two quarks and two antiquarks — for the specific case of four different quark flavours within the framework of a well-defined limit of quantum chromodynamics characterized by the correlated growth without bound of the number of colour degrees of freedom and approach to zero of the coupling constant of the strong interactions. On the one hand, the assumption that the tetraquarks of the kind defined above show up, as poles in the amplitudes for the scattering of two ordinary mesons with flavour quantum numbers that match those of these tetraquarks, already at lowest possible order in an expansion in inverse powers of the number of colours implies the existence of two types of flavour-exotic tetraquarks, distinguishable by their dominant transitions to two ordinary mesons. On the other hand, we are aware of merely a single, unique colour-singlet arrangement of two quarks and two antiquarks capable of tolerating a compact flavour-exotic tetraquark, a bound state of colour-antisymmetric diquark and antidiquark. In view of these two clearly contradictory observations, we are led to the plausible conclusion that, within the considered limiting case of quantum chromodynamics, a conceivable explanation of the riddle might consist in the non-existence of any flavour-exotic tetraquarks in form of narrow states.

XIII Quark Confinement and the Hadron Spectrum — Confinement2018
31 July – 6 August 2018
Maynooth University, Ireland

[*]Speaker.

PoS(Confinement2018)087

1. Identifying Tetraquark-Phile Feynman Diagram Contributions in Large-N_c Limit

Recently, we embarked on a systematic study [1–3] of basic qualitative properties of tetraquark mesons, $T = (\bar{q}_a\, q_b\, \bar{q}_c\, q_d)$, regarded as bound states of two quarks q_b, q_d and two antiquarks $\bar{q}_a, \bar{q}_c$ of flavour quantum numbers $a,b,c,d \in \{u,d,s,c,b\}$ and masses m_a, m_b, m_c, m_d, respectively, by trying to utilize tetraquark appearances in the scattering of two ordinary mesons, of momenta p_1 and p_2, to two ordinary mesons, of momenta p'_1 and p'_2. Suppressing all aspects of spin and parity, we adopt as interpolating operator of a given ordinary meson $M_{\bar{a}b}$ some quark-bilinear current $j_{\bar{a}b} \equiv \bar{q}_a\, q_b$ whose vacuum–meson matrix element defines the leptonic decay constant of this meson, $f_{M_{\bar{a}b}}$, according to

$$\langle 0|\, j_{\bar{a}b}\, |M_{\bar{a}b}\rangle \equiv f_{M_{\bar{a}b}} \neq 0 \ . \tag{1.1}$$

Let us confine ourselves to *compact* tetraquarks, tightly bound (in contrast to molecular-type) states.

For these investigations [1–3], our *tool* of choice is a particular limiting case of a generalization of quantum chromodynamics (QCD) to an *arbitrary* number N_c of colour degrees of freedom. QCD is a gauge theory relying on SU(3), with all quarks in its 3-dimensional fundamental representation. Large-N_c QCD [4,5] is a theory invariant under transformations of the gauge group SU(N_c), defined by a related approach of N_c to infinity and its strong coupling g_s or fine-structure constant α_s to zero,

$$g_s \propto \frac{1}{\sqrt{N_c}} \xrightarrow[N_c\to\infty]{} 0 \qquad \Longleftrightarrow \qquad \alpha_s \equiv \frac{g_s^2}{4\pi} \propto \frac{1}{N_c} \xrightarrow[N_c\to\infty]{} 0 \ ,$$

where all quarks transform according to the N_c-dimensional fundamental representation of SU(N_c). Compared with QCD, its large-N_c limit exhibits a considerably reduced complexity. Among others, it immediately predicts, as the large-N_c behaviour of an ordinary-meson leptonic decay constant [5],

$$f_{M_{\bar{a}b}} \propto \sqrt{N_c} \qquad \text{for} \quad N_c \to \infty \ .$$

Instigated by Ref. [6], various aspects of tetraquarks have been discussed along similar lines [7–10].

Our actual *targets* for the application of the limit $N_c \to \infty$ (and a $1/N_c$ expansion thereabout) are appropriate four-current Green functions. From these, we derive the amplitudes for the scattering of two ordinary mesons into two ordinary mesons, both pairs of mesons carrying, of course, the flavour quantum numbers of any tetraquark in the focus of our interest. These scattering amplitudes we then inspect for the presence of a pole betraying the existence of a compact-tetraquark intermediate state.

From our point of view, our first *task* in this enterprise is the formulation of rigorous criteria for the selection of those Feynman diagrams that offer the perspective of exhibiting singularities related to four-quark intermediate states that eventually can contribute to the formation of a tetraquark pole. Feynman diagrams conforming to this requirement are called tetraquark-phile [11–14]. We propose two necessary, but not sufficient, consistency criteria expressed in terms of the Mandelstam variable

$$s \equiv (p_1 + p_2)^2 = (p'_1 + p'_2)^2 \ .$$

1. A *tetraquark-phile* Feynman diagram must depend *nontrivially*, *viz.*, *non-polynomially*, on s.
2. A *tetraquark-phile* Feynman diagram must support an adequate four-quark intermediate state and develop a corresponding branch cut, starting at a branch point[1] $\hat{s} = (m_a + m_b + m_c + m_d)^2$.

[1] For each Feynman diagram under consideration, the existence or non-existence of such a singularity can be decided straightforwardly by means of the Landau equations [15]. See, *e.g.*, Ref. [2, App. A] for a variety of illustrative examples.

PoS(Confinement2018)087

2. *Flavour-Exotic* Tetraquarks: Pairwise Appearance in Ordinary-Meson Scattering

Next, we should specify the quark-flavour composition of the compact tetraquarks we set out to study. A list of conceivable flavour combinations in tetraquarks can be found in Table 1 of Ref. [14]. Here, we adhere to the genuinely exotic case of the flavours of all four (anti-)quarks being disparate.

2.1 Two Categories of Scattering Processes Exhibiting Unlike Behaviour in the Limit $N_c \to \infty$

The possible end-product of any scattering of two mesons $M_{\bar a b}$ and $M_{\bar c d}$ to two ordinary mesons might be either the same two mesons, $M_{\bar a b}$ and $M_{\bar c d}$, or two different mesons, $M_{\bar a d}$ and $M_{\bar c b}$, resulting from a redistribution of the available quark flavours. Within the envisaged quest for tetraquark poles in scattering amplitudes, we hence have to take into account two disjoint classes of Green functions:

$$\text{flavour-preserving correlators} \qquad \langle T(j_{\bar a b}\, j_{\bar c d}\, j^\dagger_{\bar a b}\, j^\dagger_{\bar c d})\rangle\,, \qquad \langle T(j_{\bar a d}\, j_{\bar c b}\, j^\dagger_{\bar a d}\, j^\dagger_{\bar c b})\rangle\,; \tag{2.1a}$$

$$\text{flavour-reshuffling correlators} \qquad \langle T(j_{\bar a d}\, j_{\bar c b}\, j^\dagger_{\bar a b}\, j^\dagger_{\bar c d})\rangle\,. \tag{2.1b}$$

The actual large-N_c dependence of the tetraquark-phile contributions (identified by a subscript T) to these Green functions at leading order in their series expansion in powers of $1/N_c$ [1] can be read off from (of course, N_c-subleading) Feynman diagrams of the type illustrated by the examples in Fig. 1.

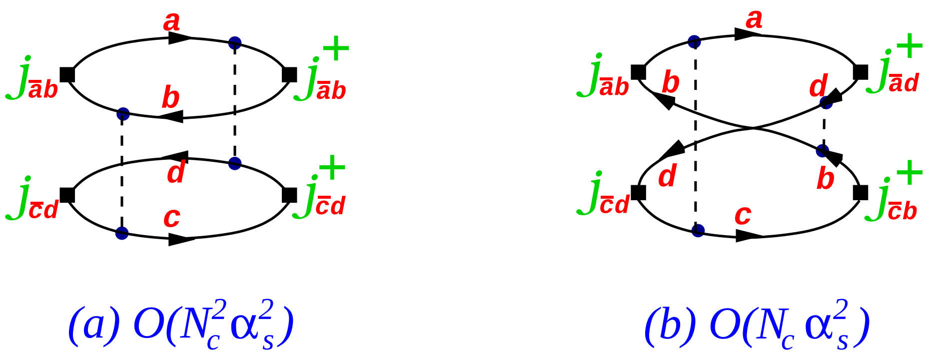

Figure 1: Typical representatives of N_c-leading tetraquark-phile Feynman diagrams potentially developing a flavour-exotic tetraquark pole in either flavour-preserving (a) or flavour-reshuffling (b) scattering amplitudes.

From the topological point of view, the discussion of tetraquark-phile correlators is presumably facilitated if imagining any encountered Feynman diagram as residing on a cylinder [3] with surface spanned or even bordered by quark lines. Let us now look at our two classes of scattering processes.

2.2 Flavour-Preserving Scattering at Leading Tetraquark-Phile Order of its $1/N_c$ Expansion

In the flavour-preserving channel, the N_c-leading Feynman diagrams (Fig. 2) involve two gluon exchanges, *i.e.*, four quark–gluon vertices, equivalent to perturbative order α_s^2, and two colour loops (contributing a factor N_c^2): the resulting N_c dependence of such a contribution is $O(\alpha_s^2 N_c^2) = O(N_c^0)$. The addition of a further gluon amounts to two additional quark–gluon vertices, raising the power of α_s by one: embedding this gluon in the cylinder surface increases the number of colour loops by one (providing a further factor N_c) and implies the earlier N_c dependence $O(\alpha_s^3 N_c^3) = O(N_c^0)$ [Fig. 3(a)]; if, however, the arrangement is such that this gluon no longer fits to the cylinder surface, the number of colour loops gets *reduced* by one, and likewise the N_c order to $O(\alpha_s^3 N_c) = O(N_c^{-2})$ [Figs. 3(b,c)].

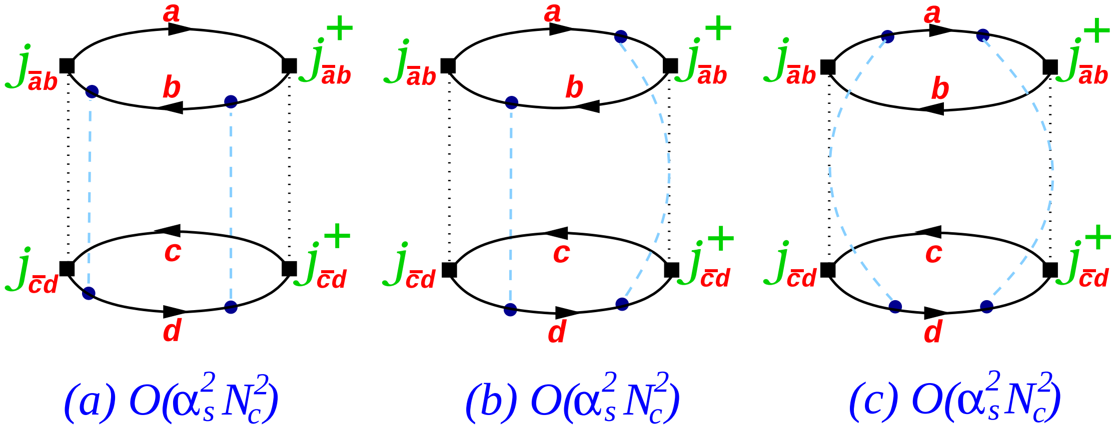

Figure 2: Cylinder interpretation of the flavour-retaining N_c-leading tetraquark-phile Feynman diagrams [3]. Dashed blue lines: planar gluons (of least tetraquark-phile number two). Dotted black lines: cylinder surface.

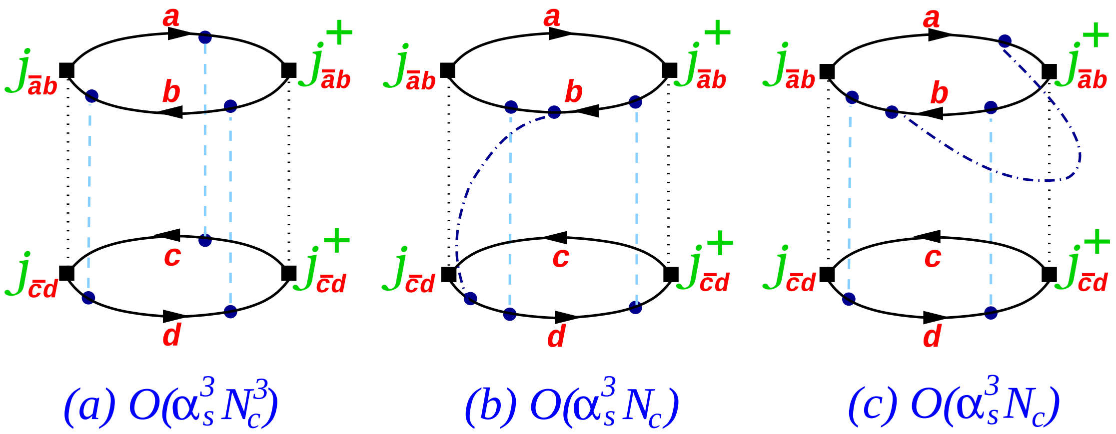

Figure 3: Amending Fig. 2(a) by one further (a) planar gluon (dashed) or (b,c) nonplanar gluon (dot-dashed).

Unfolding the imagined cylinder surfaces to planes (Fig. 4), by breaking up the two quark loops in the Feynman diagrams of Fig. 3, explains the distinction of the notions planar or nonplanar gluon.

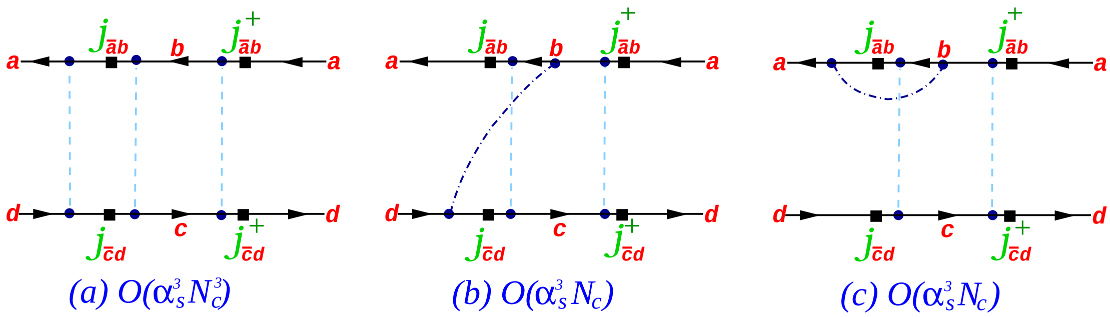

Figure 4: Planar interpretation of the Feynman diagrams of Fig. 3, demanding identification of points at $\pm\infty$.

Since we have to deal with the two classes of available scattering channels (2.1a) and (2.1b), we take the liberty of foreseeing notationally, already from the very beginning, the possible existence of two different tetraquarks T_A and T_B of the same exotic flavour content, with masses m_{T_A} and m_{T_B}, and respective large-N_c preferred coupling to one of the ordinary-meson pairs $M_{\bar{a}b}+M_{\bar{c}d}$ and $M_{\bar{a}d}+M_{\bar{c}b}$. For the N_c-*leading tetraquark-phile* contributions to flavour-preserving Green functions, in terms of generic decay constants f_M and tetraquark–two-ordinary-meson transition amplitudes A we then get

$$\langle T(j_{\bar{a}b}\,j_{\bar{c}d}\,j^\dagger_{\bar{a}b}\,j^\dagger_{\bar{c}d})\rangle_{\mathrm{T}} = f_M^4\left(\frac{|A(M_{\bar{a}b}\,M_{\bar{c}d}\leftrightarrow T_A)|^2}{p^2-m^2_{T_A}}+\frac{|A(M_{\bar{a}b}\,M_{\bar{c}d}\leftrightarrow T_B)|^2}{p^2-m^2_{T_B}}\right)+\cdots = O(N_c^0)\ , \quad (2.2)$$

$$\langle T(j_{\bar{a}d}\,j_{\bar{c}b}\,j^\dagger_{\bar{a}d}\,j^\dagger_{\bar{c}b})\rangle_{\mathrm{T}} = f_M^4\left(\frac{|A(M_{\bar{a}d}\,M_{\bar{c}b}\leftrightarrow T_A)|^2}{p^2-m^2_{T_A}}+\frac{|A(M_{\bar{a}d}\,M_{\bar{c}b}\leftrightarrow T_B)|^2}{p^2-m^2_{T_B}}\right)+\cdots = O(N_c^0)\ . \quad (2.3)$$

2.3 Flavour-Regrouping Scattering at Lowest Tetraquark-Phile Order in its $1/N_c$ Expansion

In contrast to the above flavour-retaining case, in the flavour-reshuffling channel the N_c-leading Feynman diagrams (Fig. 5) need one nonplanar gluon exchange. Tetraquark-phile planar options do not exist. The N_c-leading tetraquark-phile contributions to flavour-rearranging Green functions read

$$\langle T(j_{\bar{a}d}\,j_{\bar{c}b}\,j^\dagger_{\bar{a}b}\,j^\dagger_{\bar{c}d})\rangle_{\mathrm{T}} = f_M^4\left(\frac{A(M_{\bar{a}b}\,M_{\bar{c}d}\leftrightarrow T_A)\,A(T_A\leftrightarrow M_{\bar{a}d}\,M_{\bar{c}b})}{p^2-m^2_{T_A}}\right.$$
$$\left.+\frac{A(M_{\bar{a}b}\,M_{\bar{c}d}\leftrightarrow T_B)\,A(T_B\leftrightarrow M_{\bar{a}d}\,M_{\bar{c}b})}{p^2-m^2_{T_B}}\right)+\cdots = O(N_c^{-1})\ . \quad (2.4)$$

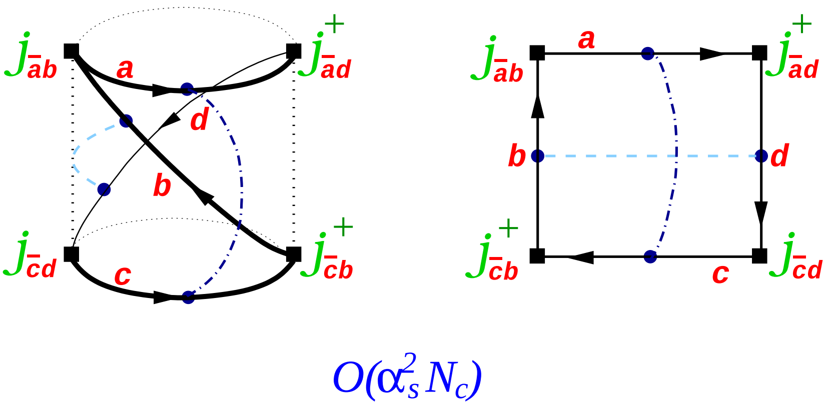

Figure 5: Cylindric (left) and unfolded (right) portrayal of a flavour-rearranging N_c-*leading tetraquark-phile* Feynman diagram [3], involving a planar and a nonplanar gluon distinguished by dashed vs. dot-dashed lines.

2.4 Constraining the Spectra of Tetraquarks Formed at N_c-Leading Tetraquark-Phile Order

Assuming the tetraquark masses $m_{T_{A,B}}$ not to grow with N_c without bound but to remain finite in the limit $N_c\to\infty$, let us now explore some implications for the resulting tetraquark spectra. Lacking convincing arguments why tetraquarks cannot emerge at the largest N_c order tolerated by the criteria of Sect. 1, we expect them to reveal their presence indeed at the leading tetraquark-phile order of N_c.

The most self-evident starting point in the attempt to satisfy the large-N_c behaviour derived, for flavour-exotic tetraquarks, in Sects. 2.2 and 2.3 is to assume the existence of only a single tetraquark state T. However, upon setting $T_A = T_B = T$, Eqs. (2.2), (2.3), and (2.4) collapse to the requirements

$$A(T \leftrightarrow M_{\bar{a}b} M_{\bar{c}d}) = O(N_c^{-1}) , \qquad A(T \leftrightarrow M_{\bar{a}d} M_{\bar{c}b}) = O(N_c^{-1}) ,$$

$$A(T \leftrightarrow M_{\bar{a}b} M_{\bar{c}d}) A(T \leftrightarrow M_{\bar{a}d} M_{\bar{c}b}) = O(N_c^{-3})$$

on the three involved transition amplitudes A: the prevailing mutual contradiction is pretty evident.[2]

In contrast, allowing for two tetraquarks $T_{A,B}$, of same flavour content but different couplings to ordinary mesons, enables us to offer a solution to the requests implied by Eqs. (2.2), (2.3), and (2.4):

$$\underbrace{A(T_A \leftrightarrow M_{\bar{a}b} M_{\bar{c}d}) = O(N_c^{-1})}_{\Longrightarrow \qquad \Gamma(T_A) = O(N_c^{-2})} \quad \stackrel{N_c}{>} \quad A(T_A \leftrightarrow M_{\bar{a}d} M_{\bar{c}b}) = O(N_c^{-2}) ,$$

$$A(T_B \leftrightarrow M_{\bar{a}b} M_{\bar{c}d}) = O(N_c^{-2}) \quad \stackrel{N_c}{<} \quad \underbrace{A(T_B \leftrightarrow M_{\bar{a}d} M_{\bar{c}b}) = O(N_c^{-1})}_{\Longrightarrow \qquad \Gamma(T_B) = O(N_c^{-2})} .$$

Accordingly, under the various assumptions mentioned above we are *unavoidably* led to consider as rather probable the existence of (at least) two — more precisely, pairwise existence of — differently decaying tetraquarks, T_A and T_B, of genuinely exotic quark-flavour composition. Their N_c-dominant decay channels control the identical large-N_c behaviour of their total decay widths $\Gamma(T_A)$ and $\Gamma(T_B)$,

$$\Gamma(T_A) = O(N_c^{-2}) , \qquad \Gamma(T_B) = O(N_c^{-2}) ,$$

and, consequently, the nature of these flavour-exotic tetraquark states: they are both narrow mesons.

3. *Compact* Flavour-Exotic Tetraquarks: Formation Mechanisms vs. Large-N_c QCD

With respect to the colour configuration inside the generic genuinely flavour-exotic tetraquarks $T = (\bar{q}_a\, q_b\, \bar{q}_c\, q_d)$, we see two possibilities for the two-step formation of any such colour-singlet state from two quarks and two antiquarks in the (N_c-dimensional) fundamental representation of SU(N_c): The formation of two colour-singlet *quark–antiquark* states, followed by the formation of a (loosely bound) molecular-type tetraquark, for which there are two options $(\bar{q}_a\, q_b)\,(\bar{q}_c\, q_d)$ and $(\bar{q}_a\, q_d)\,(\bar{q}_c\, q_b)$, or the formation of a diquark and an antidiquark, followed by the formation of a compact tetraquark, for which there is, however, only a single option in the genuinely flavour-exotic case, $(\bar{q}_a\, \bar{q}_c)\,(q_b\, q_d)$, as the latter mechanism necessitates the (anti-)diquarks to transform according to the antisymmetric $N_c\,(N_c - 1)/2$-dimensional representation of SU(N_c). We are hence confronted with two conflicting findings: On the one hand, the inspection of scattering amplitudes at N_c-leading order [1,2] suggests the *pairwise* existence of flavour-exotic tetraquarks, distinguishable by their preferred decay modes. On the other hand, among the described two-phase creation processes of colour-singlet bound states of two quarks and two antiquarks there is just *one* promising candidate for the formation of *compact* tetraquarks. A solution to this riddle may be the *nonexistence* of *compact flavour-exotic* tetraquarks.

[2] Trivially, the addition of a single quark loop to a gluon line modifies the N_c dependence of a given Feynman diagram by one unit. This observation may be abused in order to bring the large-N_c scaling of these two classes of tetraquark-phile contributions into agreement. However, such ad-hoc modification renders any conclusion drawn from inspection of some large-N_c behaviour meaningless. We consequently disregard such unmotivated way of adjusting one's desired N_c scaling.

Acknowledgement. D. M. is supported by the Austrian Science Fund (FWF), Project P29028-N27.

References

[1] W. Lucha, D. Melikhov, and H. Sazdjian, Phys. Rev. D **96** (2017) 014022, arXiv:1706.06003 [hep-ph].

[2] W. Lucha, D. Melikhov, and H. Sazdjian, Eur. Phys. J. C **77** (2017) 866, arXiv:1710.08316 [hep-ph].

[3] W. Lucha, D. Melikhov, and H. Sazdjian, Phys. Rev. D **98** (2018) 094011, arXiv:1810.09986 [hep-ph].

[4] G. 't Hooft, Nucl. Phys. B **72** (1974) 461.

[5] E. Witten, Nucl. Phys. B **160** (1979) 57.

[6] S. Weinberg, Phys. Rev. Lett. **110** (2013) 261601, arXiv:1303.0342 [hep-ph].

[7] M. Knecht and S. Peris, Phys. Rev. D **88** (2013) 036016, arXiv:1307.1273 [hep-ph].

[8] T. D. Cohen and R. F. Lebed, Phys. Rev. D **90** (2014) 016001, arXiv:1403.8090 [hep-ph].

[9] L. Maiani, A. D. Polosa, and V. Riquer, J. High Energy Phys. 06 (2016) 160, arXiv:1605.04839 [hep-ph].

[10] L. Maiani, A. D. Polosa, and V. Riquer, Phys. Rev. D **98** (2018) 054023, arXiv:1803.06883 [hep-ph].

[11] W. Lucha, D. Melikhov, and H. Sazdjian, PoS (EPS-HEP2017) 390, arXiv:1709.02132 [hep-ph].

[12] W. Lucha, D. Melikhov, and H. Sazdjian, PoS (Hadron 2017) 233, arXiv:1711.03925 [hep-ph].

[13] W. Lucha, D. Melikhov, and H. Sazdjian, EPJ Web Conf. **191** (2018) 04003, arXiv:1807.09630 [hep-ph].

[14] W. Lucha, D. Melikhov, and H. Sazdjian, EPJ Web Conf. **192** (2018) 00044, arXiv:1808.05519 [hep-ph].

[15] L. D. Landau, Nucl. Phys. **13** (1959) 181.