

Strong Isospin Breaking in the Decay Constants of Heavy–Light Mesons from Local-Duality QCD Sum Rules

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We explore, by a rather peculiar limit of QCD sum rules subject to a Borel transformation (realized by allowing the related Borel mass parameter to approach infinity), how the small difference of the masses of up and down quark translates into the leptonic decay constants of pseudoscalar or vector heavy–light mesons. For the charmed and bottom mesons, we find that the decay constants of their *lowest-lying* charged and neutral representatives should differ by an amount of the order of 1 MeV.

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1. Isospin Breaking in Weak Decay Constants from Isospin-Violating Quark Masses

The comparatively small but definitely nonzero difference of the masses of up and down quarks $(m_d - m_u)(2 \text{ GeV}) \approx 2.5 \text{ MeV}$ [1] causes a mismatch of the leptonic decay constants of heavy–light mesons. We discuss this phenomenon, for the case of the D , D^* , B , and B^* mesons, by considering a target-oriented limiting case [2–5] of the standard formulation of the QCD sum-rule framework [6].

2. QCD Sum-Rule Approach to Mesons, Borel Transformation, Local-Duality Limit

To begin with, let us rather cursorily recall the QCD sum-rule description [6] of hadron systems formed by the strong interactions. Consider a heavy–light meson bound state $(\bar{q}Q)$, in the following generically called H_q , of a heavy quark $Q = c, b$ of mass m_Q , and a light quark $q = u, d, s$ of mass m_q . For any such meson, the basic characteristics presently in the focus of our interest are its mass, M_{H_q} , and its leptonic decay constant, f_{H_q} . Meson properties of this kind are related, by QCD sum rules, to the fundamental parameters of the underlying quantum field theory, QCD, *viz.*, strong fine-structure coupling α_s , quark masses, and vacuum condensates $\langle \bar{q}q \rangle, \dots$, reflecting the nonperturbative side of the coin. Application of a Borel transformation from momentum to a new variable dubbed the Borel parameter, τ , casts the QCD sum rules in the focus of our efforts into a particularly convenient form:

$$f_{H_q}^2 \left(M_{H_q}^2 \right)^N \exp(-M_{H_q}^2 \tau) = \int_{(m_Q+m_q)^2}^{s_{\text{eff}}^{(N)}(\tau, m_Q, m_q, \alpha_s)} ds \exp(-s \tau) s^N \rho(s, m_Q, m_q, \alpha_s | m_{\text{sea}}) + \Pi_{\text{power}}^{(N)}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \dots). \quad (2.1)$$

The right-hand side of this relation involves a variety of ingredients governed, in principle, by QCD:

- The spectral density $\rho(s, m_Q, m_q, \alpha_s | m_{\text{sea}})$ encompasses all purely perturbative contributions, derivable by series expansion in powers of $\alpha_s(\mu)$ (depending on the renormalization scale μ),

$$\rho(s, m_Q, m_q, \alpha_s | m_{\text{sea}}) = \rho_0(s, m_Q, m_q) + \frac{\alpha_s(\mu)}{\pi} \rho_1(s, m_Q, m_q, \mu) + \frac{\alpha_s^2(\mu)}{\pi^2} \rho_2(s, m_Q, m_q, \mu | m_{\text{sea}}) + O(\alpha_s^3). \quad (2.2)$$

Starting at order α_s^2 , it depends also on the masses of all sea quarks, collectively labelled m_{sea} .

- The power corrections $\Pi_{\text{power}}^{(N)}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \dots)$ describe nonperturbative contributions by means of vacuum condensates of products of quark and gluon fields, brought into the game by Wilson’s operator product expansion [7], accompanied by powers of τ , whence that name.
- The effective threshold $s_{\text{eff}}^{(N)}(\tau, m_Q, m_q, \alpha_s)$ delimits, as its lower boundary, the range of values of our integration variable s where, according to the postulate of global quark–hadron duality, mutual cancellation of perturbative-QCD and hadron contributions takes place. This quantity definitely depends, in general, also on the Borel parameter τ [8–12]. Taking into account its τ dependence increases considerably the accuracy of one’s QCD sum-rule predictions [13–16].
- The non-negative integer $N = 0, 1, 2, \dots$ can be used to optimize the study: it decides upon the share of power corrections and effective threshold in the entire nonperturbative contributions.

The Borel parameter τ is a free variable yet to be determined or chosen. Within the formulation of conventional Borelized QCD sum rules, the meaningful region of τ gets constrained, from below, by insisting on a utilizably large ground-state contribution and, from above, by requiring reasonably small power-correction contributions. In contrast, the local-duality limit is defined by letting $\tau \rightarrow 0$.

If, for a convenient choice of N , all power corrections vanish in the local-duality limit, that is, if

$$\lim_{\tau \rightarrow 0} \Pi_{\text{power}}^{(\widehat{N})}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \dots) = 0 \quad \text{for some } \widehat{N},$$

the generic Borelized QCD sum rule (2.1) reduces to a spectral integral over its perturbative density:

$$f_{H_q}^2 \left(M_{H_q}^2 \right)^{\widehat{N}} = \int_{(m_Q+m_q)^2}^{s_{\text{eff}}^{(\widehat{N})}(m_Q, m_q, \alpha_s)} ds s^{\widehat{N}} \rho(s, m_Q, m_q, \alpha_s | m_{\text{sea}}), \quad (2.3)$$

$$s_{\text{eff}}^{(\widehat{N})}(m_Q, m_q, \alpha_s) \equiv s_{\text{eff}}^{(\widehat{N})}(0, m_Q, m_q, \alpha_s).$$

In this case, the effective threshold $s_{\text{eff}}^{(\widehat{N})}(m_Q, m_q, \alpha_s)$ must take care of all the nonperturbative effects.

3. Illustrative Special Case: Pseudoscalar and Vector-Meson Weak Decay Constants

Let us now boil down the general QCD sum-rule approach to mesons briefly sketched in Sect. 2 to our actual targets, heavy-light pseudoscalar and vector mesons, *i.e.*, $H_q = P_q, V_q$, of momentum p . Their weak decay constants, f_{P_q, V_q} , result from matrix elements of the relevant axial-vector or vector heavy–light quark current: for a pseudoscalar meson P_q , its leptonic decay constant f_{P_q} is defined by

$$\langle 0 | \bar{q}(0) \gamma_\mu \gamma_5 Q(0) | P_q(p) \rangle = i f_{P_q} p_\mu;$$

for a vector meson V_q , with polarization vector $\varepsilon_\mu(p)$, its weak decay constant f_{V_q} can be found from

$$\langle 0 | \bar{q}(0) \gamma_\mu Q(0) | V_q(p) \rangle = f_{V_q} M_{V_q} \varepsilon_\mu(p).$$

For all such mesons, the spectral densities are available up to order $O(\alpha_s m_q)$ and $O(\alpha_s^2 m_q^0)$ [17–20]. The applicability of the local-duality limit is endangered by the potential existence of terms of order

$$\tau^{N-2} \log(\tau) \quad (3.1)$$

in the power corrections. Accordingly, let us inspect, in Figs. 1 and 2, for the cases $N = 0, 1, 2, 3$, the τ dependence of the power corrections and spectral densities, the former being suitably redefined by

$$\overline{\Pi}_{\text{power}}^{(N)}(\tau) \equiv \Pi_{\text{power}}^{(N)}(\tau, \dots) \frac{\exp(M_{H_q}^2 \tau)}{\left(M_{H_q}^2 \right)^N}. \quad (3.2)$$

In accordance with the behaviour expectable from expression (3.1), all the power corrections vanish for $N = 0$, approach a finite value for $N = 1$, and develop a singularity for $N = 2, 3$ in the limit $\tau \rightarrow 0$:

$$\lim_{\tau \rightarrow 0} \Pi_{\text{power}}^{(0)}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \dots) = 0,$$

$$\lim_{\tau \rightarrow 0} |\Pi_{\text{power}}^{(1)}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \dots)| < \infty,$$

$$\lim_{\tau \rightarrow 0} |\Pi_{\text{power}}^{(2,3)}(\tau, m_Q, m_q, \alpha_s, \langle \bar{q}q \rangle, \dots)| = \infty.$$

The local-duality limit of $\widehat{N} = 0$ QCD sum rules for pseudoscalar and vector mesons is well-defined.

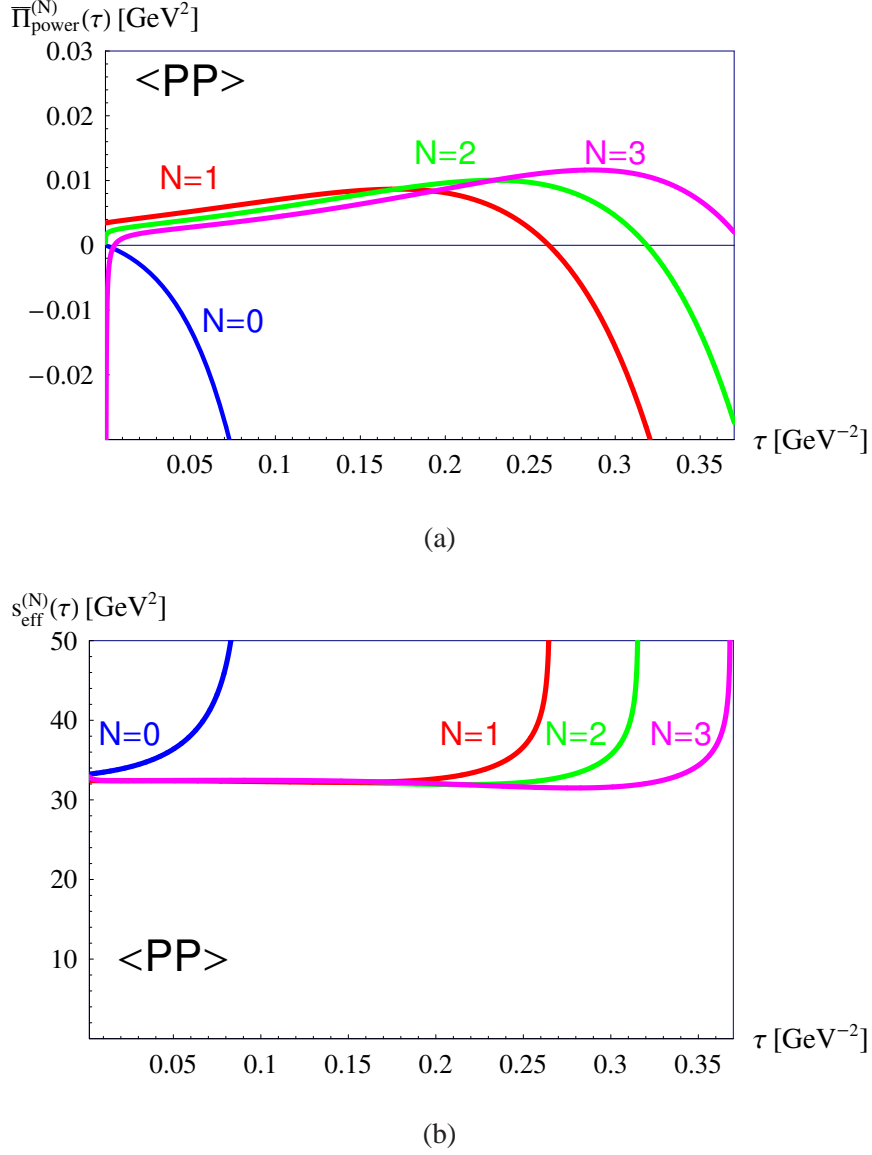


Figure 1: Pseudoscalar mesons: τ dependence of (a) power corrections (3.2) and (b) effective threshold $s_{\text{eff}}^{(N)}$.

Choosing $\hat{N} = 0$ simplifies the QCD sum rule (2.3) to a relation for only the decay constant f_{H_q} :

$$f_{H_q}^2 = \int_{(m_Q+m_q)^2}^{s_{\text{eff}}(m_Q, m_q, \alpha_s)} ds \rho_{H_q}(s, m_Q, m_q, \alpha_s | m_{\text{sea}}) \equiv F_{H_q}(s_{\text{eff}}(m_q), m_q | m_{\text{sea}}), \quad H_q = P_q, V_q. \quad (3.3)$$

We denote the function encoding all light-quark-mass dependence by the Greek letter F (digamma).

4. Strong Isospin Breaking in Charmed- and Bottom-Meson Weak Decay Constants

Heavy–light-meson decay-constant studies [21–25] relying on the conventional QCD sum-rule framework with Borel-parameter-upgraded effective threshold draw the behaviour of the latter from knowledge about meson *masses* in our focus. In the local-duality approach, upon noting that [2,4,5]

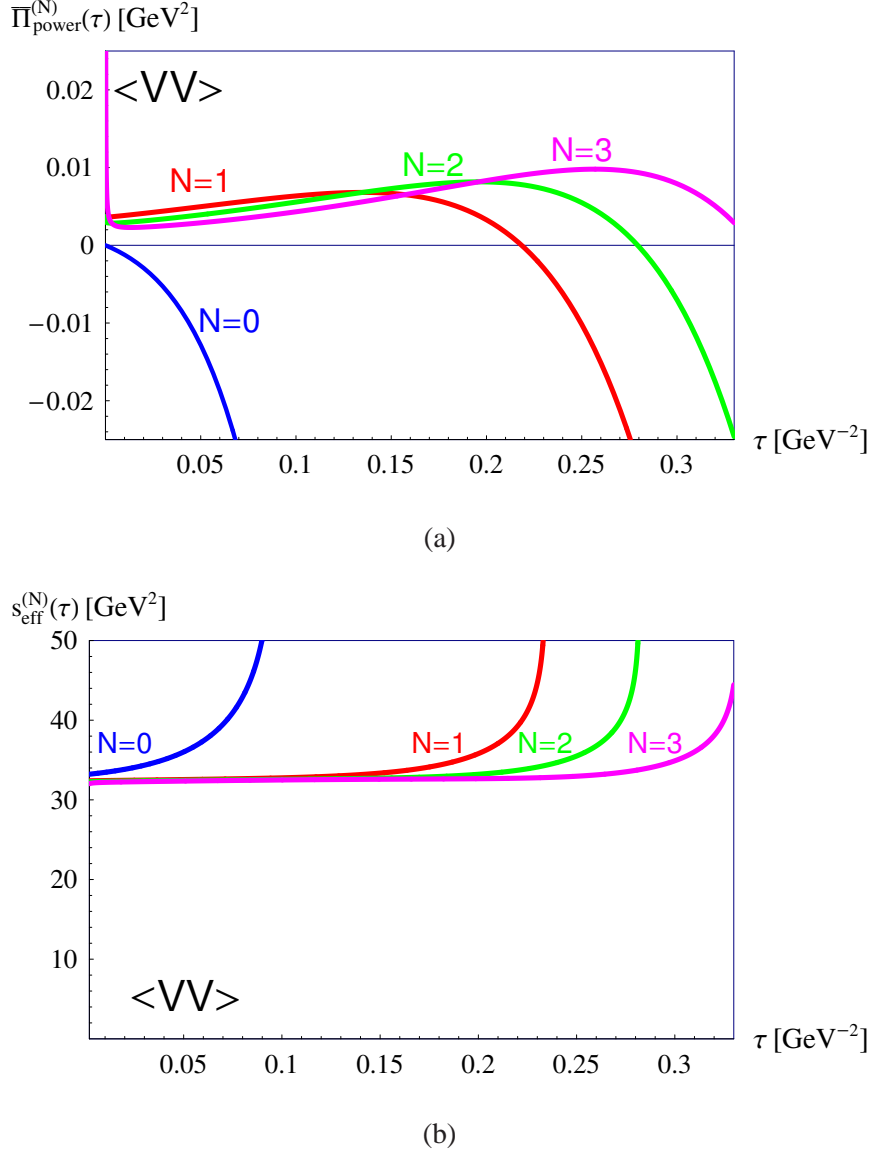


Figure 2: Vector mesons: τ dependence of (a) the power corrections (3.2) and (b) the effective threshold $s_{\text{eff}}^{(N)}$.

in the difference of heavy–light decay constants the dependence on the strange sea quarks drops out,

$$\begin{aligned}
 & F_{H_d}(s_{\text{eff}}(m_d), m_d | m_{\text{sea}}) - F_{H_u}(s_{\text{eff}}(m_u), m_u | m_{\text{sea}}) \\
 &= F_{H_d}(s_{\text{eff}}(m_d), m_d | m_{\text{sea}} = 0) - F_{H_u}(s_{\text{eff}}(m_u), m_u | m_{\text{sea}} = 0) + \mathcal{O}\left(\frac{\alpha_s^2}{\pi^2}(m_d - m_u)\right), \quad (4.1)
 \end{aligned}$$

we adjust the function $f_H(m_q)$ of a continuously varying m_q , normalized to its average $f_H(m_{ud})$ with $m_{ud} \equiv \frac{1}{2}(m_u + m_d)$, for several ansatzes of increasing degree of sophistication for the problem’s core

$$z_{\text{eff}} \equiv \sqrt{s_{\text{eff}}} - m_Q - m_q, \quad (4.2)$$

to related lattice-QCD results $f_H(m_{ud})$ and $f_H(m_s)$ at a physical quark mass [26,27]. Figures 3 and 4 then yield, in nearly perfect agreement with the conventional findings [28], the f_{H_q} differences [2–5]

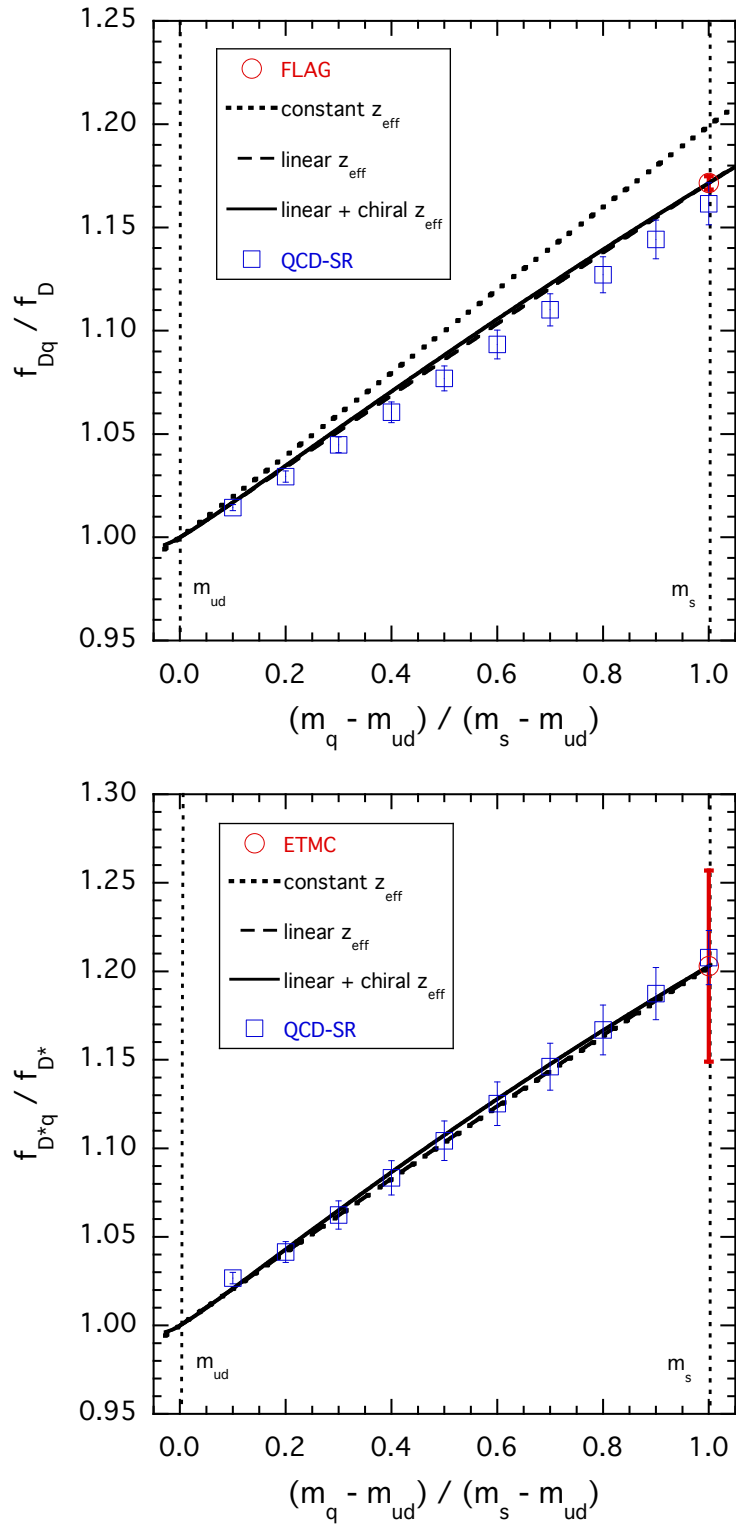


Figure 3: Behaviour of the decay constants $f_{D_q^{(*)}} \equiv f_{D_q^{(*)}}(m_q)$ of charmed pseudoscalar and vector mesons, in units of $f_{D^{(*)}} \equiv f_{D^{(*)}}(m_{ud})$, for the light-quark mass m_q varying in the interval $(0, m_s)$, resulting from the three ansatzes considered by Ref. [2] (dubbed “constant”, “linear” and “linear + chiral”) for our effective-threshold function z_{eff} defined in Eq. (4.2) [2–5], compared with the results of a conventional QCD sum-rule study [28].

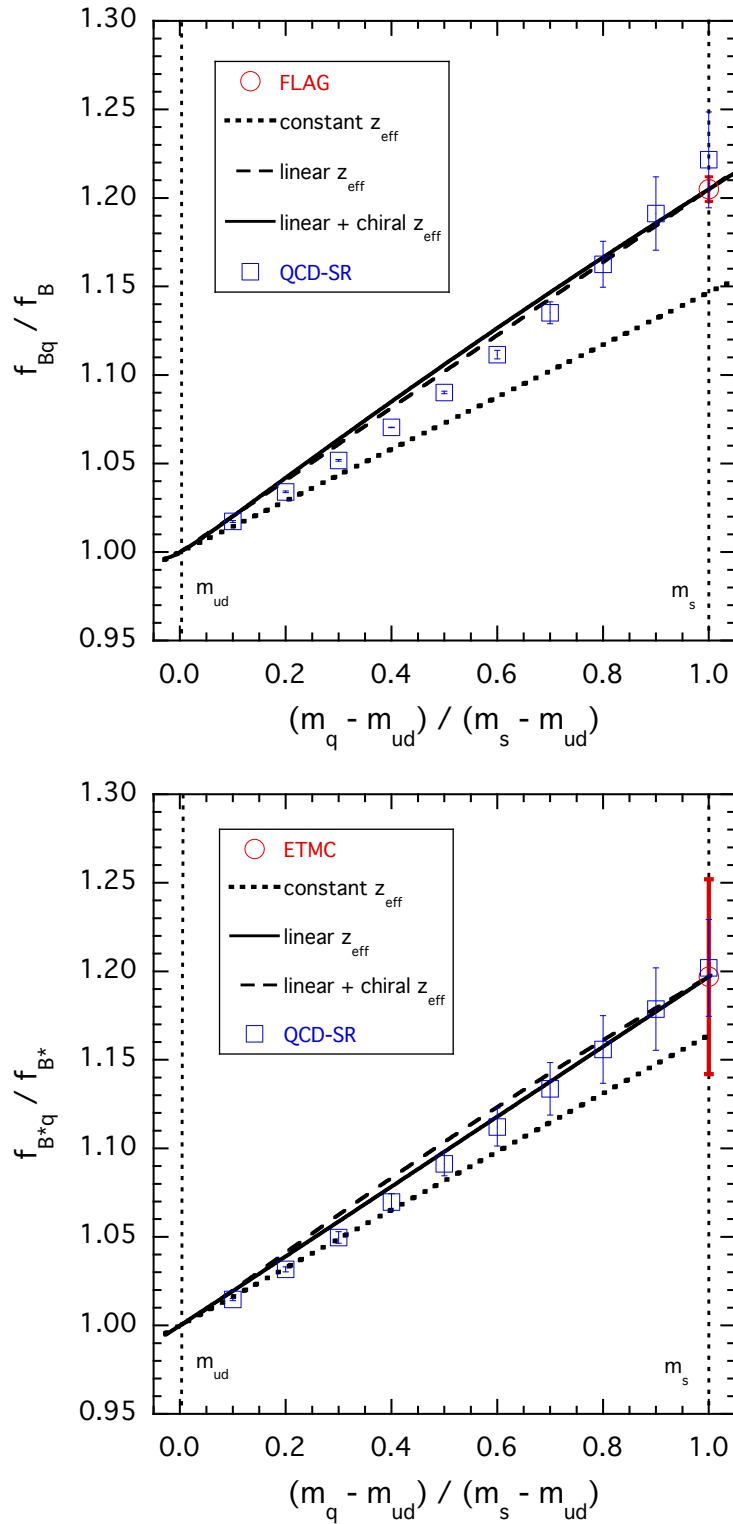


Figure 4: Behaviour of the decay constants $f_{B_q^{(*)}} \equiv f_{B_q^{(*)}}(m_q)$ of both pseudoscalar and vector bottom mesons, in units of $f_{B^{(*)}} \equiv f_{B_q^{(*)}}(m_{ud})$, for the light-quark mass m_q varying in the interval $(0, m_s)$, arising from the three ansatzes considered by Ref. [2] (dubbed “constant”, “linear” and “linear + chiral”) for our effective-threshold function z_{eff} defined in Eq. (4.2) [2–5], compared with the results of a conventional QCD sum-rule study [28].

$$\begin{aligned}
f_{D^\pm} - f_{D^0} &= (0.96 \pm 0.09) \text{ MeV} , & f_{D^{*\pm}} - f_{D^{*0}} &= (1.18 \pm 0.35) \text{ MeV} , \\
f_{B^0} - f_{B^\pm} &= (1.01 \pm 0.10) \text{ MeV} , & f_{B^{*0}} - f_{B^{*\pm}} &= (0.89 \pm 0.30) \text{ MeV} .
\end{aligned}$$

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References

- [1] Particle Data Group (M. Tanabashi *et al.*), Phys. Rev. D **98** (2018) 030001.
- [2] W. Lucha, D. Melikhov, and S. Simula, Eur. Phys. J. C **78** (2018) 168, arXiv:1702.07537 [hep-ph].
- [3] W. Lucha, D. Melikhov, and S. Simula, PoS (EPS-HEP 2017) 669, arXiv:1709.02131 [hep-ph].
- [4] W. Lucha, D. Melikhov, and S. Simula, PoS (Hadron2017) 175, arXiv:1711.07899 [hep-ph].
- [5] W. Lucha, D. Melikhov, and S. Simula, preprint HEPHY-PUB 1006/18 (2018), arXiv:1808.07684 [hep-ph].
- [6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147** (1979) 385.
- [7] K. G. Wilson, Phys. Rev. **179** (1969) 1499.
- [8] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **79** (2009) 096011, arXiv:0902.4202 [hep-ph].
- [9] W. Lucha, D. Melikhov, and S. Simula, J. Phys. G **37** (2010) 035003, arXiv:0905.0963 [hep-ph].
- [10] W. Lucha, D. Melikhov, H. Sazdjian, and S. Simula, Phys. Rev. D **80** (2009) 114028, arXiv:0910.3164 [hep-ph].
- [11] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **687** (2010) 48, arXiv:0912.5017 [hep-ph].
- [12] W. Lucha, D. Melikhov, and S. Simula, Phys. Atom. Nucl. **73** (2010) 1770, arXiv:1003.1463 [hep-ph].
- [13] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **76** (2007) 036002, arXiv:0705.0470 [hep-ph].
- [14] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **657** (2007) 148, arXiv:0709.1584 [hep-ph].
- [15] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **671** (2009) 445, arXiv:0810.1920 [hep-ph].
- [16] D. Melikhov, Phys. Lett. B **671** (2009) 450, arXiv:0810.4497 [hep-ph].
- [17] K. G. Chetyrkin and M. Steinhauser, Phys. Lett. B **502** (2001) 104, arXiv:hep-ph/0012002.
- [18] K. G. Chetyrkin and M. Steinhauser, Eur. Phys. J. C **21** (2001) 319, arXiv:hep-ph/0108017.
- [19] M. Jamin and B. O. Lange, Phys. Rev. D **65** (2002) 056005, arXiv:hep-ph/0108135.
- [20] P. Gelhausen, A. Khodjamirian, A. A. Pivovarov, and D. Rosenthal, Phys. Rev. D **88** (2013) 014015, arXiv:1305.5432 [hep-ph]; Phys. Rev. D **89** (2014) 099901(E); **91** (2015) 099901(E).
- [21] W. Lucha, D. Melikhov, and S. Simula, J. Phys. G **38** (2011) 105002, arXiv:1008.2698 [hep-ph].
- [22] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **701** (2011) 82, arXiv:1101.5986 [hep-ph].
- [23] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **88** (2013) 056011, arXiv:1305.7099 [hep-ph].
- [24] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **735** (2014) 12, arXiv:1404.0293 [hep-ph].
- [25] W. Lucha, D. Melikhov, and S. Simula, Phys. Rev. D **91** (2015) 116009, arXiv:1504.03017 [hep-ph].
- [26] FLAG Working Group (S. Aoki *et al.*), Eur. Phys. J. C **77** (2017) 112, arXiv:1607.00299 [hep-lat].
- [27] ETM Collaboration (V. Lubicz *et al.*), Phys. Rev. D **96** (2017) 034524, arXiv:1707.04529 [hep-lat].
- [28] W. Lucha, D. Melikhov, and S. Simula, Phys. Lett. B **765** (2017) 365, arXiv:1609.05050 [hep-ph].